Exercise session 4

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1 – Employment rent

Workers supply extra effort to make sure that they can keep a job that generates employment rent.
How do the following factors affect employment rent?

Employment rent = opportunity cost = value of the next best option (other employment or unemployment benefits)
Employment rent = wage – reservation wage – disutility of effort

Unemployment benefits paid for a shorter time	Employment rent ↑ (reservation wage ↓)
Announcement that the plan where you work currently will close after six months	Employment rent ↓ (I expect to be unemployed in six months) (If alternative employment is scarce, reservation wage ↓ as I will be unemployed together will all my coworkers, therefore I know what there will be a lot of competition for jobs)
A decline in the demand for the product that your company sells	No particular effect (or employment rent ↓ if firm lays off workers due to decline in demand)
The departure of your boss	Employment rent ↓ (because firm-specific assets ↓)

2 –Fixed and variable costs

R&D investments to come up with a new pharmaceutical.	FC
Advertising campaign on conventional or social media.	FC
Renting the facility for a summer cafe in a park.	FC
Employing workers to the cafe on a fixed wage contract for the whole summer.	FC
Hiring temporary help for the cafe on sunny days.	VC

3 – Price elasticity of demand…

	Elastic / Inelastic
…for oil in any given month.	Inelastic I have to continue driving no matter what
…for oil over a longer horizon.	Elastic New technologies can be developed, e.g. more efficient engines / that use different types of fuel
…for pulled corn (nyhtökaura) as more meat-like vegetarian products enter the market.	(becomes more) Elastic If I have available substitutes, I can purchase those when price of nyhtökaura increases
…for fair trade coffee versus the mainstream coffee brands. Compare the price differences at your local supermarket to the premium that fair trade producers get for their beans. What do you conclude abot the demand elasticities?	Inelastic “fair trade” is a key feature for those that purchase it for ethical reasons – substitution is only for other fair trade coffees

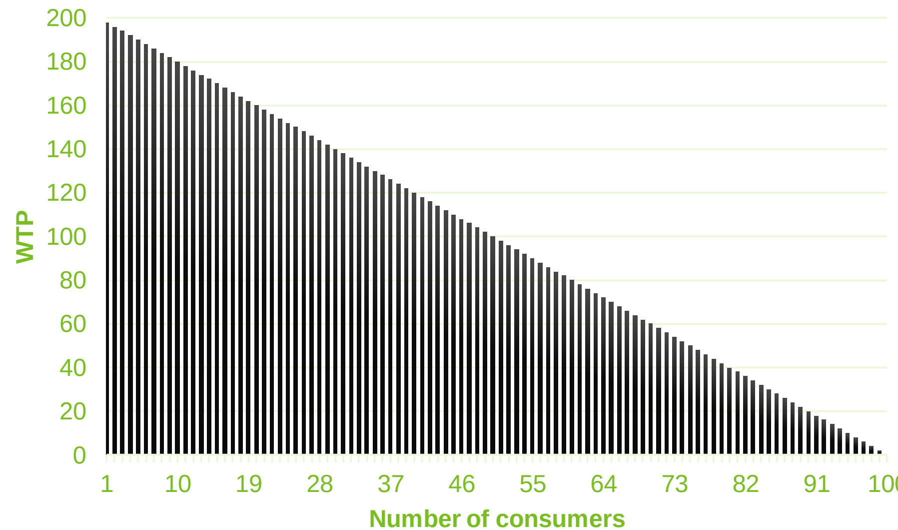
- Price elasticity of Snacks and candy is 0.295. At profit maximizing prices, the markup (a number between 0 and 1) should be equal to the inverse of the elasticity. Stores can set their prices. Should we conclude that they are not profit-maximizing or can you give other explanations?
- If the inverse of the elasticity is not within the interval 0-1, we might conclude that stores are not profit maximisers. However, the elasticity reported in the study is valid for demand “across all markets” (and reflects for example substitutability between candies and other products, which is low). However, the demand elasticity for these products in each store is much higher, as consumers may switch to a different store looking for lower prices.

4a – Demand curves

There are 100 consumers.

$$v_1 = 198; v_2 = 196; \dots; v_i = 200 - 2i; \dots; v_{100} = 0$$

- Construct the demand curve, i.e. for each Q, find price (or prices) P such that the number of consumers with wtp at least P is Q.



