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School of Electrical
Engineering

Celestial Mechanics and Satellite Orbits

Introduction to Space

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Lecture: Jaan Praks

Assignment

- Draw Earth, and a satellite orbiting the Earth.
- Draw the orbit of the satellite.
- Mark rotation direction of the Earth.
- Change your picture with your neighbour. Discuss.

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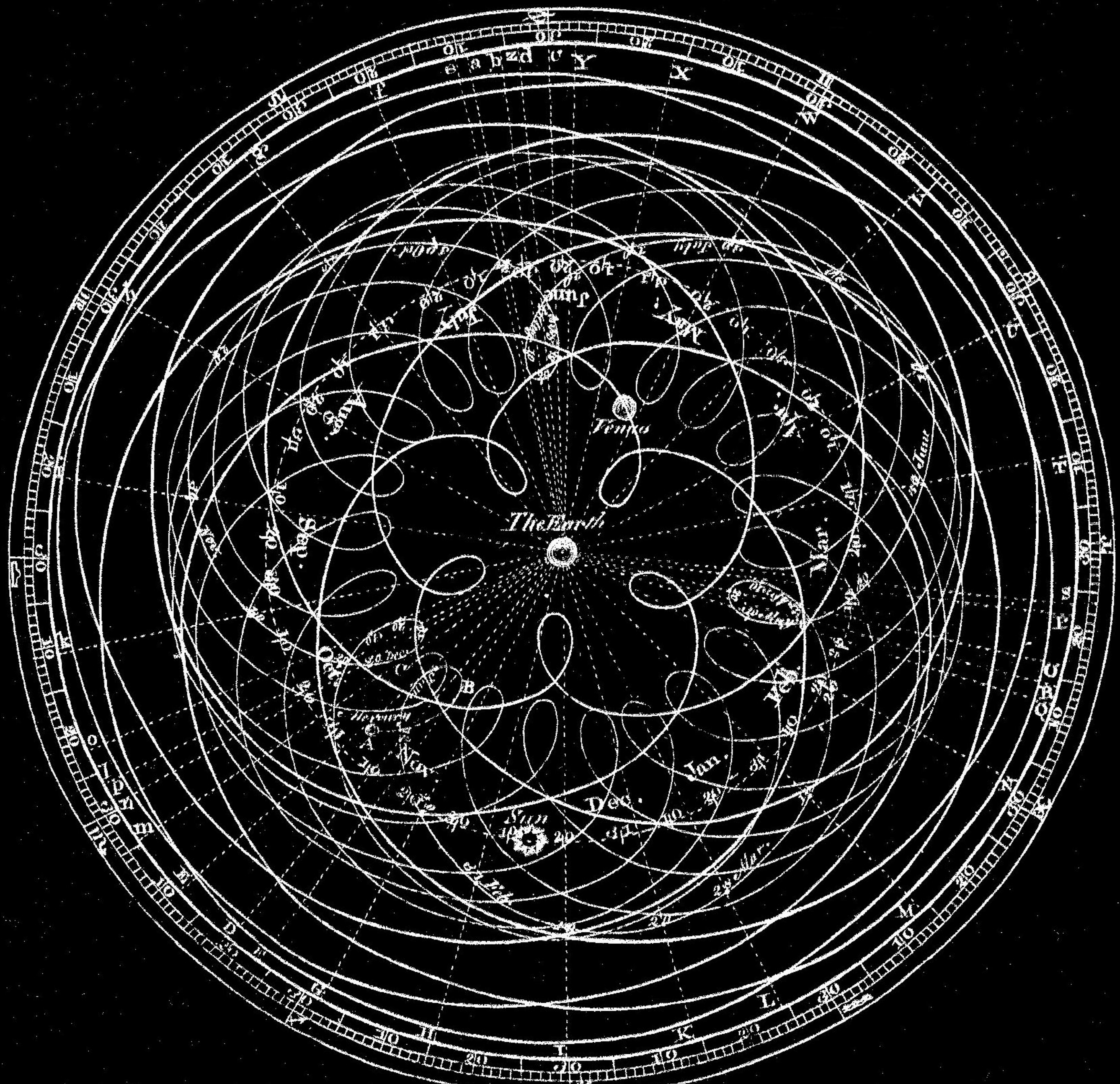
History

Schema huius præmissæ diuisionis Sphærarum.
EMPIREVM
HABITACVLVM
DEI
ET
OMNIVM
COELVM
Primu Mobile
Decimum Coelum
Crystallinum
Nonu Coelum
Firmamentum
Octauum
SATVRNI
COELV
IOVIS
MARTIS
SOLIS
VENERIS
MERCVRII
LVNÆ
© Justin Ng/HotSpot Media

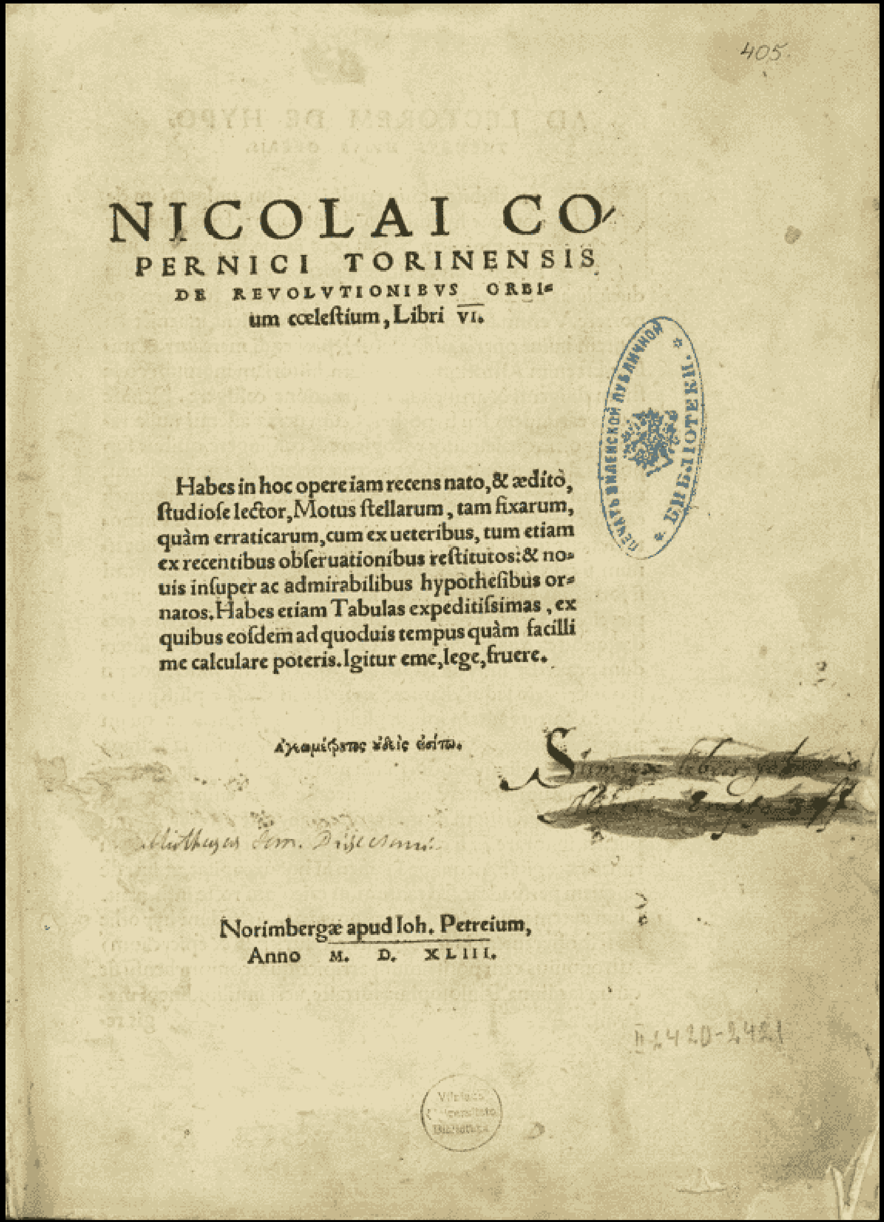
Schema huius præmissæ diuisionis Sphærarum .

(Photograph ©2007–08 Tunç Tezel.)

Planets, stars with will of their own…



The Earth
Venus
Sun
Jan.
Dec.
Feb.
Mar.



405

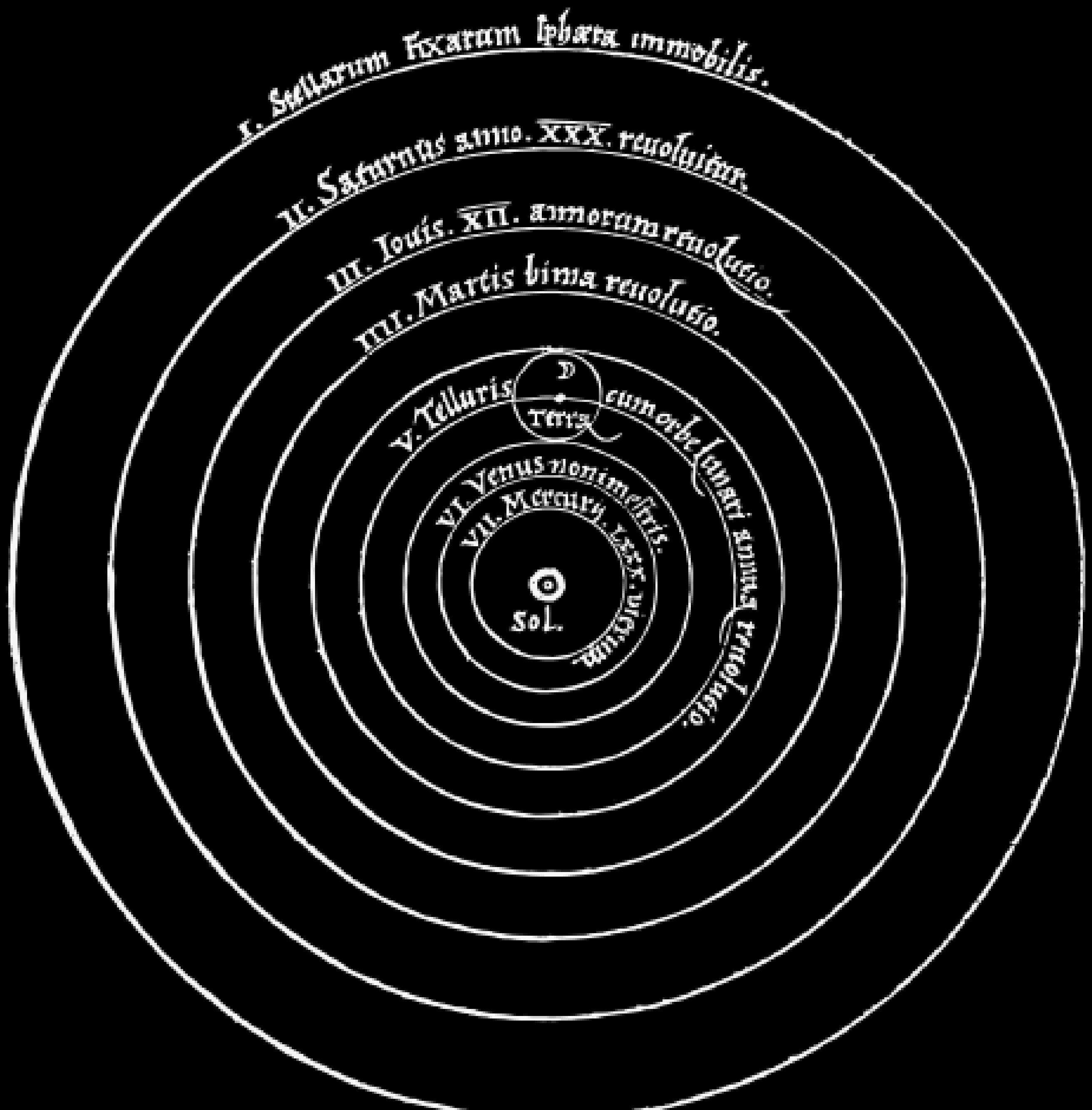
NICOLAI CO-
PERNICI TORINENSIS
DE REVOLVTIONIBVS ORBI-
um cœlestium, Libri VI.

Habes in hoc opere iam recens nato, & ædito, studiose lector, Motus stellarum, tam fixarum, quàm erraticarum, cum ex ueteribus, tum etiam ex recentibus obseruationibus restitutos: & nouis insuper ac admirabilibus hypothesibus ornatos. Habes etiam Tabulas expeditissimas, ex quibus eosdem ad quoduis tempus quàm facillime calculare poteris. Igitur eme, lege, fruere.

Ἀγεωμέτρητος οὐδεὶς εἰσίτω.

Norimbergæ apud Ioh. Petreium,
Anno M. D. XLIII.

Nicolaus Copernicus 1473 - 1543

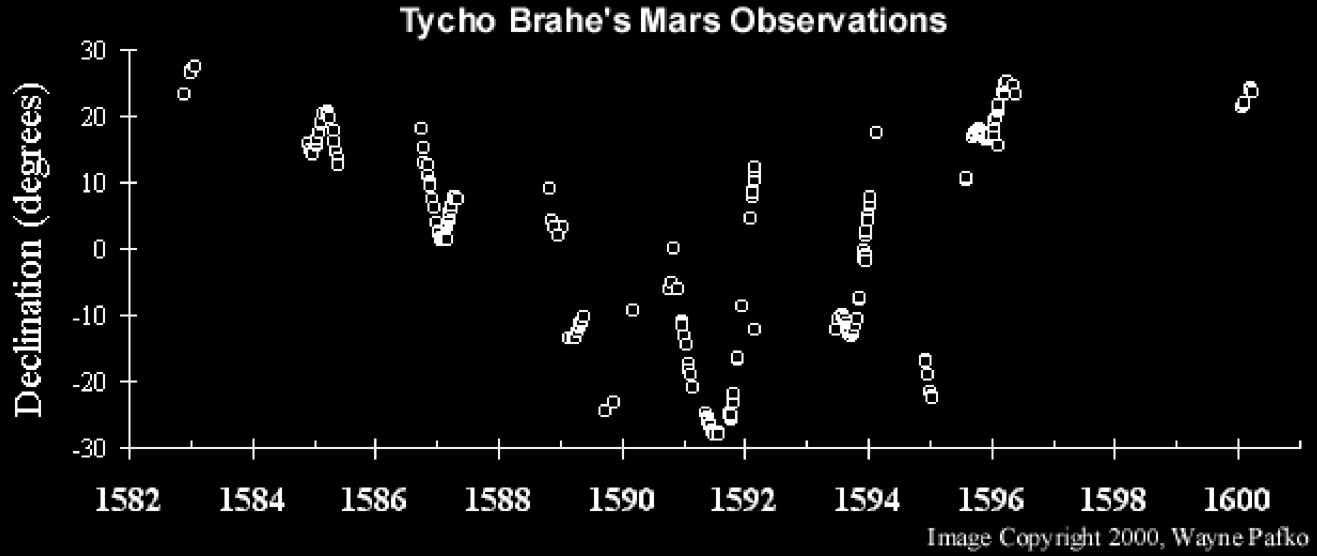

I. Stellarum Fixarum sphæra immobilis.
II. Saturnus anno. XXX. reuolvitur.
III. Iouis. XII. annorum reuolutio.
IIII. Martis bima reuolutio.
V. Telluris cum orbe lunari annua reuolutio.
Terra
VI. Venus nonimestris.
VII. Mercurij. LXXX. dierum.
Sol.

Tycho Brahe (1546 – 1601)

Danish nobleman and astronomer.

Passionate about planetary motion.

Made the most accurate measurements of planetary movements. (without a telescope!)