4b – Demand curves

Compare the demand curve you obtained in part a to the line $P(Q) = 200 - 2Q$ in the (Q, P) -coordinate system. [they are essentially the same curves, but this is continuous]

- Compute the marginal revenue curve for this continuous demand curve.

First, we compute the revenue curve:

$$R(Q) = Q \cdot P(Q) = Q \cdot (200 - 2Q) = 200Q - 2Q^2$$

Then, the marginal revenue:

$$\mathrm{MR(Q)} = \frac{dR(Q)}{dQ} = 200 - 4Q$$

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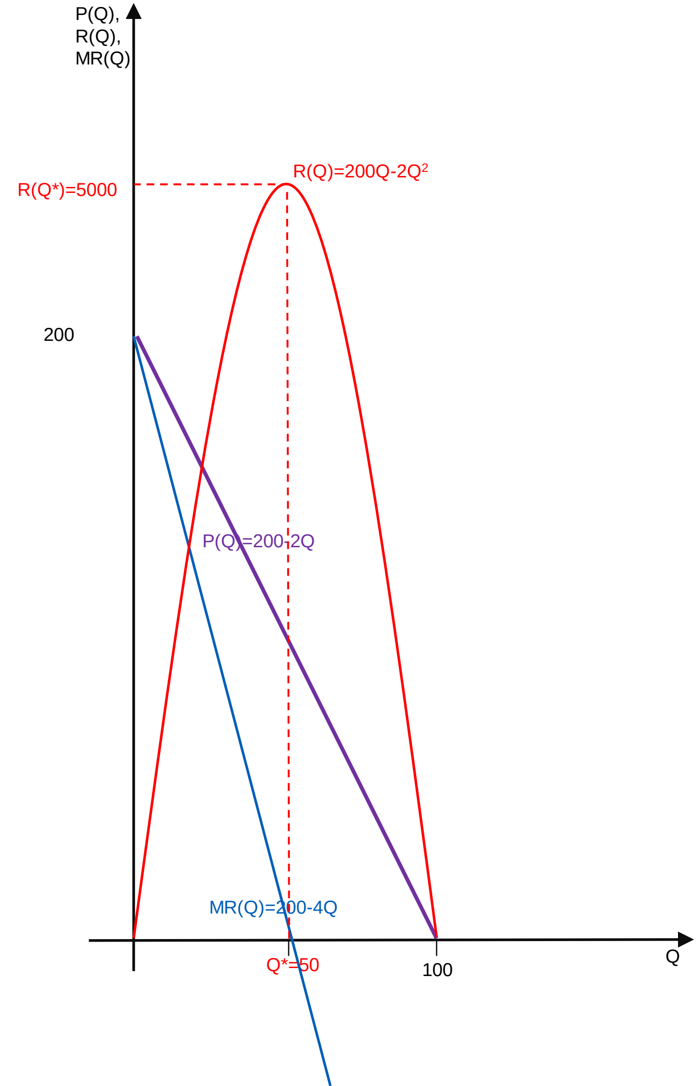
<u>EXTRA:</u>

To find quantity at which revenue is maximized, we equate the marginal revenue to 0:

$$Q^*: \mathrm{MR}(Q^*) = 200 - 4Q^* = 0 \rightarrow Q^* = 50$$

Now, we can calculate the maximum revenue:

$$\mathrm{R}(Q^*) = 200 \cdot 50 - 2 \cdot 50^2 = 5000$$



4c – Demand curves

Demand curve:

$$P(Q) = 150 - Q \; if \; Q \leq 100;$$
$$P(Q) = 0 \; if \; Q > 100$$

- Compute the marginal revenue curve for this continuous demand curve.

First, we compute the revenue curve:

$$R(Q) = Q \cdot P(Q) = Q \cdot (150 - Q) = 150Q - Q^2$$

Then, the marginal revenue:

$$\mathrm{MR(Q)} = \frac{dR(Q)}{dQ} = 150 - 2Q$$

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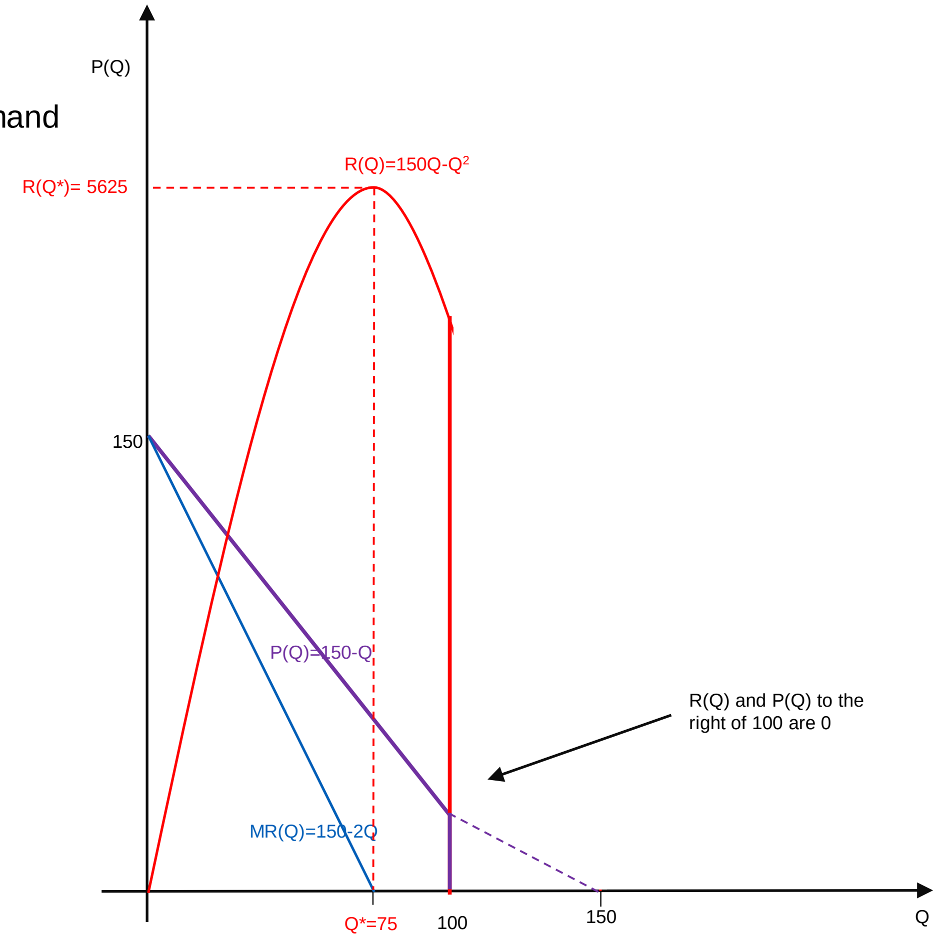
<u>EXTRA:</u>

To find quantity at which revenue is maximized, we equate the marginal revenue to 0:

$$Q^*: \mathrm{MR}(Q^*) = 150 - 2Q^* = 0 \rightarrow Q^* = 75$$

Now, we can calculate the maximum revenue:

$$\mathrm{R}(Q^*) = 150 \cdot 75 - 75^2 = 5625$$



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4d – Demand curves

$For\ any\ 100 < A < 200$: consider the demand curve

$P^A(Q) = A - \frac{A-100}{100} 2Q\ if\ Q \leq 100; P^A(Q) = 0\ if\ Q > 100.$

- Compute the marginal revenue curve for this demand.

First, we compute the revenue curve:

$$R(Q) = Q \cdot P^A(Q) = Q \cdot \left(A - \frac{A-100}{100} 2Q\right) = QA - \frac{A-100}{50} Q^2$$

Then, the marginal revenue:

$$\mathrm{MR(Q)} = \frac{dR(Q)}{dQ} = A - 2\frac{A-100}{50} Q = A - \frac{A-100}{25} Q$$

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<u>EXTRA:</u>

To find quantity at which revenue is maximized, we equate the marginal revenue to 0:

$$Q^*: \mathrm{MR}(Q^*) = A - \frac{A-100}{25} Q^* = 0 \rightarrow Q^* = \frac{25A}{A-100}$$

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THE ECONOMIC MEANING WHEN A=100?

$$P^A(Q) = 100 - \frac{100-100}{100} 2Q = 100$$

When A=100, demand is infinitely elastic at price 100 up to Q=100.