Kepler’s laws

Based on observations of Tycho Brahe 1546 – 1601

I. Each of the planets moves on an elliptical path with the Sun at one focus of the ellipse (1609)

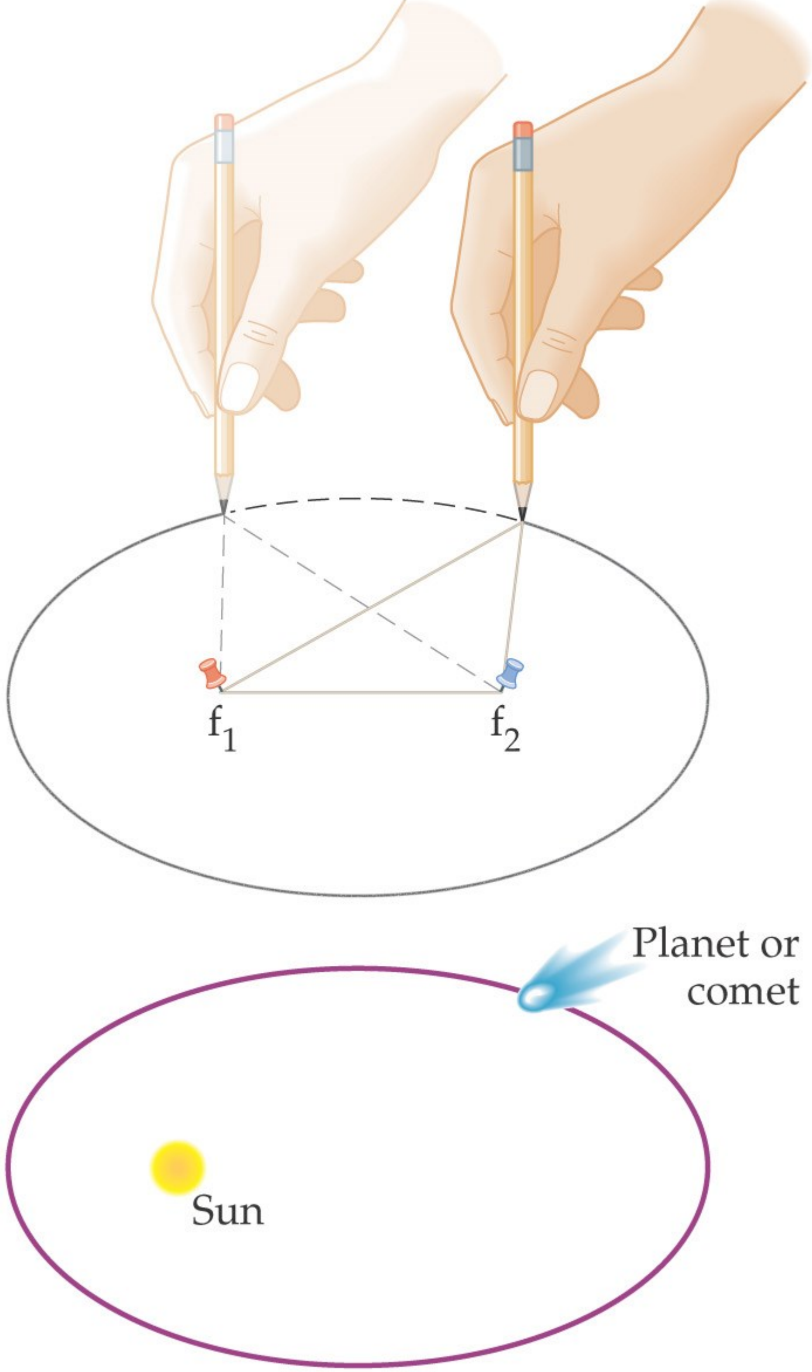
II. For each of the planets, the straight line connecting the planet to the Sun sweeps out equal areas in equal times (1609)

III. The squares of the periods of the planets are proportional to the cubes of the major axes of their orbits (1619)

Johannes Kepler (1571 - 1630)

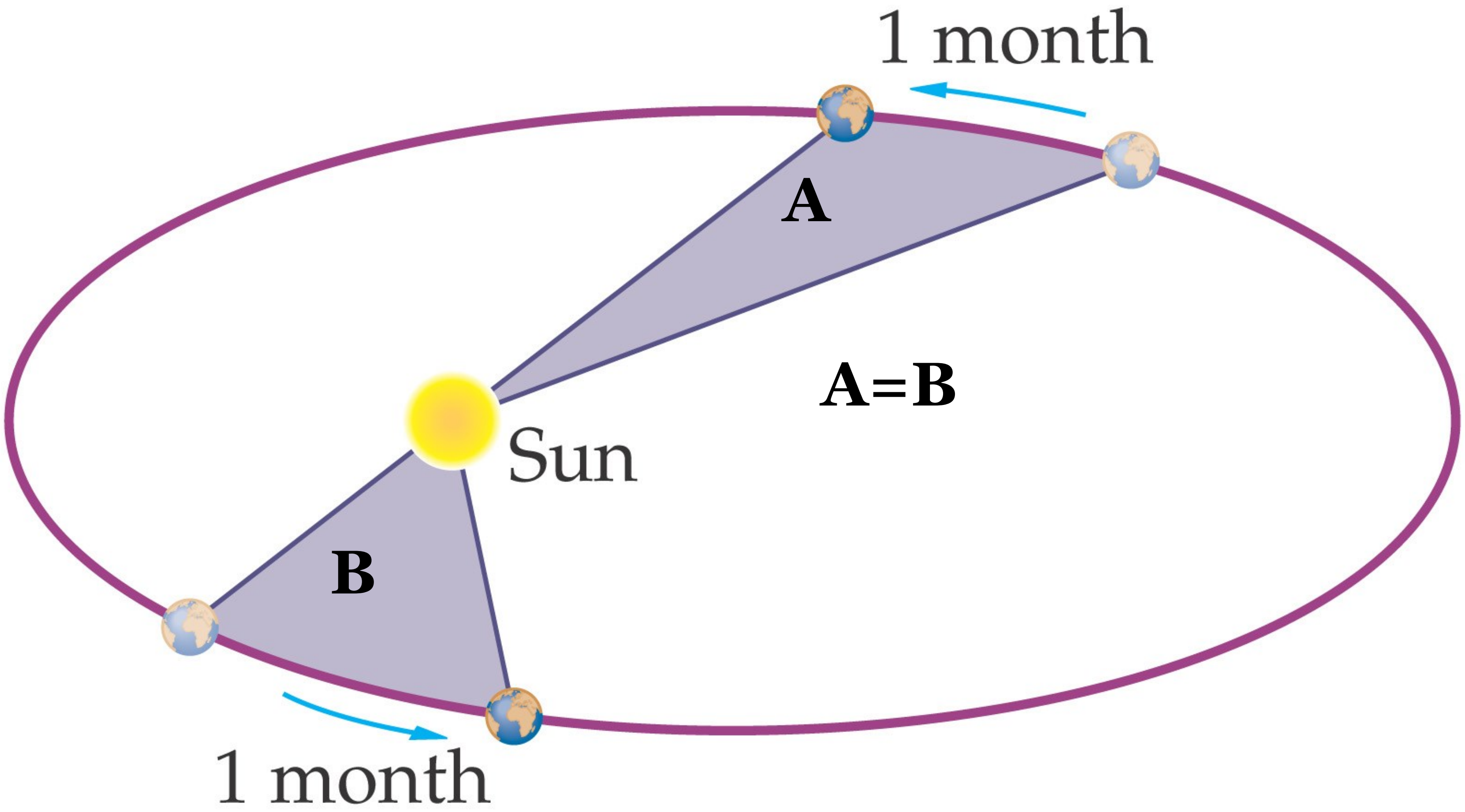
Kepler's

1



Kepler's

2

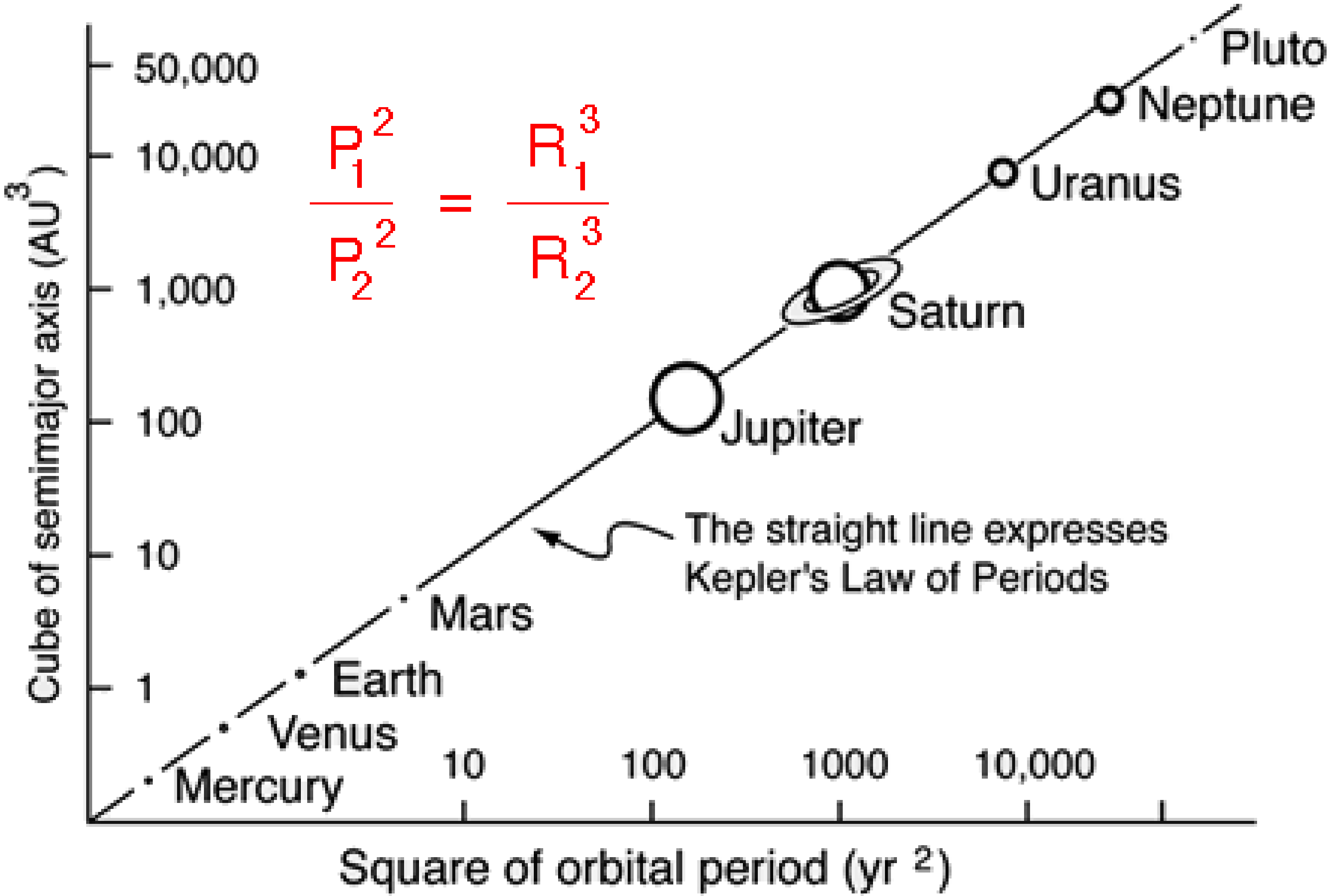


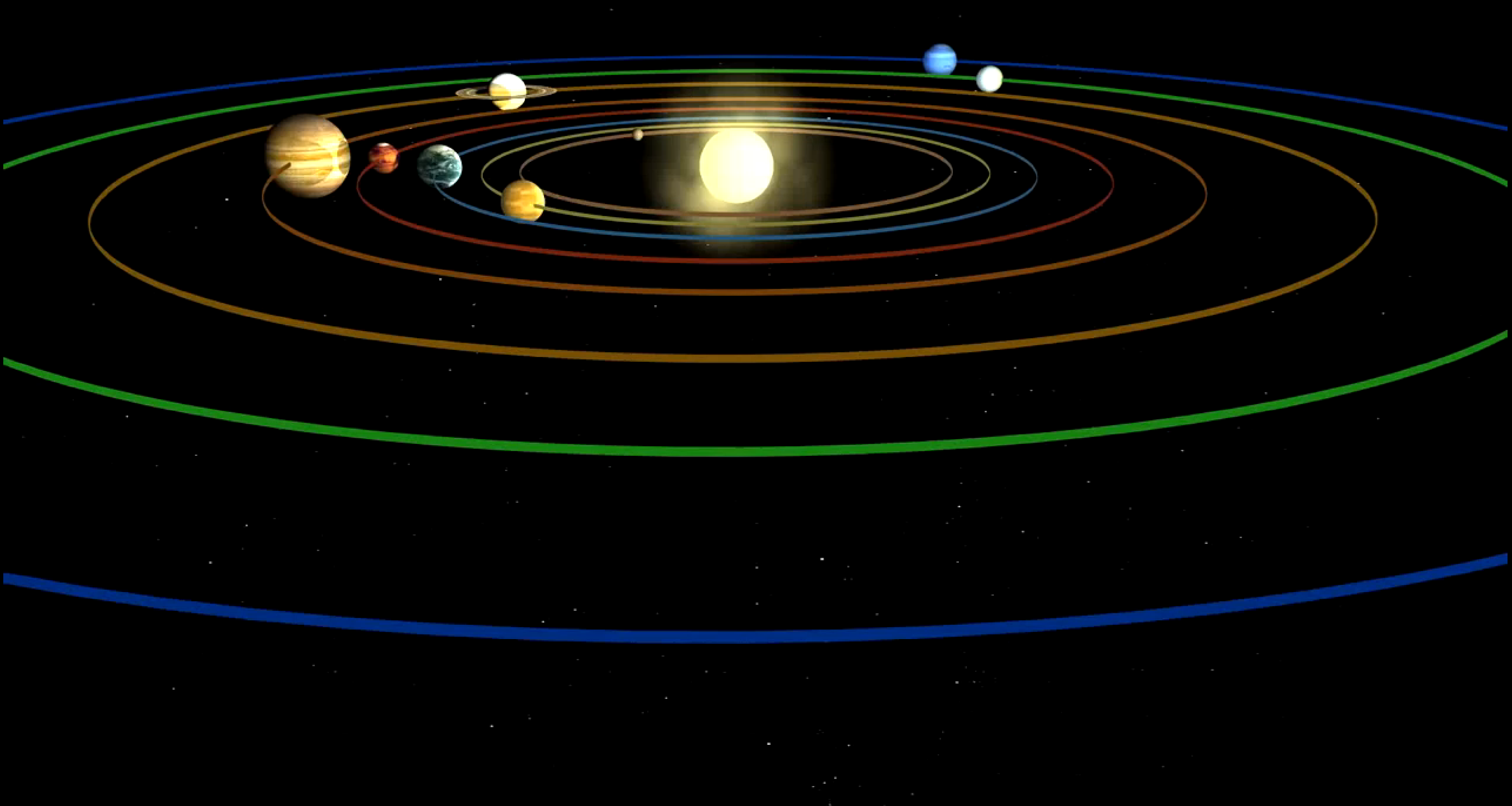
Probable examination question!

Kepler's

3

$$\frac{P_1^2}{P_2^2} = \frac{R_1^3}{R_2^3}$$





The Solar System

Newtonian Mechanics

Study of orbits of natural and artificial bodies in space

Based on Newton's laws *(transl. Andrew Motte, 1729)*:

"**LAW I.** *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*"

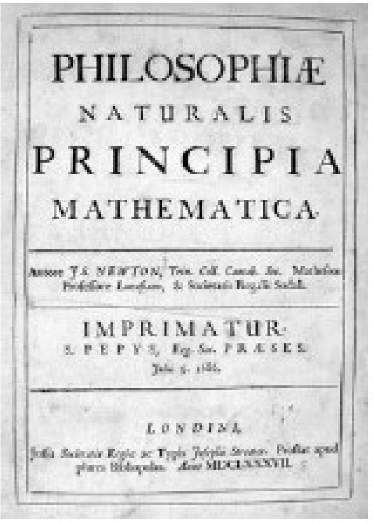
"**LAW II.** *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

$$\frac{dp}{dt} = \boldsymbol{F} \quad (p \equiv mv)$$

"**LAW III.** *To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*"

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21}$$

Isaac Newton
(1642 – 1727)


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

IMPRIMATUR

LONDINI

Principia, 1687

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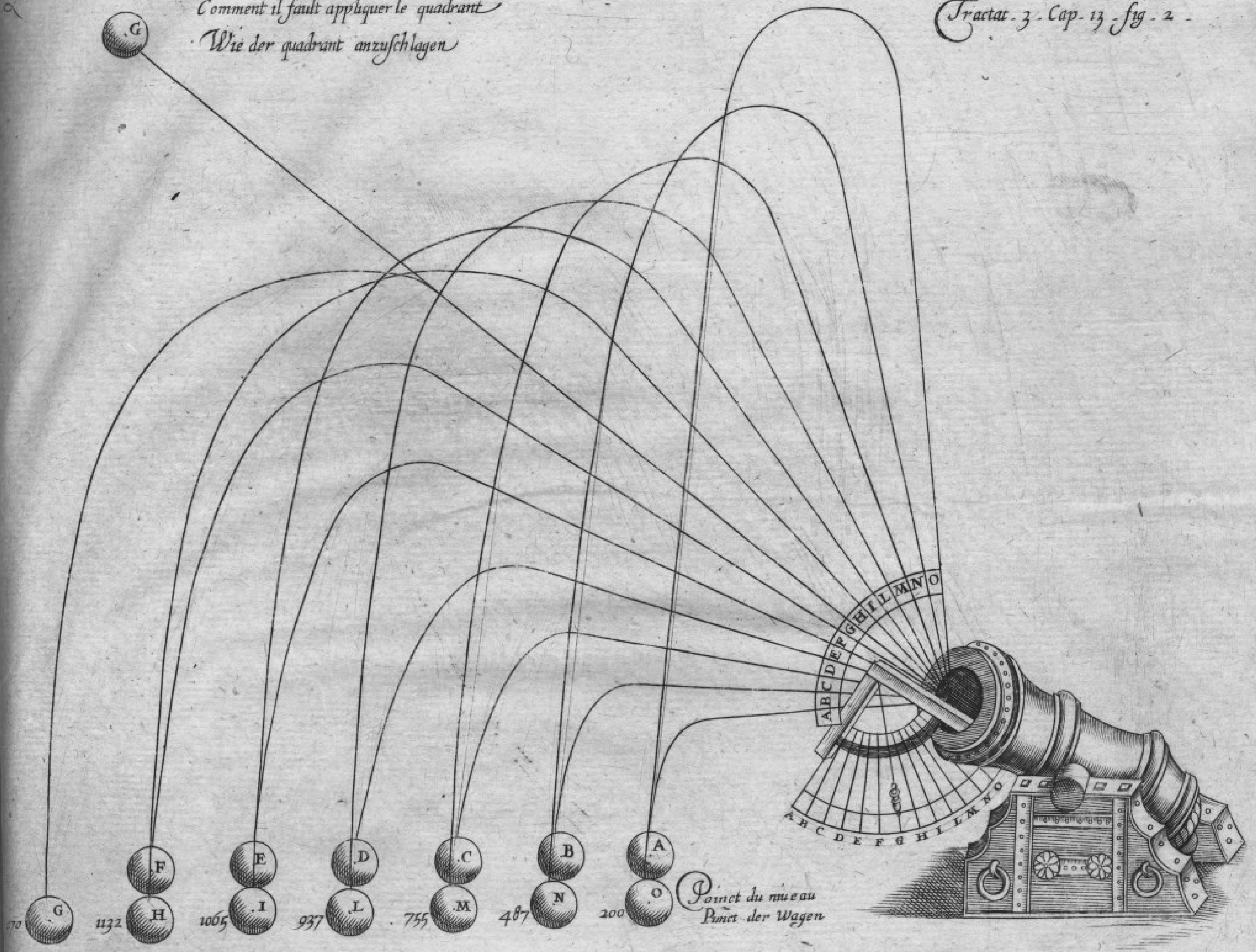
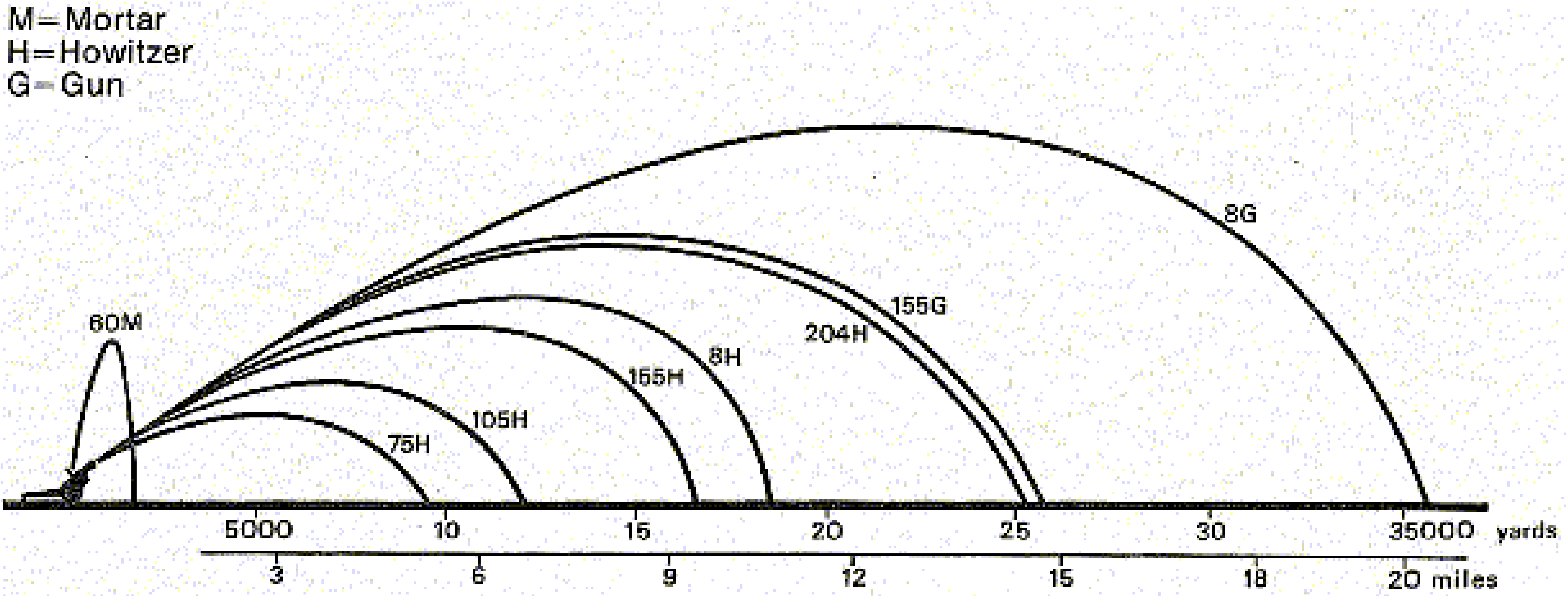
Comment il fault appliquer le quadrant
Wie der quadrant anzuschlagen