5a – Profits

- Compute the profit maximizing Q for the demand curves given in parts b. and c. when F = 800 and c = 5.

$$C(Q) = F + c \cdot Q = 800 + 5Q$$

Profits are revenue minus costs (we computed revenues earlier):

$$\Pi_B(Q_B) = R_B(Q_B) - C(Q_B) = 200Q_B - 2{Q_B}^2 - (800 + 5Q_B) = 195Q_B - 2{Q_B}^2 - 800$$
$$\Pi_C(Q_C) = R_C(Q_C) - C(Q_C) = 150Q_C - {Q_C}^2 - (800 + 5Q_C) = 145Q_C - {Q_C}^2 - 800$$

We now derive marginal profits:

$$M\Pi_B(Q_B) = \frac{d\Pi_B(Q_B)}{dQ_B} = 195 - 4Q_B$$
$$M\Pi_C(Q_C) = \frac{d\Pi_C(Q_C)}{dQ_C} = 145 - 2Q_C$$

We set marginal profits to 0 to find the Q at which profits are maximized:

$$M\Pi_B(Q_B^*) = 195 - 4Q_B^* = 0 \rightarrow Q_B^* = 48.75$$
$$M\Pi_C(Q_C^*) = 145 - 2Q_C^* = 0 \rightarrow Q_C^* = 72.5$$

We obtain the same result if we equate marginal cost with marginal revenue:

$$MC_B(Q_B^*) = MR_B(Q_B^*): 5 = 200 - 4Q_B^* \rightarrow Q_B^* = 48.75$$
$$MC_C(Q_C^*) = MR_C(Q_C^*): 5 = 150 - 2Q_C^* \rightarrow Q_C^* = 72.5$$

5a – Profits

- Compute also the associated prices and profits.
- For which population is the profit larger? How does your answer change if F = 1200?

Prices at such quantities are given by the demand curve.

$$P_B(Q_B^*) = 200 - 2Q_B^* = 200 - 2 \cdot 48.75 = 102.5$$
$$P_C(Q_C^*) = 150 - Q_C^* = 150 - 72.5 = 77.5$$

Profits at such quantities are:

$$\Pi_B(Q_B^*) = 195 \cdot Q_B^* - 2{Q_B^*}^2 - 800 = 195 \cdot 48.75 - 2 \cdot 48.75^2 - 800 = 3953.125$$
$$\boldsymbol{\Pi_C(Q_C^*) = 145 \cdot Q_C^* - {Q_C^*}^2 - 800 = 145 \cdot 72.5 - 72.5^2 - 800 = 4456.25}$$

Which we can also calculate from:

$$\Pi_B(Q_B^*) = R_B(Q_B^*) - C(Q_B^*) = Q_B^* \cdot P(Q_B^*) - C(Q_B^*) = 48.75 \cdot 102.5 - (800 + 5 \cdot 48.75) = 3953.125$$
$$\Pi_C(Q_C^*) = R_C(Q_C^*) - C(Q_C^*) = Q^* \cdot P(Q_C^*) - C(Q_C^*) = 72.5 \cdot 77.5 - (800 + 5 \cdot 72.5) = 4456.25$$

If F=1200, marginal costs or optimum quantities do not change. Only, total profits are 400 lower at the maximum.

$$\Pi_B(Q_B^*) = 3553.125$$
$$\Pi_C(Q_C^*) = 4056.25$$

5b – Profits

- Now c=40 and F=100. Which market has a higher profit at this marginal cost?

$$C(Q) = F + c \cdot Q = 100 + 40Q$$

Profits are now:

$$\Pi_B(Q_B) = R_B(Q_B) - C(Q_B) = 200Q_B - 2{Q_B}^2 - (100 + 40Q_B) = 160Q_B - 2{Q_B}^2 - 100$$
$$\Pi_C(Q_C) = R_C(Q_C) - C(Q_C) = 150Q_C - {Q_C}^2 - (100 + 40Q_B) = 110Q_C - {Q_C}^2 - 100$$

Marginal profits:

$$M\Pi_B(Q_B) = \frac{d\Pi_B(Q_B)}{dQ_B} = 160 - 4Q_B$$
$$M\Pi_C(Q_C) = \frac{d\Pi_C(Q_C)}{dQ_C} = 110 - 2Q_C$$

We set marginal profits to 0 to find the Q at which profits are maximized:

$$M\Pi_B(Q_B^*) = 160 - 4Q_B^* = 0 \rightarrow Q_B^* = 40$$
$$M\Pi_C(Q_C^*) = 110 - 2Q_C^* = 0 \rightarrow Q_C^* = 55$$

Profits at such quantities are:

$$\boldsymbol{\Pi_B(Q_B^*) = 160 \cdot 40 - 2 \cdot 40^2 - 100 = 3100}$$
$$\Pi_C(Q_C^*) = 110 \cdot 55 - 55^2 - 100 = 2925$$