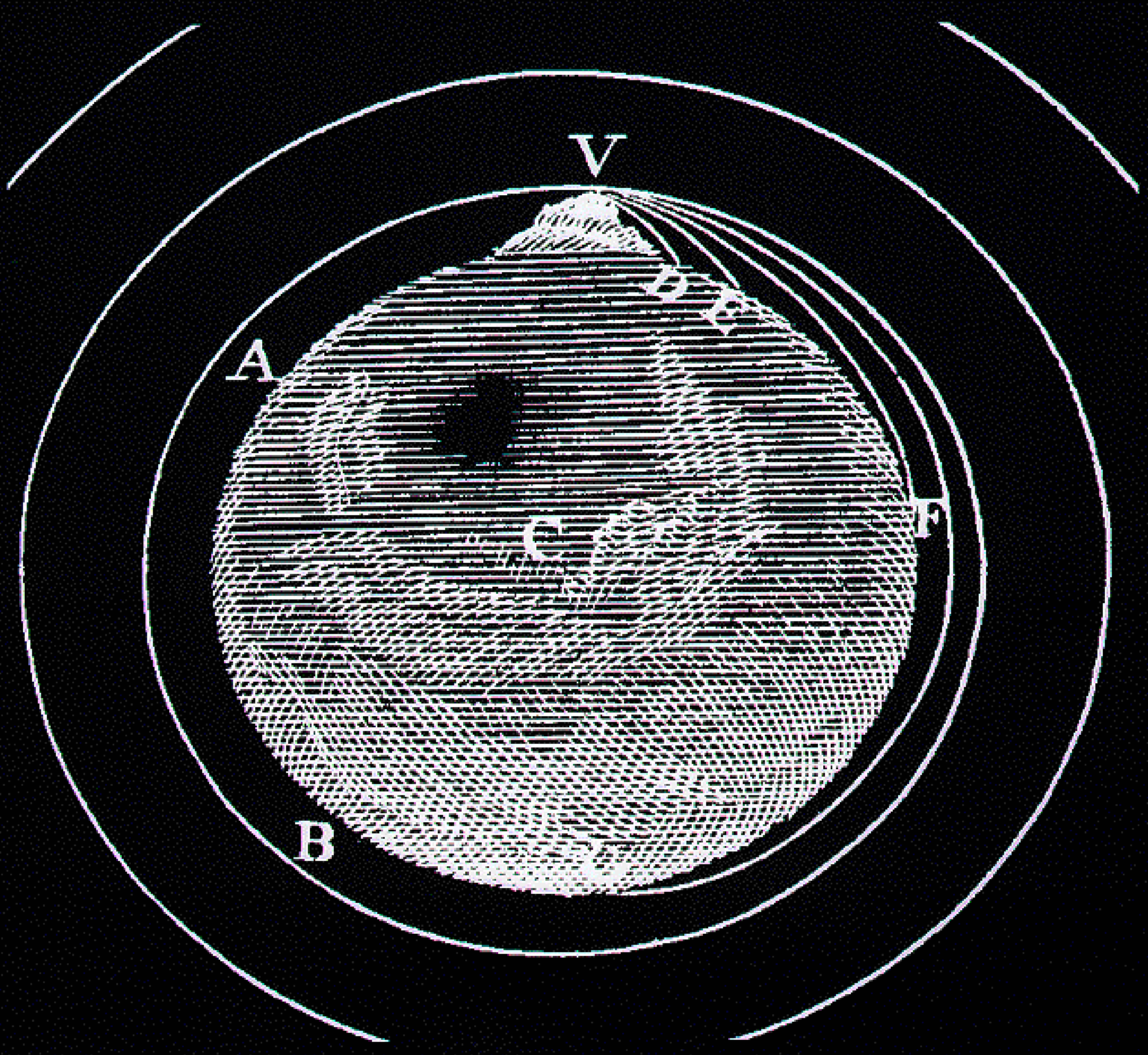
Tractat. 3. Cap. 13. fig. 2.





M=Mortar
H=Howitzer
G=Gun
60M
75H
105H
155H
8H
204H
155G
8G
5000
10
15
20
25
30
35000
yards
3
6
9
12
15
18
20 miles

after dinner, the weather being warm, we went
into the garden, & drank thea und
some appletrees; only he, & my
other discourse, he told me, he
same situation, as when formi
gravitation came into his mind
apple always descend perpendic
ground, thought he to himself; o
of an apple, as he sat in a contem
why shd it not go sideways, or up
stantly to the earths center? as
sen is, that the earth draws it.
drawing power in matter. & the
ing power in the matter of the e
the earths center, not in any sid
therefore dos this apple fall
or toward the center. if matter
ter; it must be in proportion of i
therefore the apple draws the
as the earth draws the apple.



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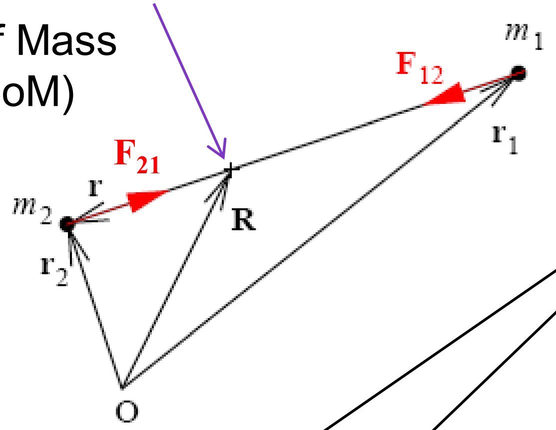
Basics of classical orbital mechanics

Two body problem: Reduction to one-body problem; central forces

$$R = \frac{m_1 \boldsymbol{r}_1 + m_2 \boldsymbol{r}_2}{m_1 + m_2}$$

Center of Mass
CM (or CoM)



$$\boldsymbol{r}_1 = \boldsymbol{R} - \frac{m_2 \boldsymbol{r}}{m_1 + m_2}$$

$$\boldsymbol{r}_2 = \boldsymbol{R} + \frac{m_1 \boldsymbol{r}}{m_1 + m_2}$$

$$m_1 \ddot{\boldsymbol{r}}_1 = \boldsymbol{F}_{12};$$
$$m_2 \ddot{\boldsymbol{r}}_2 = \boldsymbol{F}_{21} = -\boldsymbol{F}_{12} \Rightarrow$$

$$m_2 \ddot{\boldsymbol{r}}_2 - m_1 \ddot{\boldsymbol{r}}_1 = 2\boldsymbol{F}_{21}$$

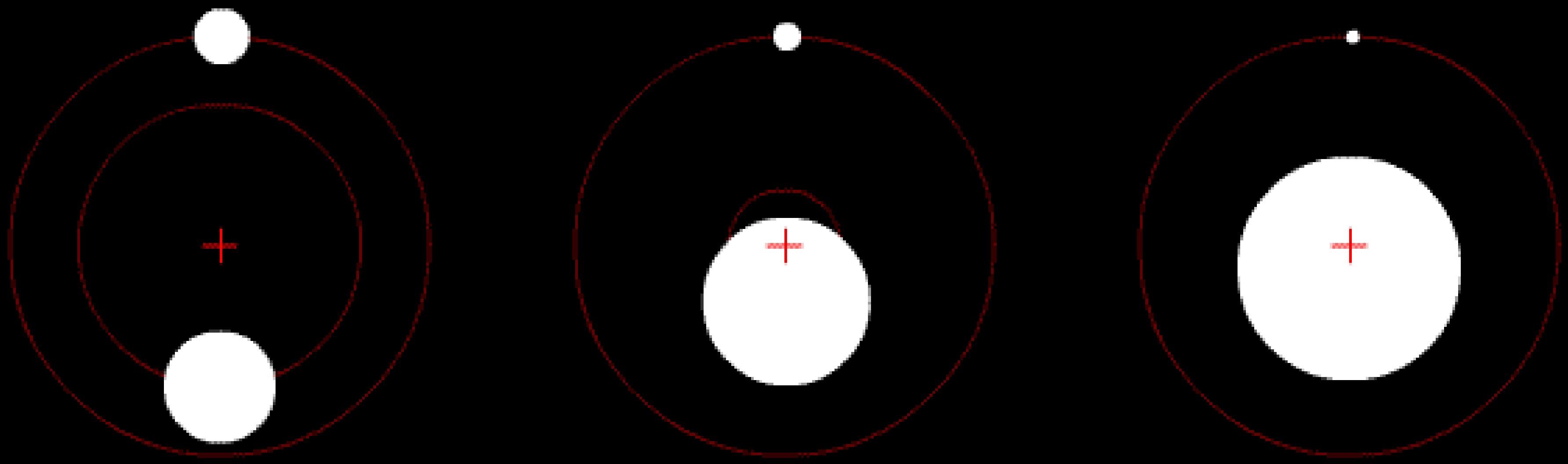
$$m \ddot{\boldsymbol{r}} = \boldsymbol{F}_{21} \qquad m = \frac{m_1 m_2}{m_1 + m_2}$$ reduced mass

$$\boldsymbol{F}_{21} = f(r)\boldsymbol{e}_r$$ central force

Example

$$\boldsymbol{F} = -\frac{GMm}{r^2}\boldsymbol{e}_r$$ Newton’s law of gravity

$$m_1 \gg m_2$$
$$m \approx m_2 \qquad \boldsymbol{r}_1 \approx \boldsymbol{R} \quad \boldsymbol{r}_2 \approx \boldsymbol{R} + \boldsymbol{r}$$



Motion under central force (e.g. gravity)

$\boldsymbol{F}(\boldsymbol{r}) = f(r)\boldsymbol{e}_r$ points to the origin

Calculate the moment of the force

$$\boldsymbol{r} \times \dot{\boldsymbol{p}} = \boldsymbol{r} \times \boldsymbol{F} = f(r)\,\boldsymbol{r} \times \boldsymbol{e}_r = 0$$

Calculate the time derivative of the angular momentum

$$\dot{\boldsymbol{L}} = \frac{d}{dt}(\boldsymbol{r} \times \boldsymbol{p}) = \boldsymbol{v} \times \boldsymbol{p} + \boldsymbol{r} \times \dot{\boldsymbol{p}} = 0$$

$\boldsymbol{L}$ is a constant of motion

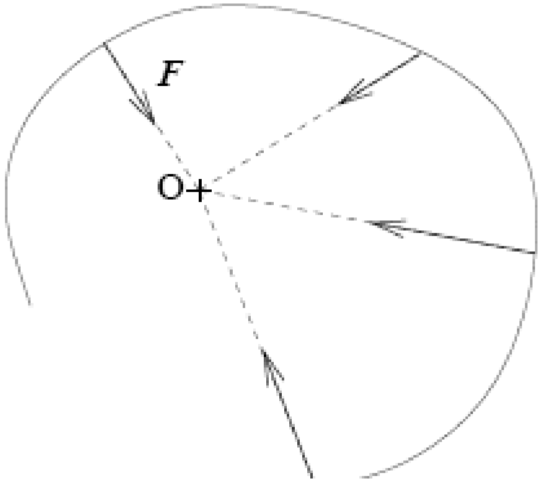
$\boldsymbol{r} \perp \boldsymbol{L}$ and $\boldsymbol{v} \perp \boldsymbol{L}$: the motion takes place on a plane

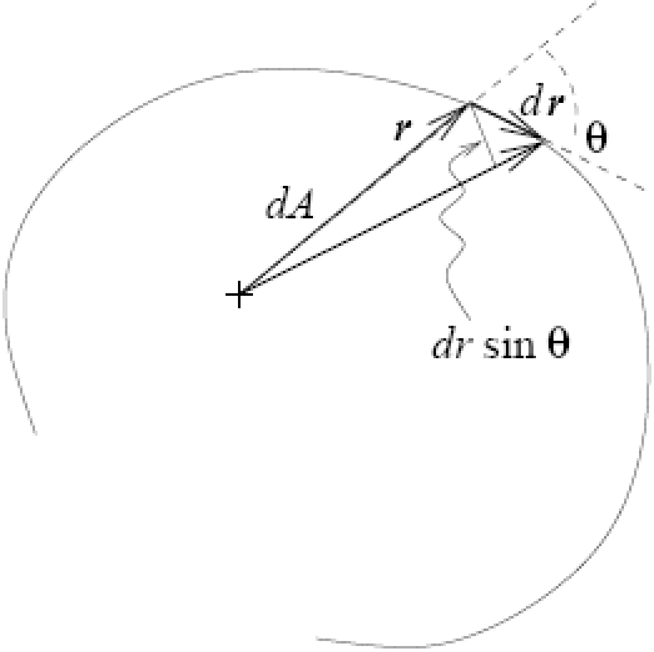
Calculate the area element

$$dA = \tfrac{1}{2}\, r \, \sin\theta \, dr = \tfrac{1}{2}|\boldsymbol{r} \times d\boldsymbol{r}|$$

$$\Rightarrow \frac{dA}{dt} = \tfrac{1}{2}|\boldsymbol{r} \times \boldsymbol{v}| = \tfrac{1}{2m}|\boldsymbol{L}|$$

The surface velocity dA/dt is constant (Kepler's second law)





Solutions of the equation of motion Conic sections

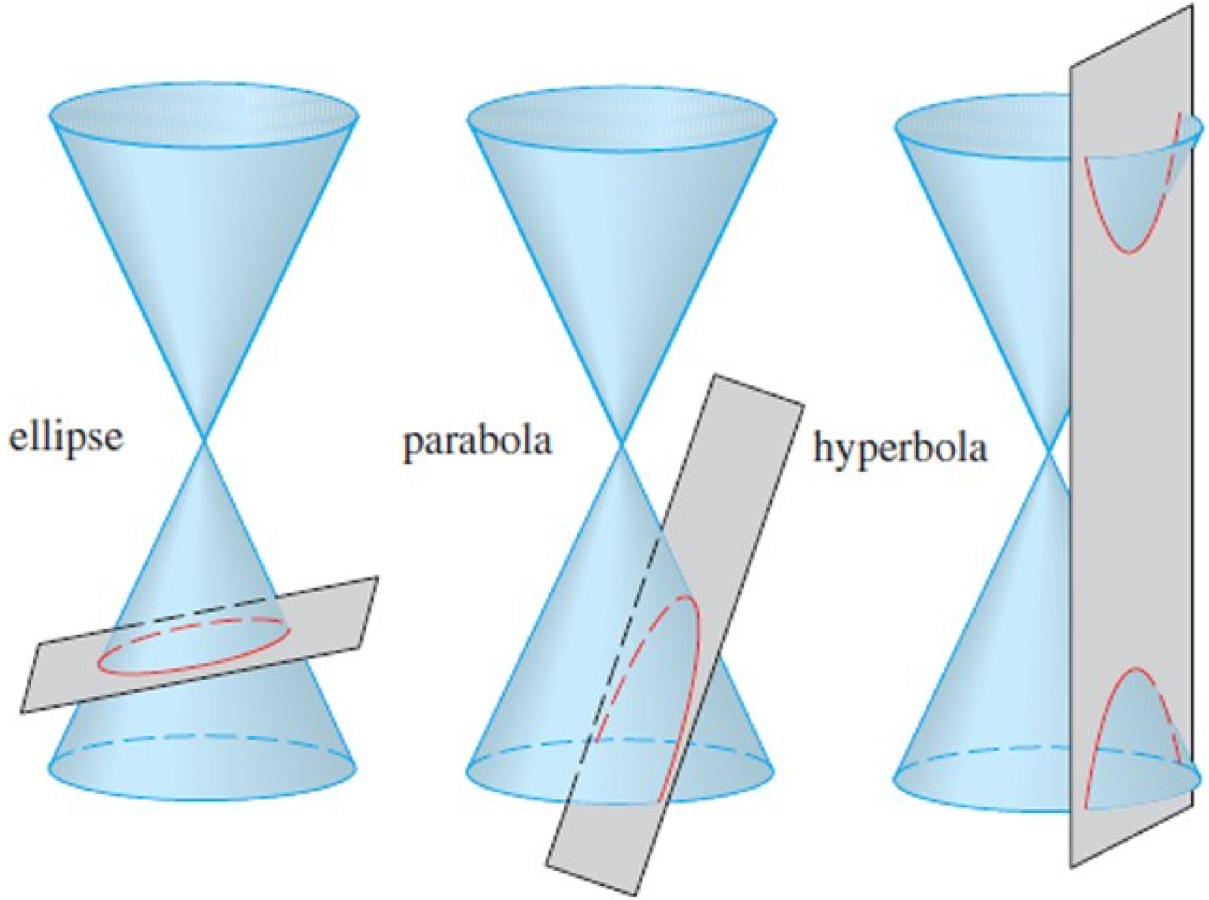
Solutions of the equation $m\ddot{\boldsymbol{r}} = -\frac{GMm}{r^2}\boldsymbol{e}_r$ are of the form

$$r = \frac{p}{1 + \varepsilon\cos\theta}; \qquad p = \frac{l^2}{mk} \qquad |\boldsymbol{L}| = l$$

$$\varepsilon = \sqrt{1 + \frac{2El^2}{mk^2}} \qquad k = GMm$$

These are known as conic sections with ellipticity:

$$\begin{cases} \varepsilon = 0 & \text{circle} \\ 0 < \varepsilon < 1 & \text{ellipse} \\ \varepsilon = 1 & \text{parabola} \\ \varepsilon > 1 & \text{hyperbola} \end{cases}$$



Trajectories

If e > 1, the trajectory is a hyperbola

(an open trajectory)

If e = 1, the trajectory is a parabola

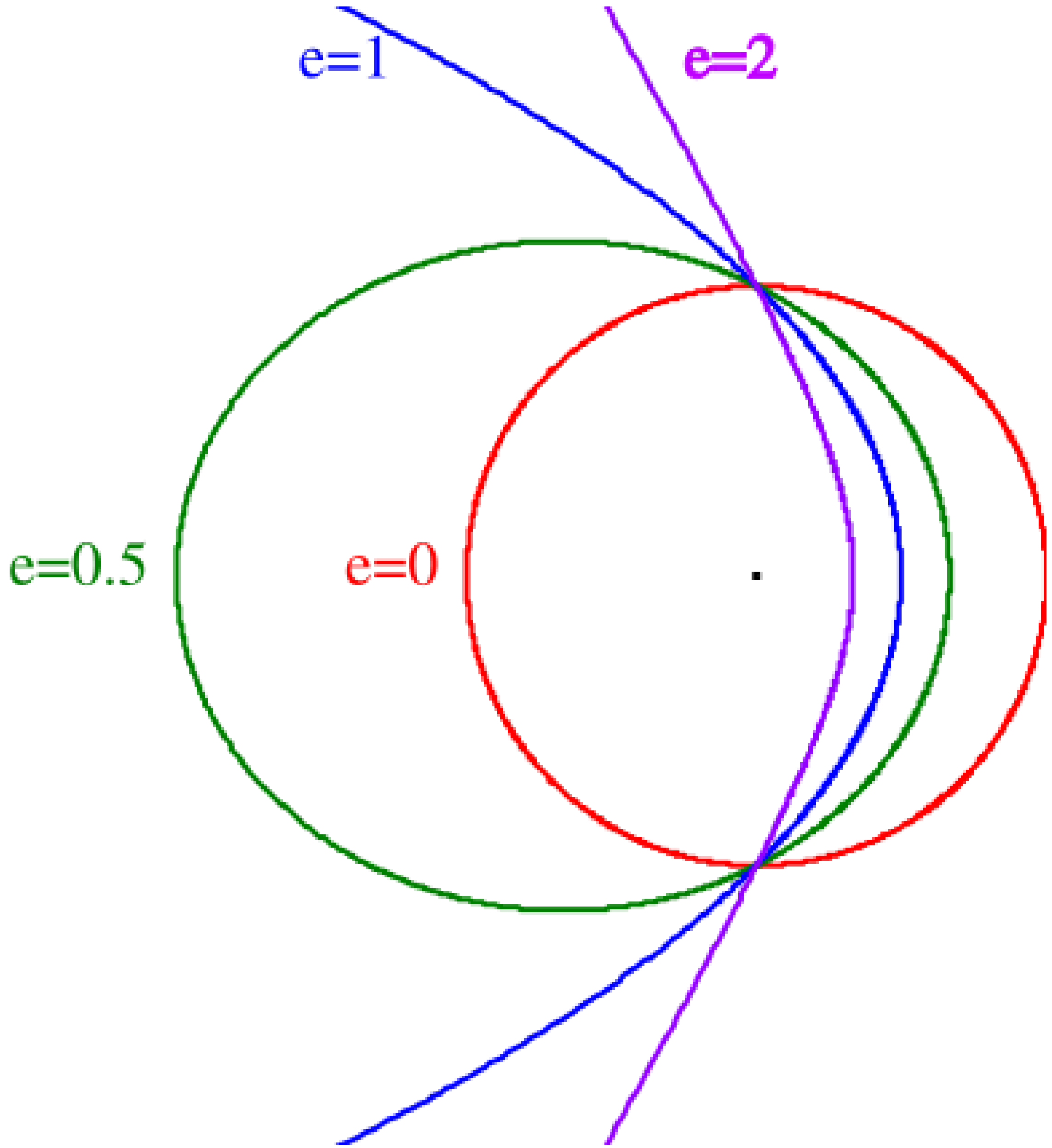
(an open trajectory)

If 0 < e < 1, the trajectory is an ellipse

(a closed trajectory; i.e., an orbit)

If e = 0, the orbit is circular.

(a closed trajectory; i.e., an orbit)
This is just a special case of the ellipse



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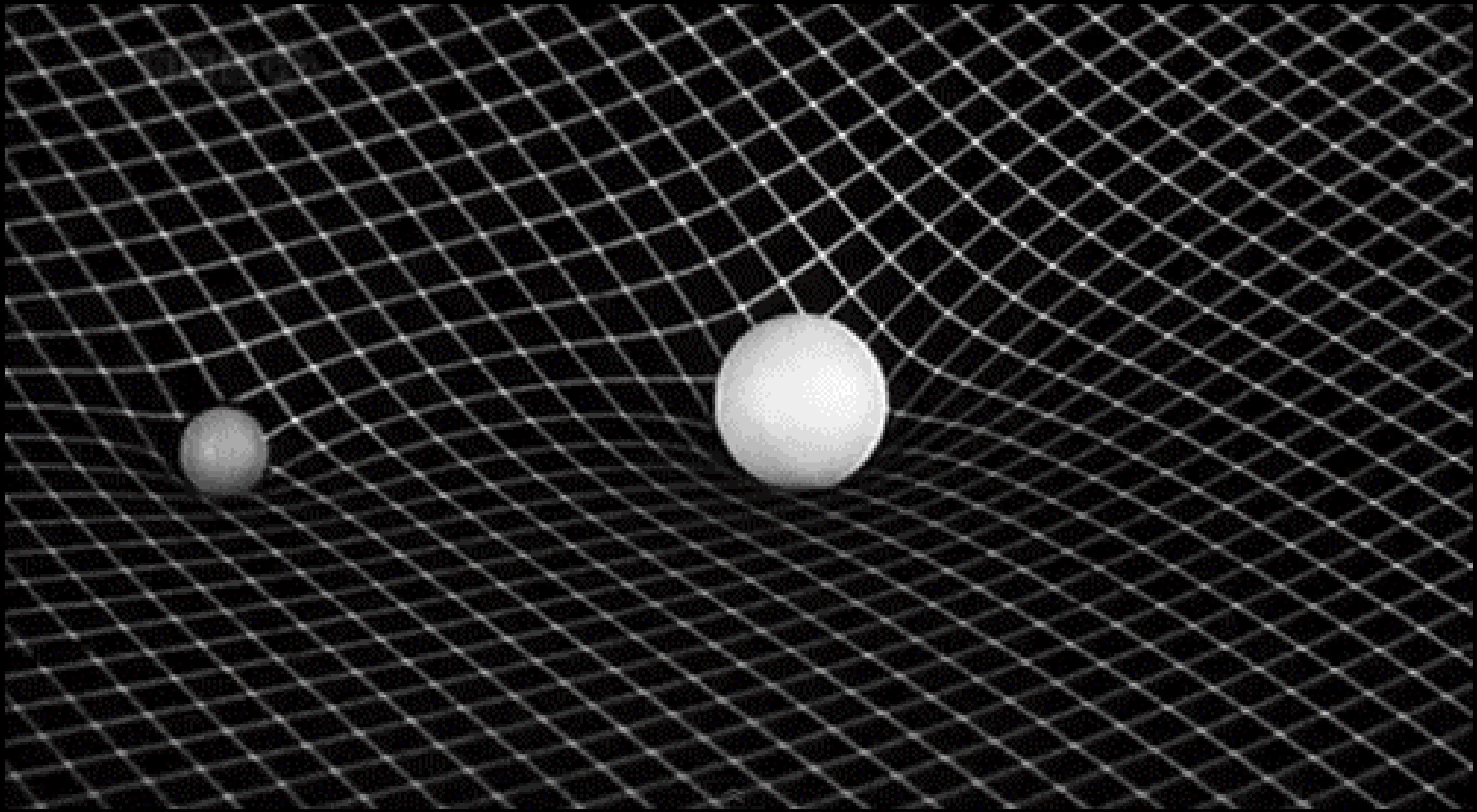
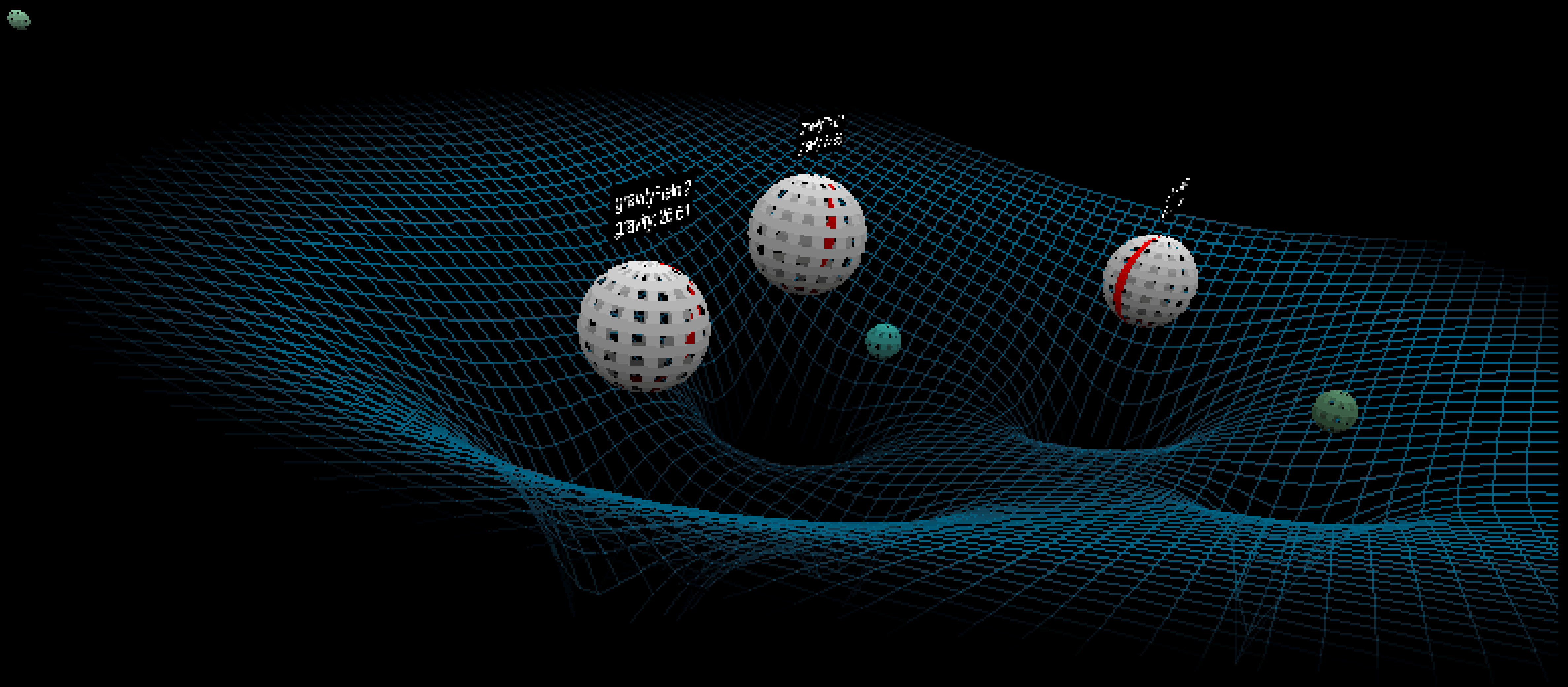
Modern interpretation of gravity

General Relativity (1915)

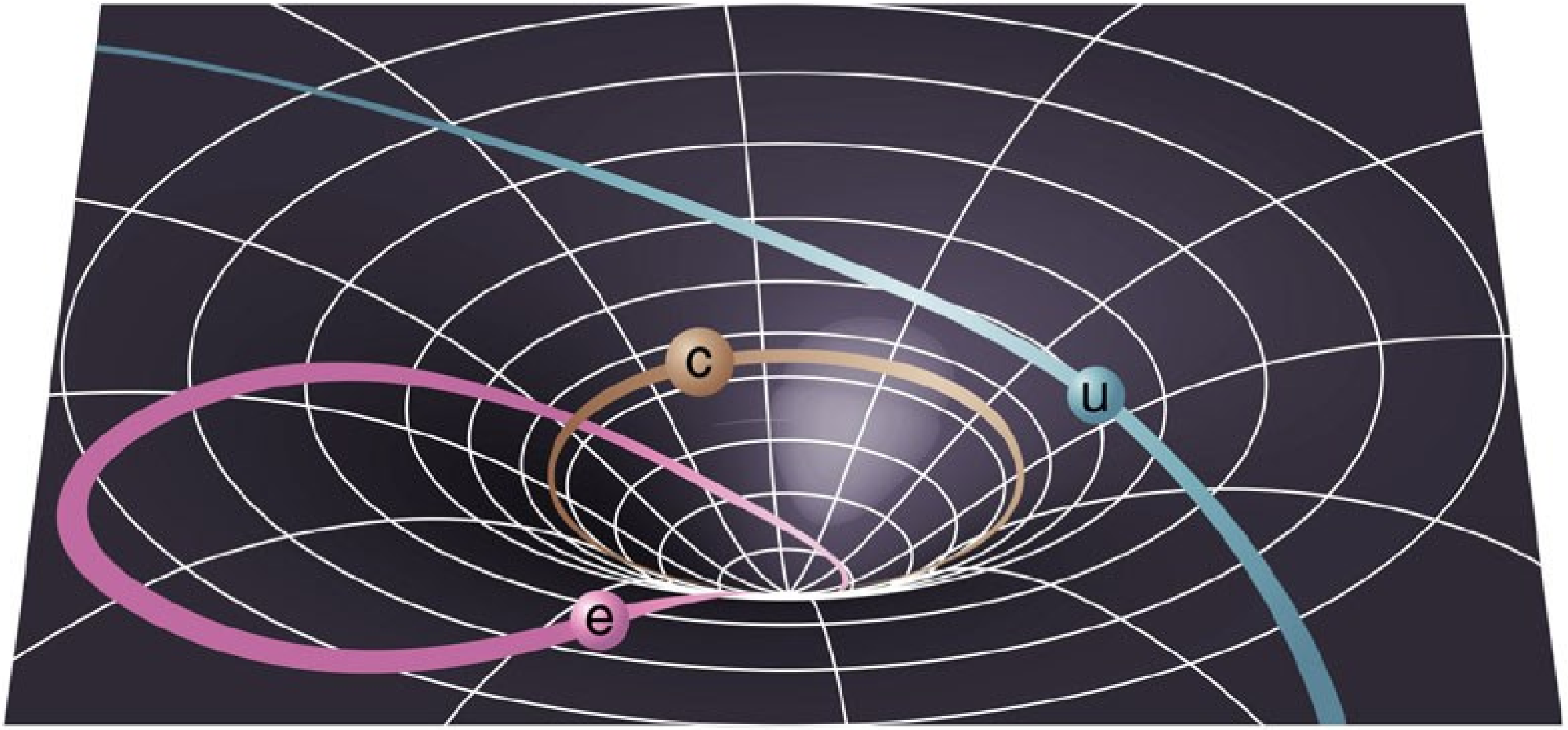
The observed gravitational effect between masses results from their warping of spacetime