5c – Profits

- Find the values of c for which selling to the population in part b. is more profitable than selling for the population of part c. in the previous exercise, if there are no fixed costs, that is if $C(Q) = c \cdot Q$

Write the profit function in the two markets and derive Q_B^* and Q_C^*:

$$\Pi_B(Q_B) = 200Q_B - 2{Q_B}^2 - cQ_B = (200 - c)Q_B - 2{Q_B}^2$$
$$\Pi_C(Q_C) = 150Q_C - {Q_C}^2 - cQ_c = (150 - c)Q_C - {Q_C}^2$$
$$M\Pi_B(Q_B) = 200 - c - 4Q_B$$
$$M\Pi_C(Q_C) = 150 - c - 2Q_C$$
$$\Pi_B(Q_B^*) = 0 \Leftrightarrow Q_B^* = 50 - \frac{c}{4}$$
$$\Pi_C(Q_C^*) = 0 \Leftrightarrow Q_C^* = 75 - \frac{c}{2}$$

Substituting Q_B^* and Q_C^* into the profit respective functions for the two markets, we have:

$$\Pi_B(Q_B^*) = (200 - c)\left(50 - \frac{c}{4}\right) - 2\left(50 - \frac{c}{4}\right)^2 = 10000 - 100c + \frac{c^2}{4} - 2\left(2500 - 25c + \frac{c^2}{16}\right) = 5000 - 50c + \frac{c^2}{8}$$

$$\Pi_C(Q_C^*) = (150 - c)\left(75 - \frac{c}{2}\right) - \left(75 - \frac{c}{2}\right)^2 = 11250 - 150c + \frac{c^2}{2} - \left(5625 - 75c + \frac{c^2}{4}\right) = 5625 - 75c + \frac{c^2}{4}$$

Imposing $\Pi_B(Q_B^*) > \Pi_C(Q_C^*)$:

$$5000 - 50c + \frac{c^2}{8} > 5625 - 75c + \frac{c^2}{4} \Leftrightarrow 5000 - 5625 - 50c + 75c + \frac{c^2}{8} - \frac{c^2}{4} > 0 \Leftrightarrow$$
$$-625 + 25c - \frac{c^2}{8} > 0$$
$$100 - 50\sqrt{2} < c < 100 + 50\sqrt{2}$$
$$29.29 < c < 170.71$$

5d – Optimal sales

- Cost function: $C(Q) = cQ$
- Demand curve: $P^A(Q) = A - \frac{A-100}{100} 2Q$
- Compute the optimal sales quantity.

I derived MR in part 4d: $\text{MR}(\text{Q}) = \frac{dR(Q)}{dQ} = A - 2\frac{A-100}{50}Q = A - \frac{A-100}{25}Q$

From the cost function, we derive $MC(Q) = \frac{dC(Q)}{dQ} = c$

Optimal sales, where profits are maximized, $\text{MR}(Q^*) = MC(Q^*)$

$$A - \frac{A - 100}{25}Q^* = c$$

$$\frac{A - 100}{25}Q^* = A - c$$

$$Q^* = \frac{25(A - c)}{A - 100}$$

5d – Optimal sales

- Will you always find a quantity where MR(Q) = MC(Q)?
- If not, what is the optimal Q in this case?

$$Q^* = \frac{25(A-c)}{A-100}$$

First of all, the denominator cannot be 0, otherwise the ratio is undefined: $A - 100 \neq 0 \Leftrightarrow A \neq 100$

Then, negative values of Q* are not economically possible: $\frac{25(A-c)}{A-100} \geq 0$

So, there are two cases where Q* exists:

1) $25(A-c) \geq 0 \cap A-100 > 0$	*2)* $25(A-c) \leq 0 \cap A-100 < 0$
$25(A-c) \geq 0$ $\Leftrightarrow (A-c) \geq 0$ $\Leftrightarrow A \geq c$	$25(A-c) \leq 0$ $\Leftrightarrow (A-c) \leq 0$ $\Leftrightarrow A \leq c$
$A-100 > 0$ $\Leftrightarrow A > 100$	$A-100 < 0$ $\Leftrightarrow A < 100$
$A \geq c \cap A > 100$	$A \leq c \cap A < 100$

As we already know that $100 < A < 200$, the only relevant restriction is that $A \geq c$.
In the case we analyzed earlier where A=100, profit maximizing Q in undefined. Firm can sell all they produce.

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