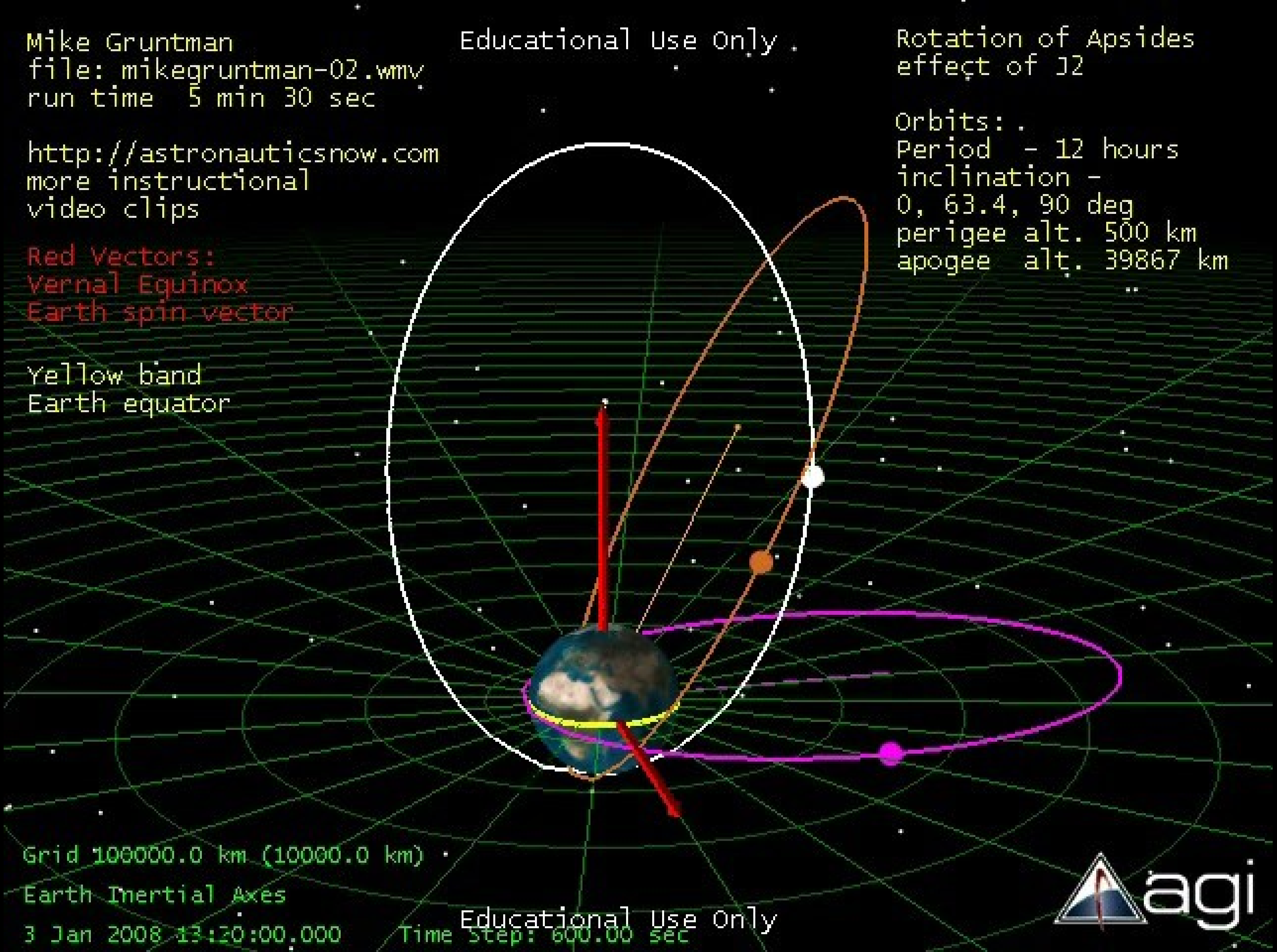


Albert Einstein



c circular orbit
e elliptical orbit
u unbound orbit




Mike Gruntman
file: mikegruntman-02.wmv
run time 5 min 30 sec
http://astronauticsnow.com
more instructional
video clips
Red Vectors:
Vernal Equinox
Earth spin vector
Yellow band
Earth equator
Educational Use Only
Rotation of Apsides
effect of J2
Orbits:
Period - 12 hours
inclination -
0, 63.4, 90 deg
perigee alt. 500 km
apogee alt. 39867 km
Grid 100000.0 km (10000.0 km)
Earth Inertial Axes
3 Jan 2008 13:20:00.000
Time Step: 600.00 sec
Educational Use Only
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Orbits and their properties

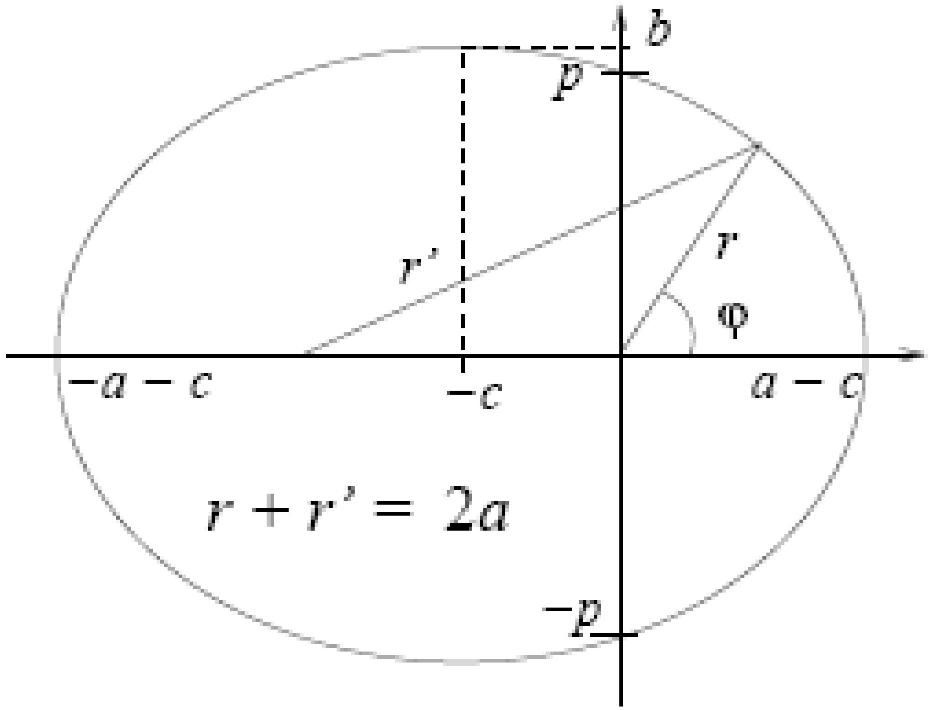
Elliptical orbits and Kepler's laws

K-I $r = \dfrac{p}{1+\varepsilon\cos\theta}$; $\varepsilon = \sqrt{1+\dfrac{2El^2}{mk^2}} < 1$;

$$2a = \frac{p}{1-\varepsilon} + \frac{p}{1+\varepsilon} = \frac{2p}{1-\varepsilon^2}$$

$$c = a - p/(1+\varepsilon) = \varepsilon a$$

$$b^2 = a^2 - c^2 = a^2(1-\varepsilon^2) = ap$$



K-II During an infinitesimal time period dt

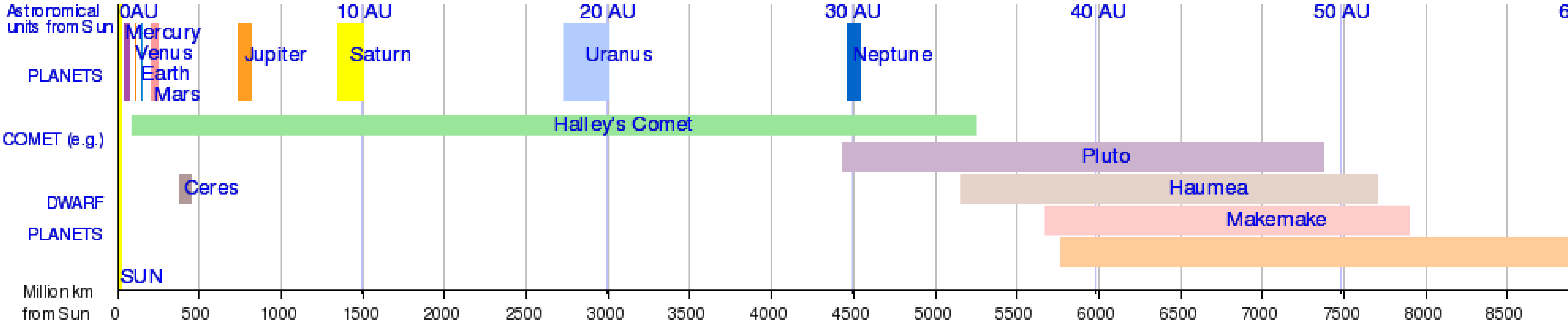
$$dA = \tfrac{1}{2} r\, r\, d\varphi \;\Rightarrow\; \dot{A} = \tfrac{1}{2} r^2 \dot{\varphi} = \frac{l}{2m} = \text{constant}$$

K-III From above $\dfrac{b^2}{a} = p = \dfrac{l^2}{mk} \;\Rightarrow\; b = \dfrac{\sqrt{a}\,l}{\sqrt{mk}}$, and the area of the ellipse is

$$A = \pi a b$$

$$A = \int_0^T \dot{A}\, dt = \frac{lT}{2m} = \pi a \frac{\sqrt{a}\,l}{\sqrt{mk}} \quad\Rightarrow\quad T = 2\pi\sqrt{\frac{m}{k}}\, a^{3/2}$$

Ellipticity of planetary orbits



Kinetic + potential energy

Orbital energy is constant

Kinetic plus potential energy

$$E = \frac{-GMm}{r} + \frac{M(v_M)^2}{2} + \frac{m(v_m)^2}{2}$$

Kinetic

$$E = \frac{-GMm}{r} + \mu \frac{v^2}{2}$$

Potential

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Parabolic and hyperbolic orbits: Escape velocity

For an open conic section $\varepsilon = \sqrt{1 + \frac{2El^2}{\mu k^2}} \geq 1 \quad \mu = \frac{mM}{m+M}$

thus $E \geq 0$.

and the minimum velocity to escape is given by $\frac{1}{2}\mu v_e^2 = \frac{GMm}{r}$

$$v_e = \sqrt{\frac{2GMm}{\mu r}} = \sqrt{\frac{2G(M+m)}{r}} \approx \sqrt{\frac{2GM}{r}}$$

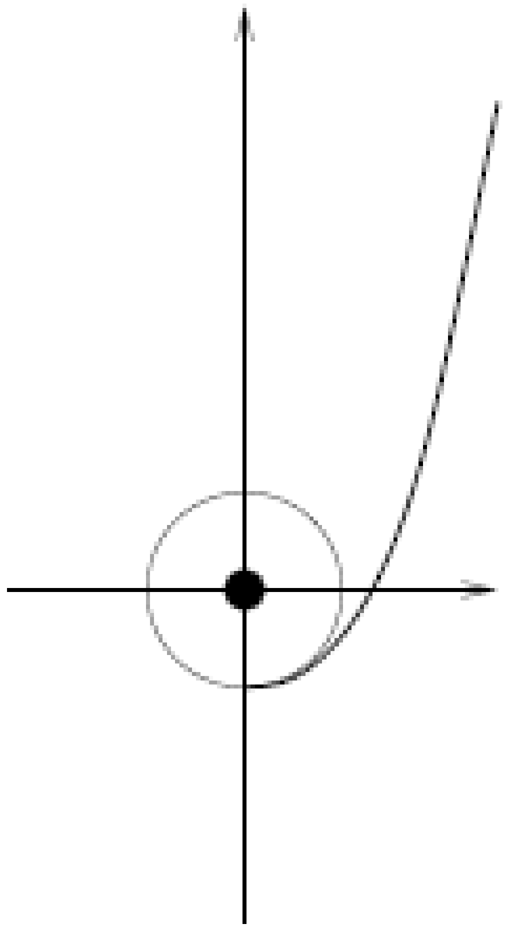
From the surface of the Earth $v_e = 11.2$ km/s
From the surface of the Sun $v_e = 618$ km/s

Escape from geostationary orbit $r = 6.6\, R_E$

$v_{\varphi 0} = 2\pi r/(24\,\text{h}) = 3.06$ km/s

$v_e = 11.2$ km/s$/\sqrt{6.6} = 4.36$ km/s

$\Delta v = v_e - v_{\varphi 0} = 1.3$ km/s in the direction of the motion



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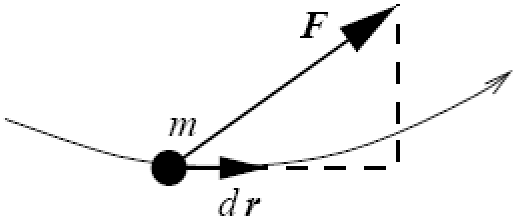
How to change the orbit

Recall the work done by a force



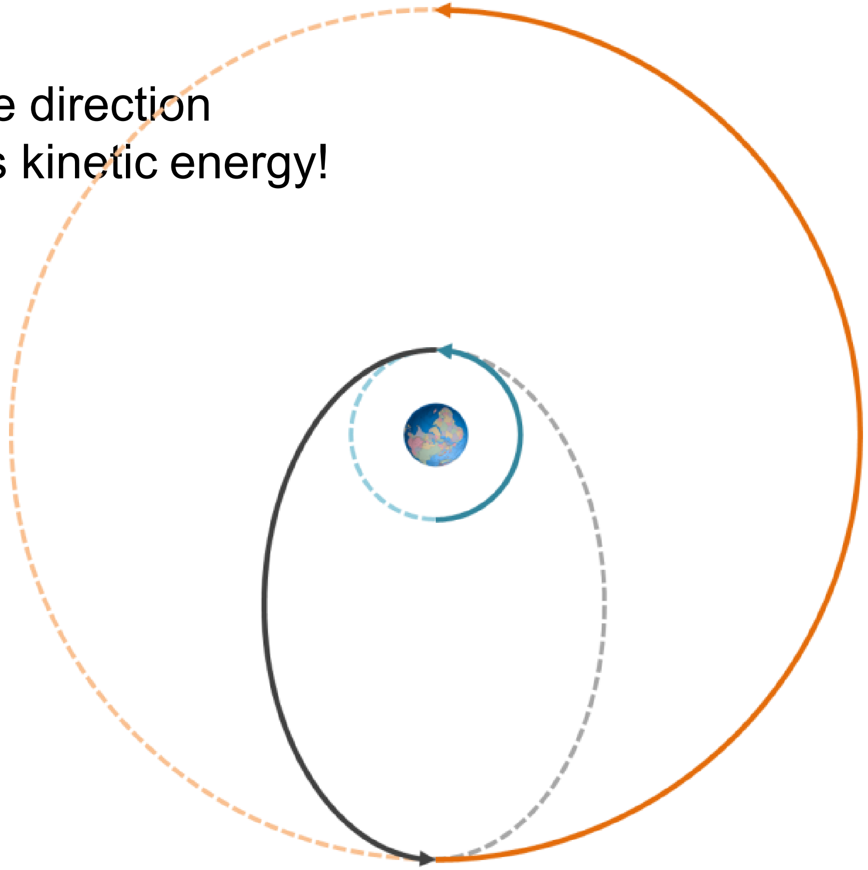
$$W = \int_{\boldsymbol{r}(t)}^{\boldsymbol{r}(t+\Delta t)} \boldsymbol{F} \cdot d\boldsymbol{r}$$

$$= \int_{t}^{t+\Delta t} \boldsymbol{F} \cdot \frac{d\boldsymbol{r}}{dt}\, dt$$

Thus only the component of force in the direction of the spacecraft motion can change its kinetic energy!

To lift the apogee, fire the rocket in perigee
To lift the perigee, fire the rocket in apogee

To reach from one circular orbit to another with least energy:
Hohmann transfer orbit



Vis-viva equation

Orbital energy is constant

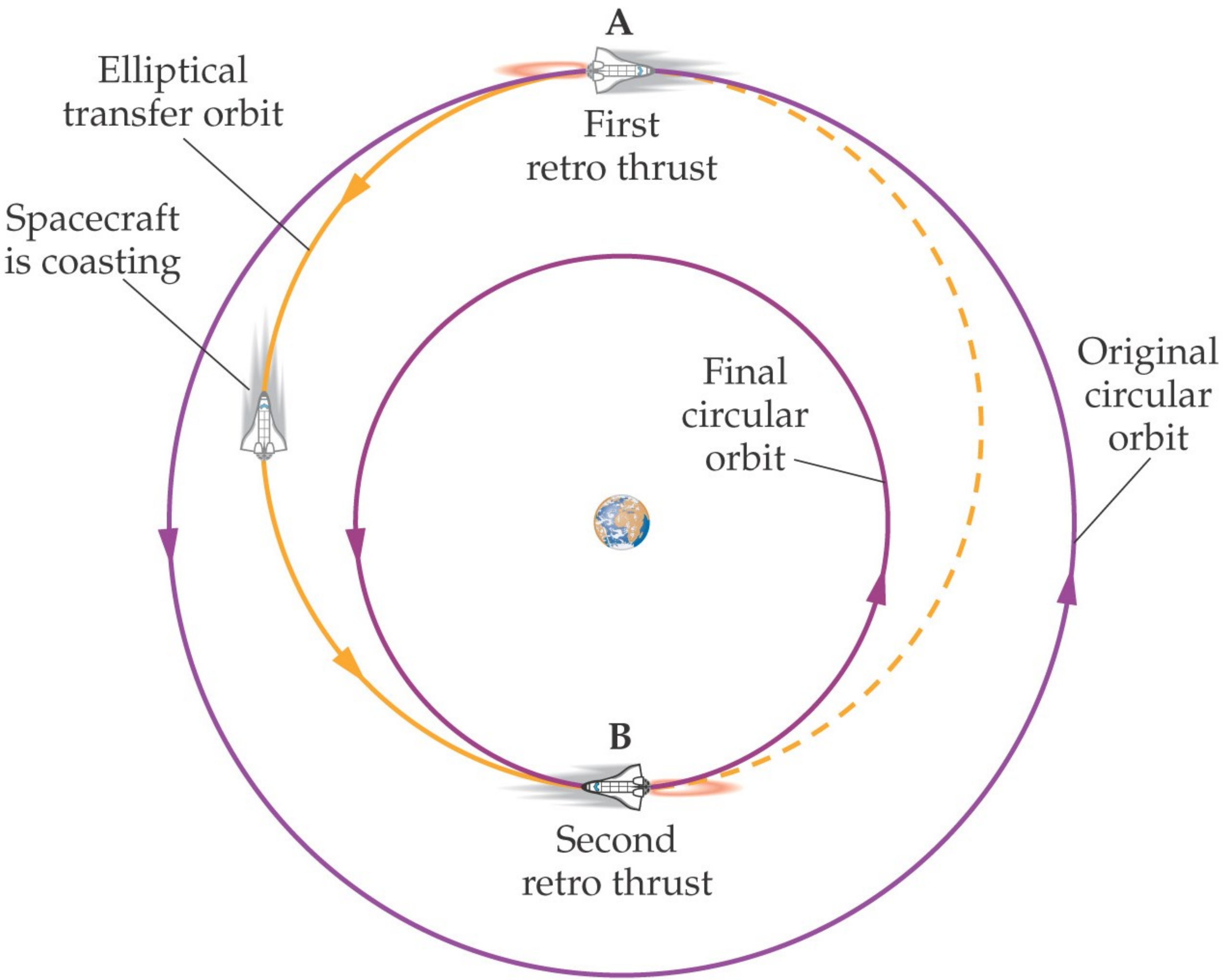
Kinetic plus potential energy

$$E = \frac{-GMm}{r} + \frac{M(v_M)^2}{2} + \frac{m(v_m)^2}{2}$$

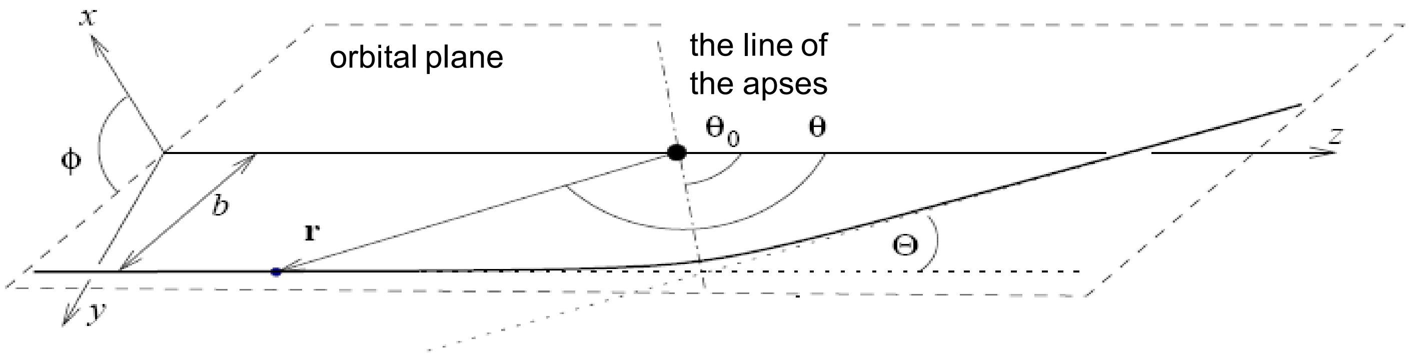
$$E = \frac{-GMm}{r} + \mu\frac{v^2}{2}$$

$$v^2 = G(M+m)\left(\frac{2}{r} - \frac{1}{a}\right)$$

Hohmann transfer orbit



Hyperbolic orbits: Scattering in the gravitational field of a planet



$$r = \frac{p}{1 + \varepsilon \cos(\theta - \theta_0)}$$

$r \to \infty$ when $\theta \to \pi$ $\quad 1 + \varepsilon \cos(\pi - \theta_0) = 0 \;\Rightarrow\; \cos\theta_0 = \frac{1}{\varepsilon}$

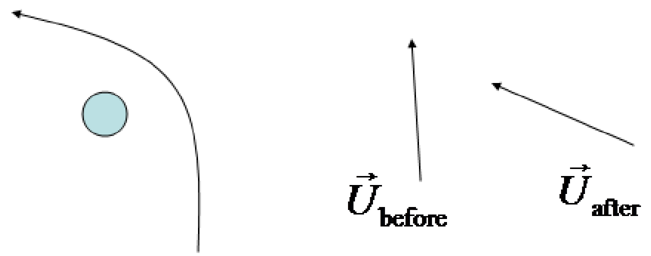
Symmetry: $\Rightarrow \Theta = \pi - 2\theta_0$

$$\cot\frac{\Theta}{2} = \sqrt{\varepsilon^2 - 1} = \sqrt{\frac{2El^2}{mk^2}} = \frac{2Eb}{|k|} \qquad b = \frac{|k|}{2E}\cot\frac{\Theta}{2}$$

impact parameter

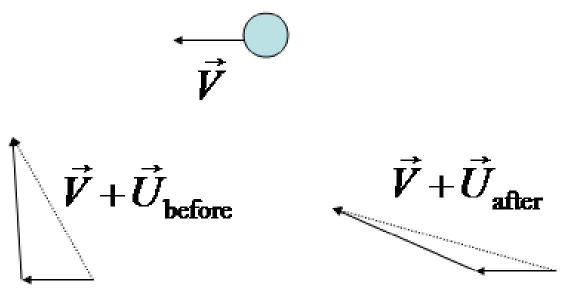
Using a planet to accelerate / decelerate a spacecraft

Frame of Reference: Moving with Planet

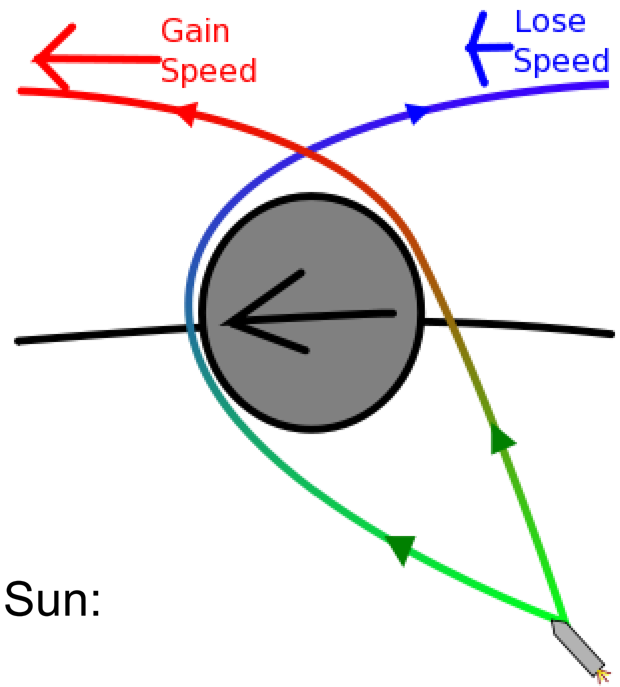


$$\left|\vec{U}_{\text{before}}\right| = \left|\vec{U}_{\text{after}}\right|$$

Frame of Reference: Planet Moving Left



$$\left|\vec{V}+\vec{U}_{\text{after}}\right| > \left|\vec{V}+\vec{U}_{\text{before}}\right|$$

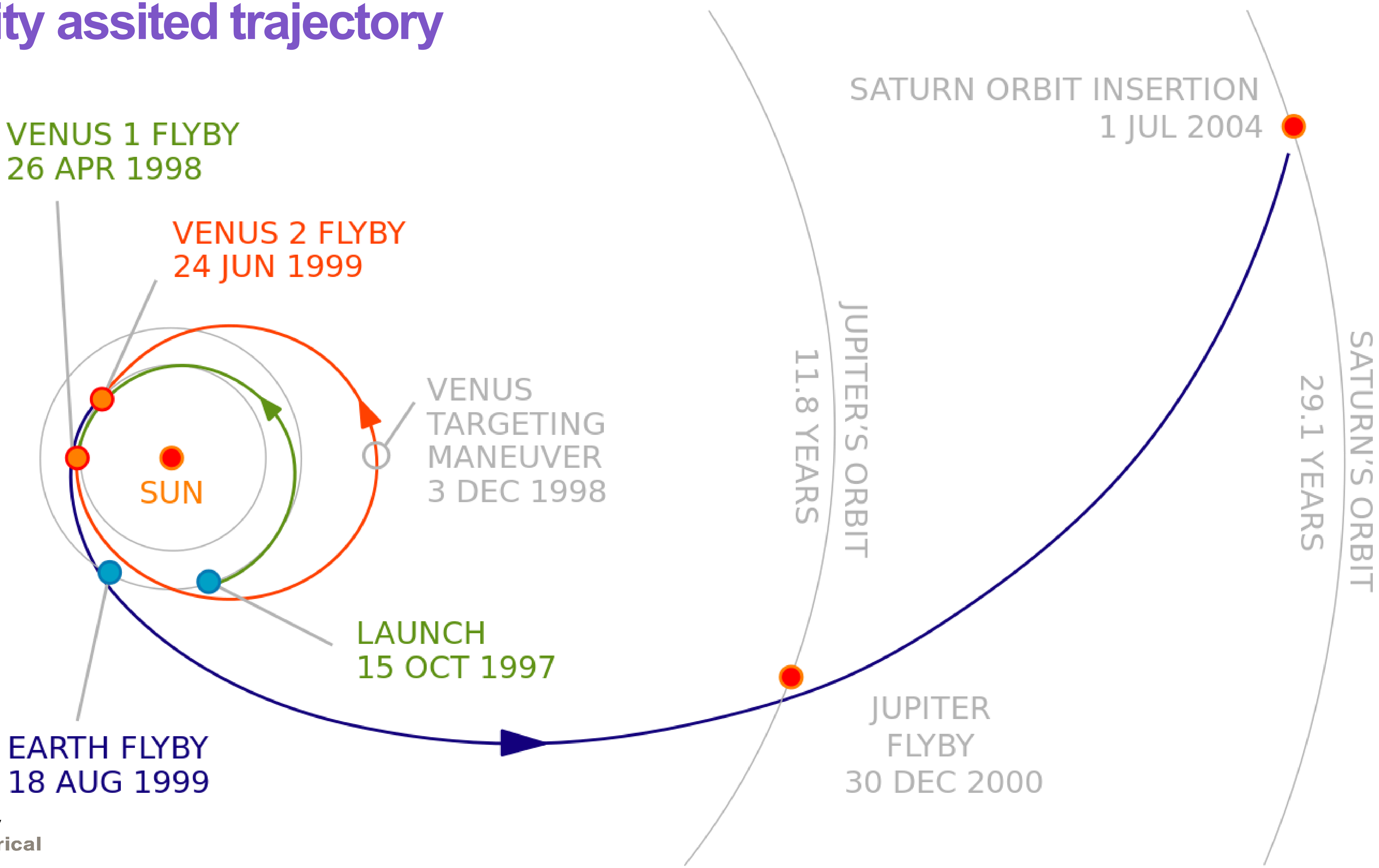


In the frame of the Sun:

To accelerate
take over the planet from behind

To decelerate
let the planet take over you

Cassini gravity assited trajectory



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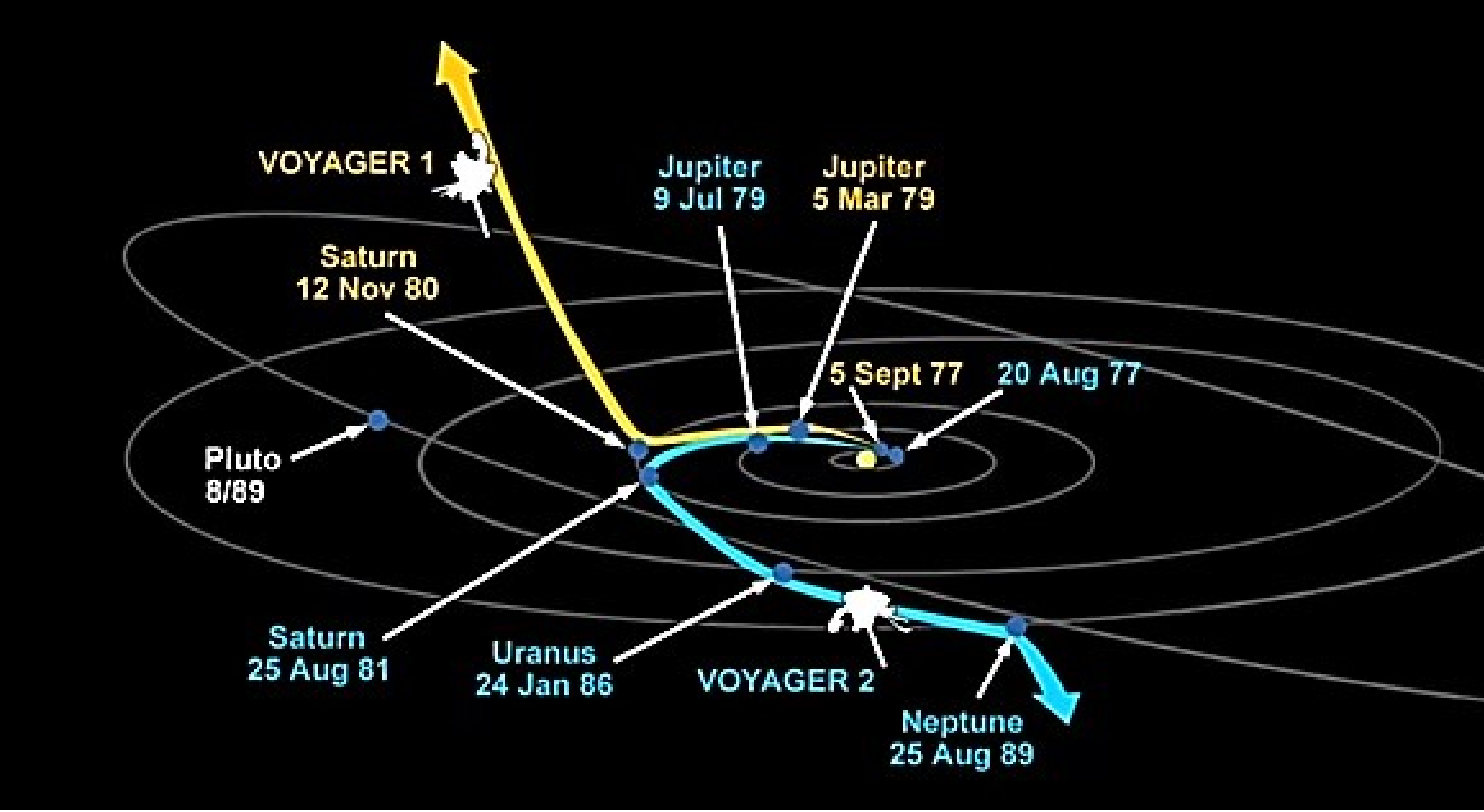
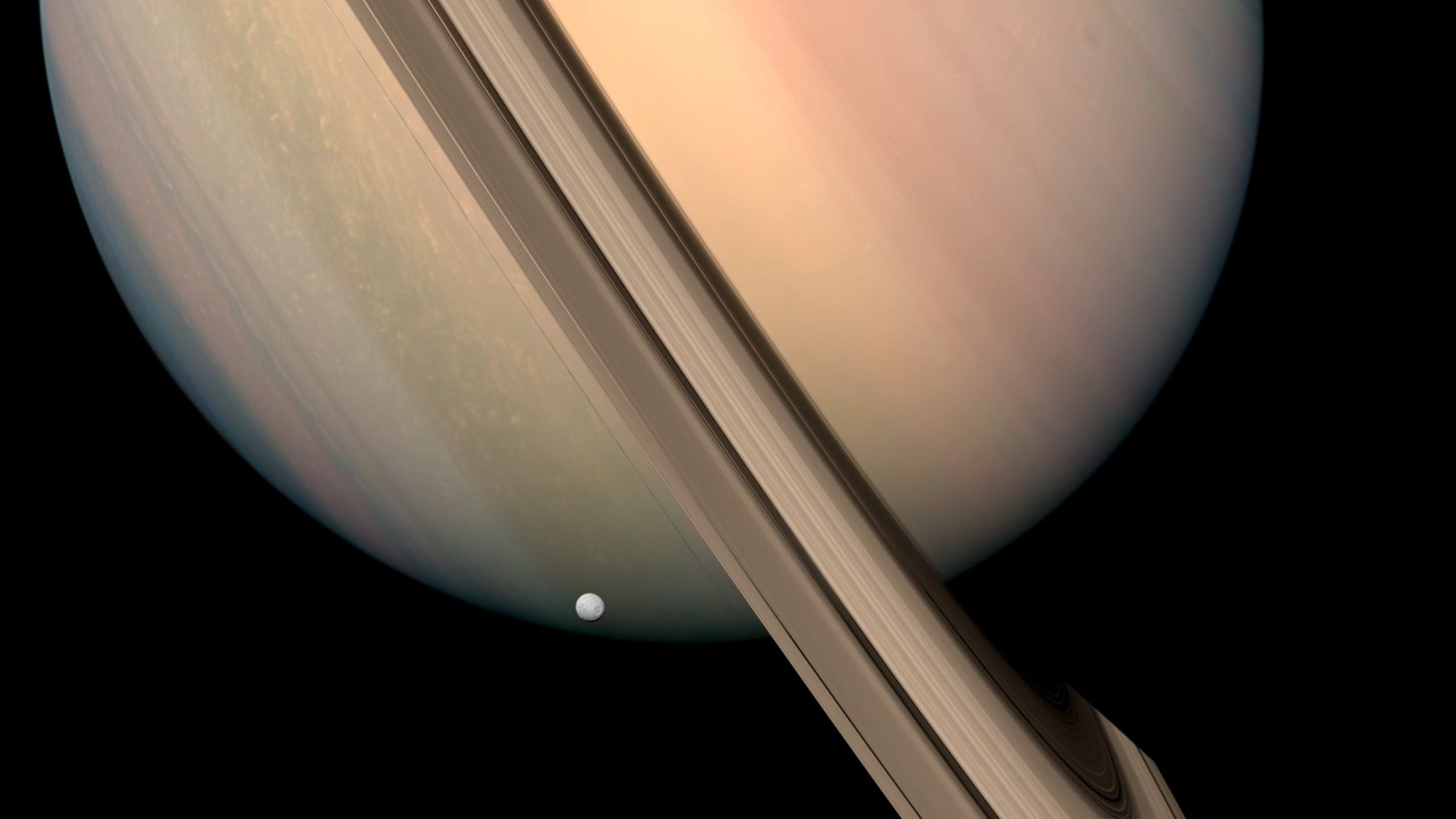

VOYAGER 1
Jupiter
9 Jul 79
Jupiter
5 Mar 79
Saturn
12 Nov 80
5 Sept 77
20 Aug 77
Pluto
8/89
Saturn
25 Aug 81
Uranus
24 Jan 86
VOYAGER 2
Neptune
25 Aug 89



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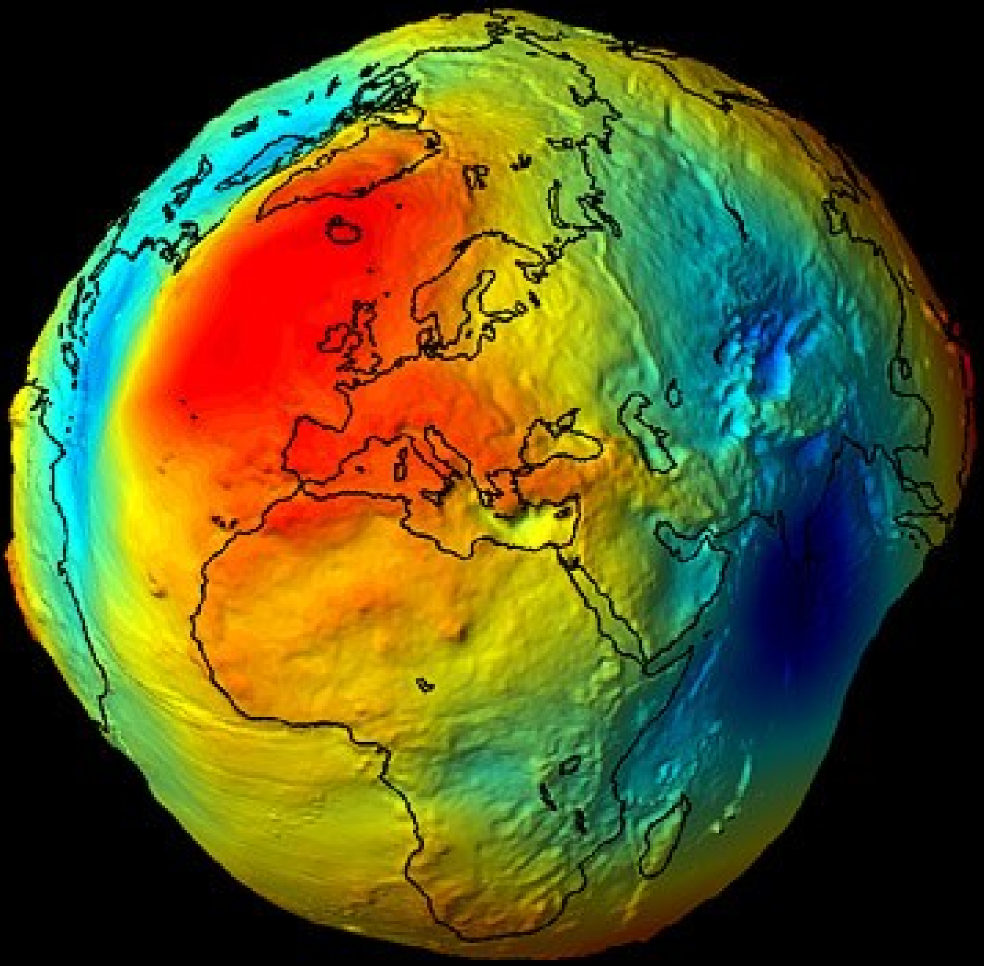
Orbits around real celestial body

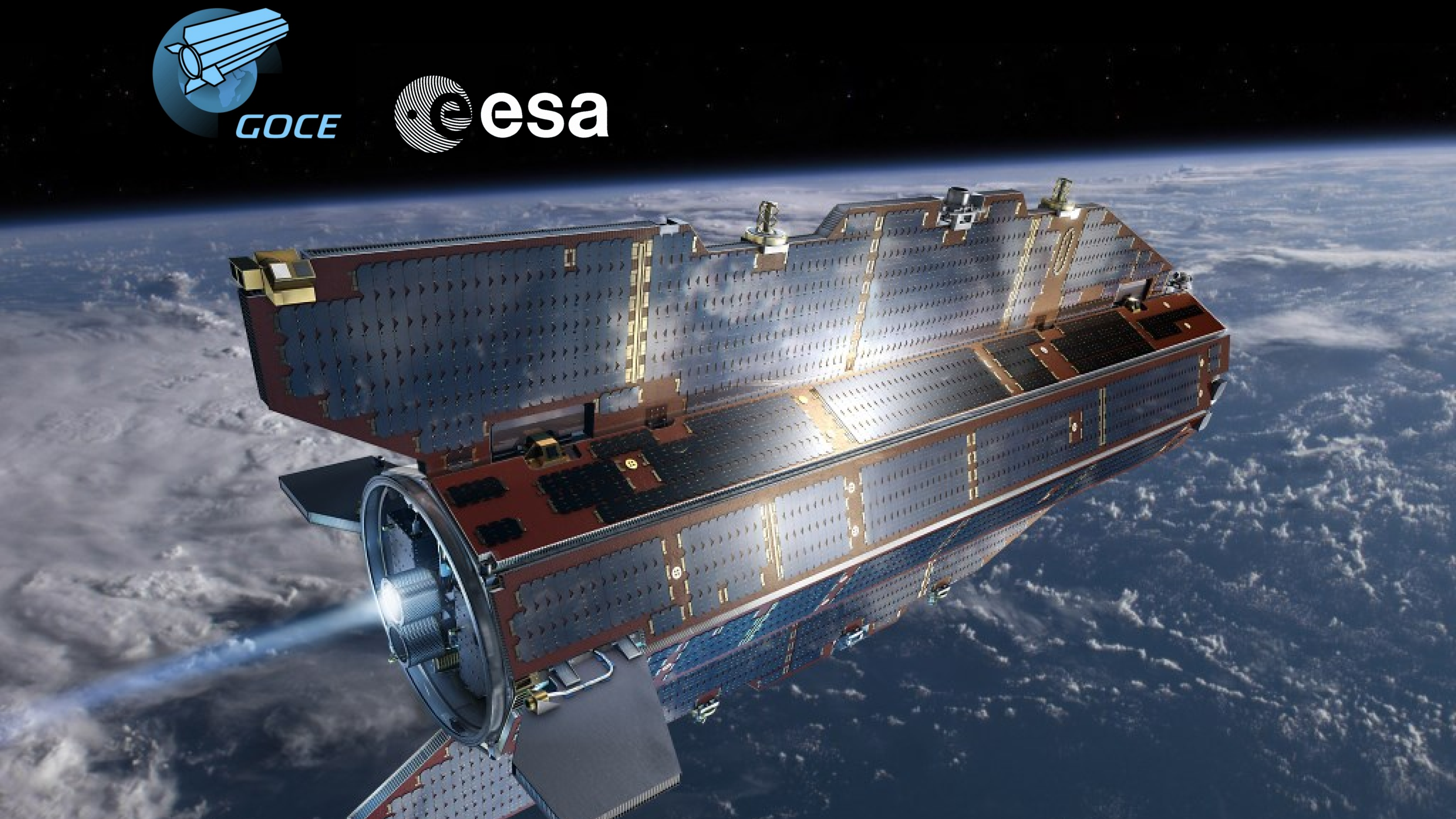
Potato #345

Kevin Abosch (2010)

Earth gravity field measured by GOCE



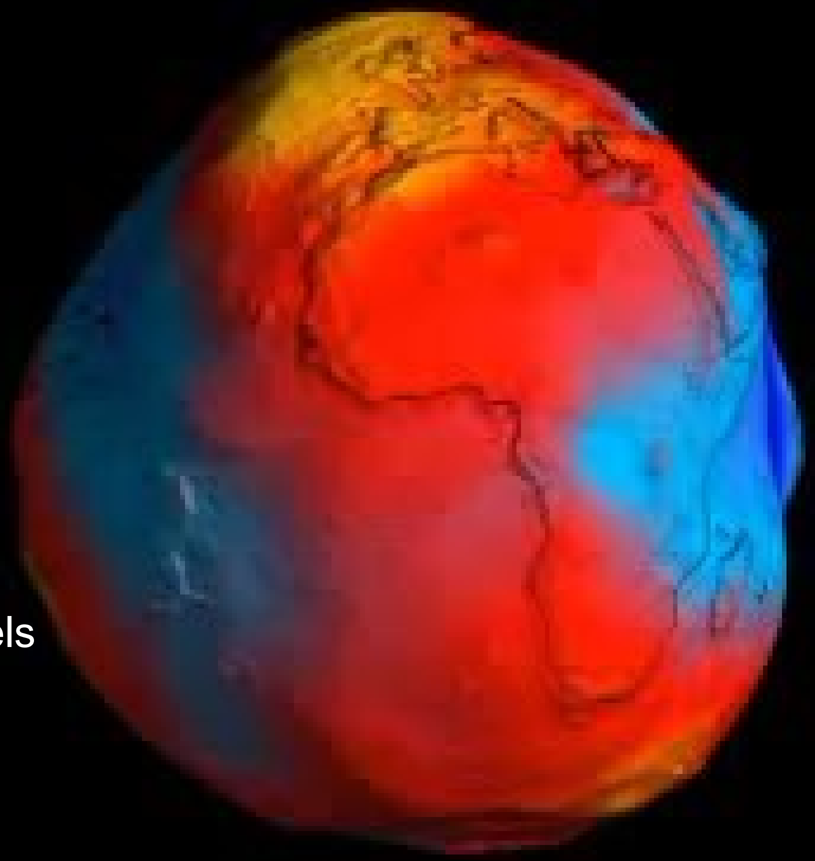
esa


GOCE
esa

Perturbations to orbits

The two-body approach is a theorist's dream
Other effective forces arise from:

- atmospheric drag
  - largest at perigee and makes an elliptical orbit more circular
  - steady deceleration on the circular orbit lowers the altitude and finally the satellites return to the atmosphere
- radiation pressure of the Sun
  - increased need for orbital corrections
  - telecommunication satellites on GEO have typically large solar panels
- inhomogeneous gravitational field
  - oblateness
  - uneven distribution of matter
- other celestial bodies
  - Moon, Jupiter



Gravitational perturbations

The gravitational potential can be represented as a spherical harmonic expansion

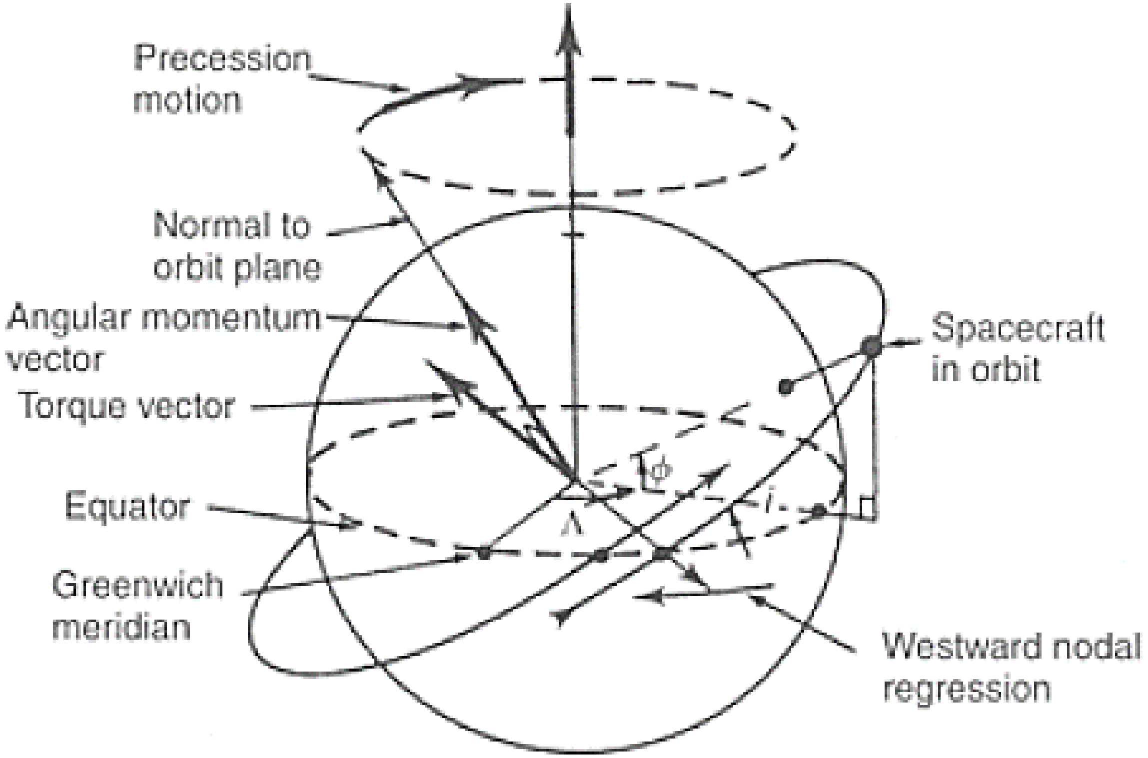
$$U(r,\Phi,\Lambda)=\frac{\mu}{r}\left\{-1+\sum_{n=2}^{\infty}\left[\left(\frac{R_E}{r}\right)^n J_n P_{n0}(\cos\Phi)+\sum_{m=1}^{n}\left(\frac{R_E}{r}\right)^n (C_{nm}\cos m\Lambda+S_{nm}\sin m\Lambda)P_{nm}(\cos\Phi)\right]\right\}$$

The most important contribution is due to the oblateness

- ➢ it turns the angular momentum vector
- ➢ the nodal line rotates

$$\Delta\Omega=-\frac{3\pi J_2 R_E^2}{p^2}\cos i\ \frac{\text{rad}}{\text{rev}}$$

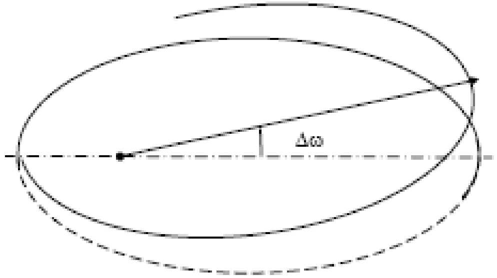
A shift of 360 deg / year results in a Sun-synchronous orbit



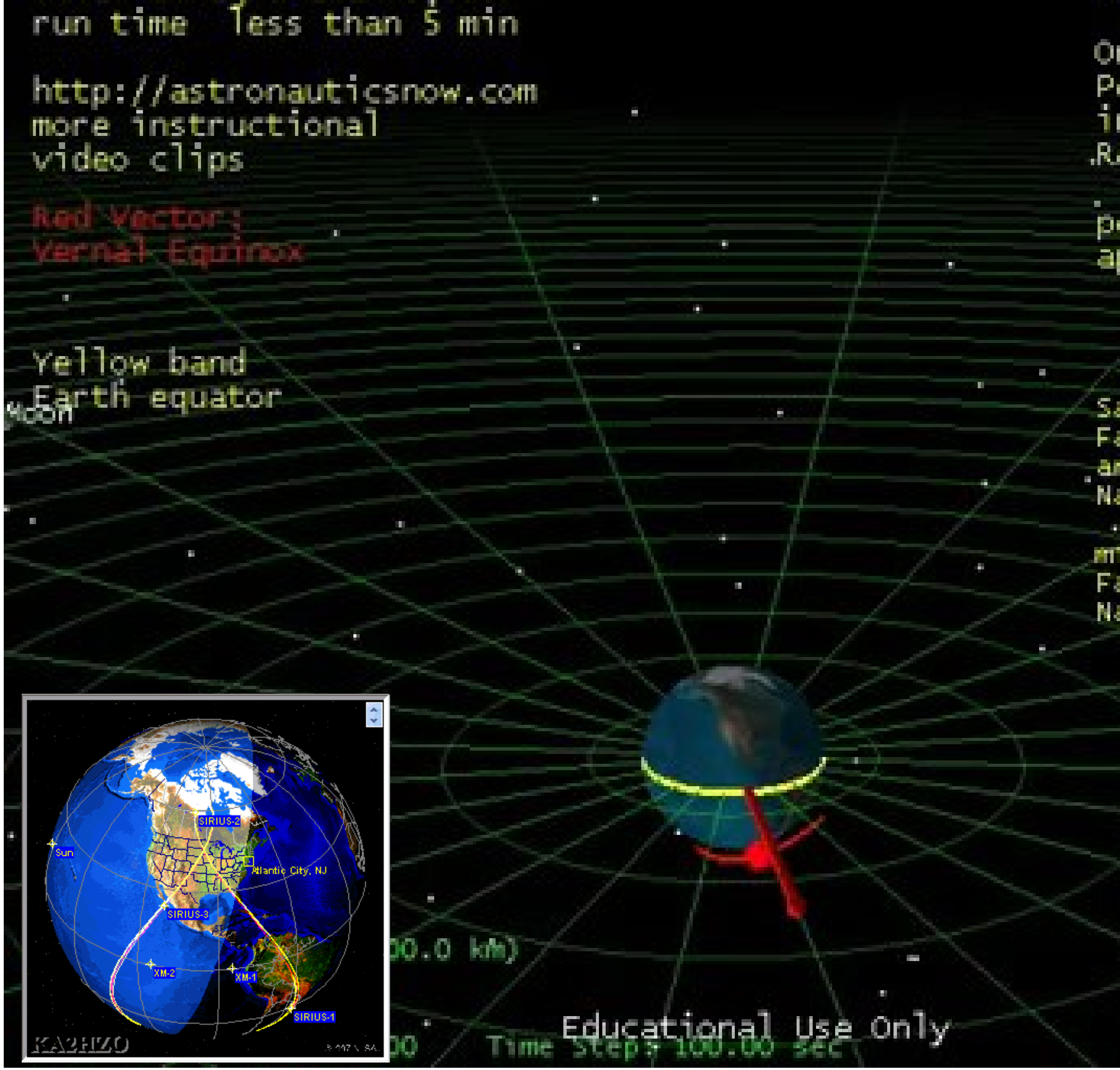
Gravitational perturbations

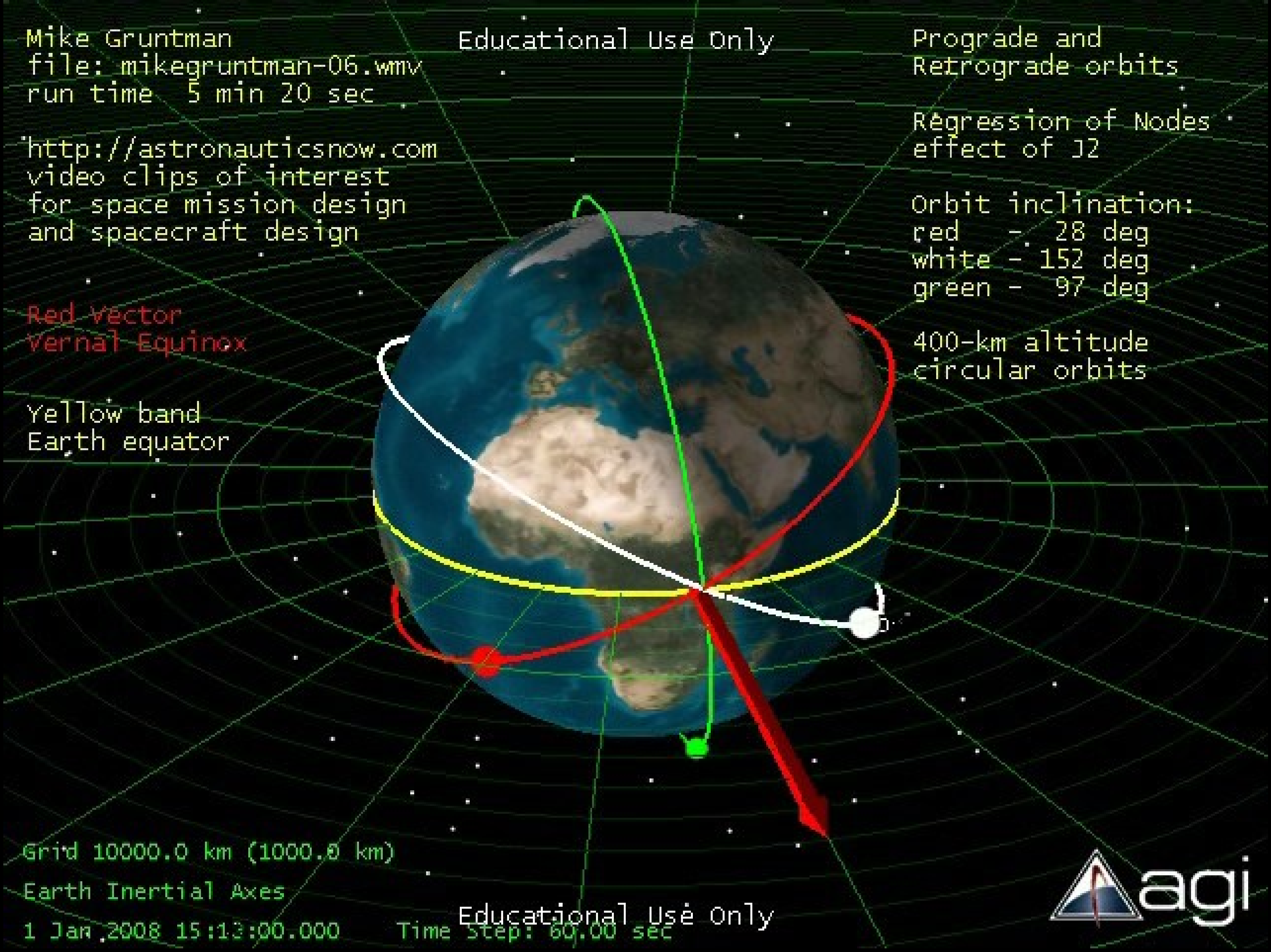
The oblateness also causes precession of the line of the apsides

$$\Delta\omega = 3\pi \frac{J_2 R_E^2}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right) \frac{\text{rad}}{\text{rev}}$$



Inclination of 63.4 deg is a special case: so-called **Molniya orbits**




Mike Gruntman
file: mikegruntman-06.wmv
run time 5 min 20 sec
http://astronauticsnow.com
video clips of interest
for space mission design
and spacecraft design
Red Vector
Vernal Equinox
Yellow band
Earth equator
Educational Use Only
Prograde and
Retrograde orbits
Regression of Nodes
effect of J2
Orbit inclination:
red - 28 deg
white - 152 deg
green - 97 deg
400-km altitude
circular orbits
Grid 10000.0 km (1000.0 km)
Earth Inertial Axes
1 Jan 2008 15:13:00.000
Time Step: 60.00 sec
Educational Use Only
agi

Effects of other celestial bodies
Three-body problem

For more than 2 bodies the equation of motion

$$m_i \ddot{\boldsymbol{r}}_i = -\sum_{j \neq i} \frac{G m_i m_j (\boldsymbol{r}_i - \boldsymbol{r}_j)}{|\boldsymbol{r}_i - \boldsymbol{r}_j|^3} \qquad i = 1, 2, \ldots, N$$

soon becomes intractable

Poincaré (ca. 1890): the system is very sensitive to initial conditions
➢ chaos (which was reinvented in the 1960s)

K. F. Sundman (1912): there is a unique solution for the 3-body problem
➢ a very slowly converging expansion in powers fo $t^{1/3}$
➢ impractical for calculations in celestial mechanics

Reduced three-body problem Lagrange points

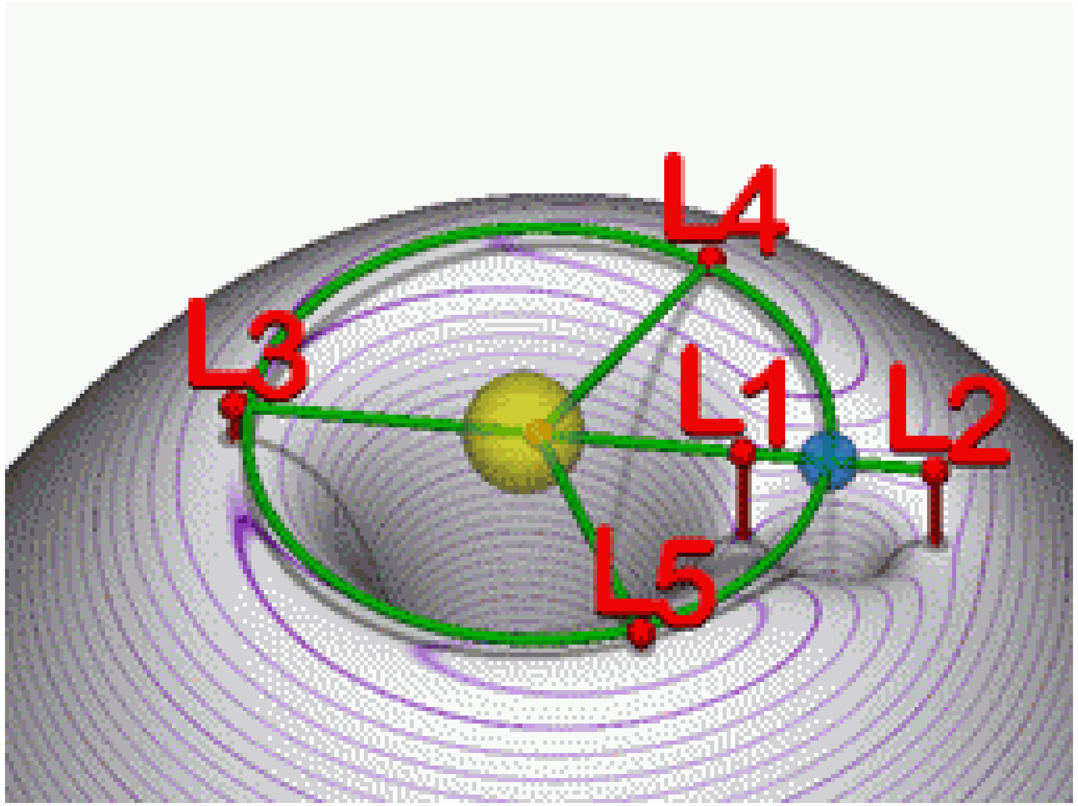
Two large and one small body

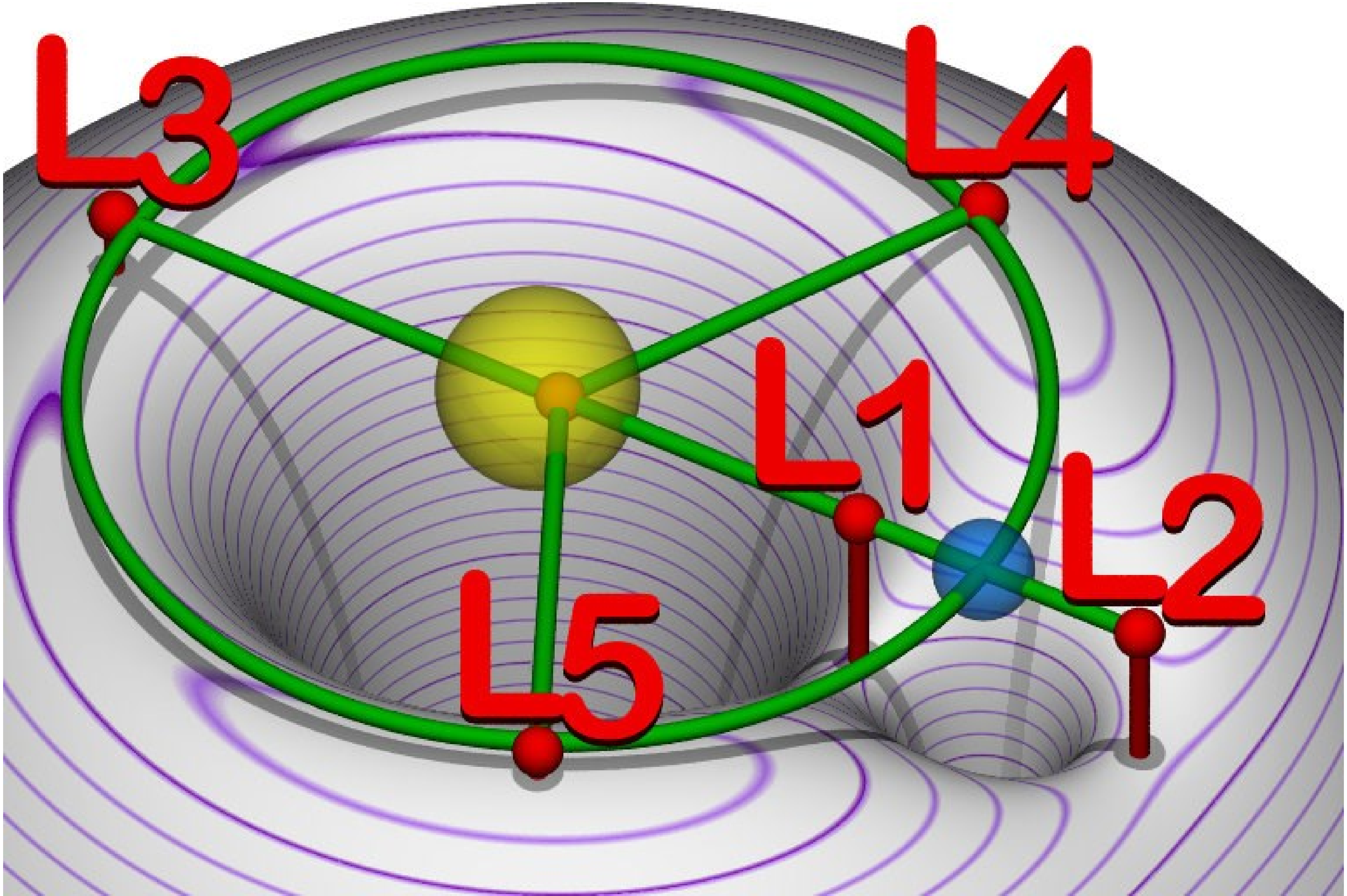
- solve the 2-body problem for the large masses
- consider the motion of the small body in the gravitational potential of the large ones.

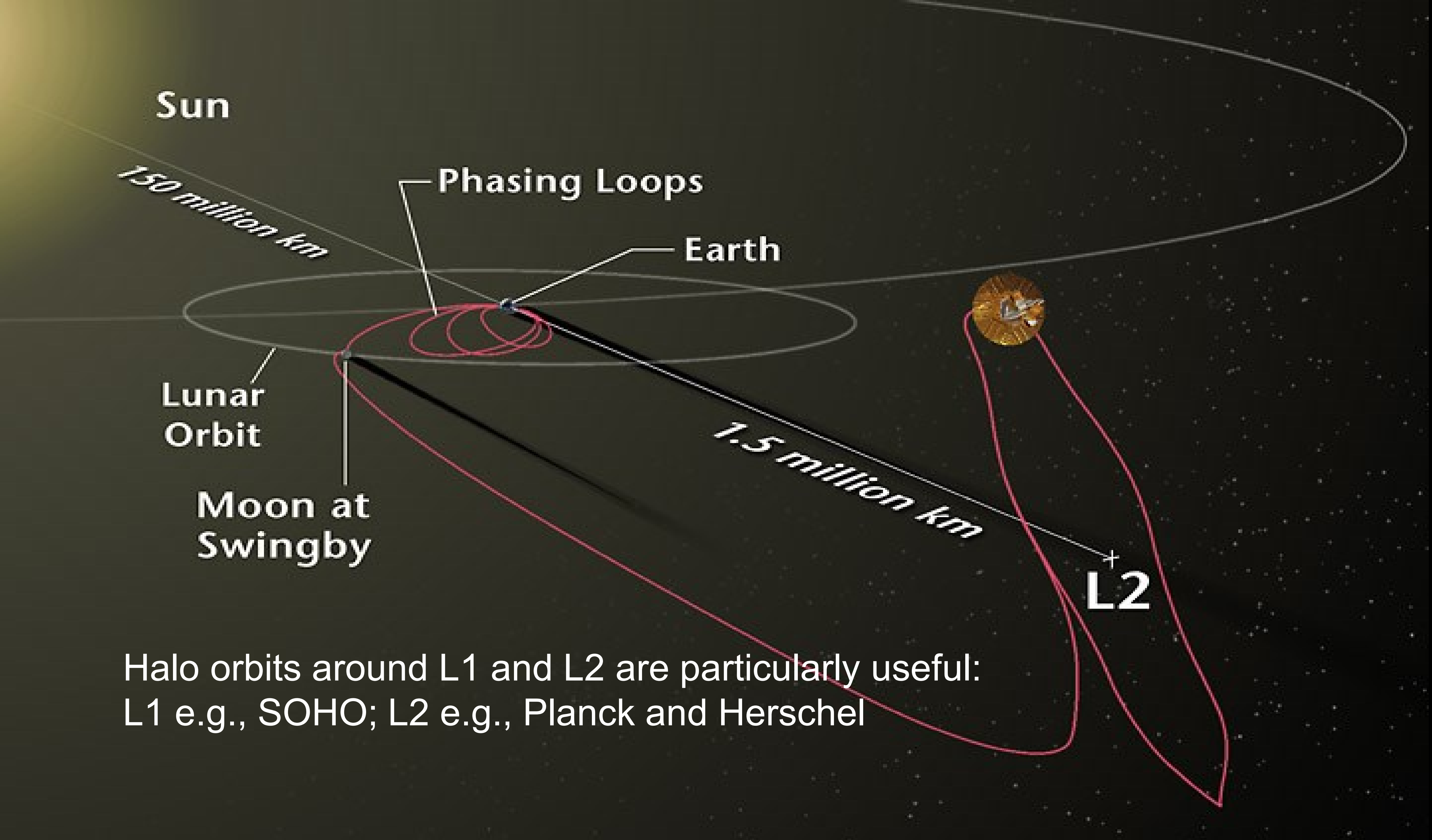
Does not work in the Sun-Earth-Moon system

but OK for Sun-Earth-spacecraft

Five Lagrange points

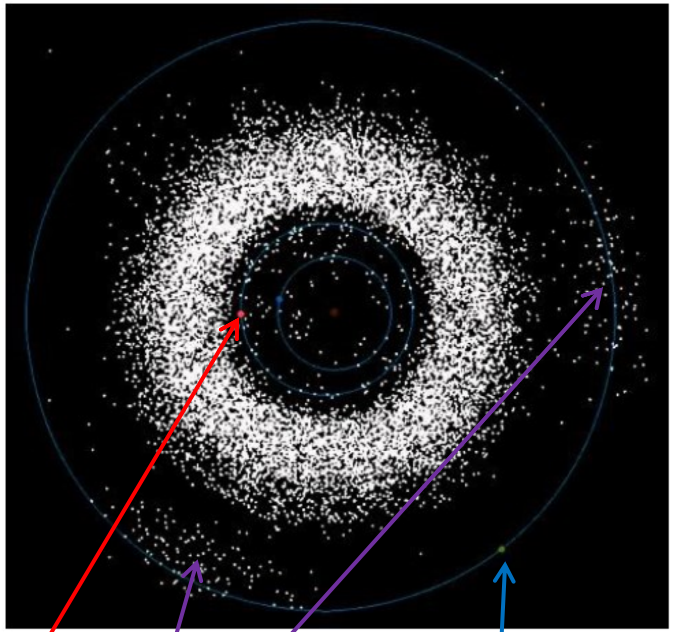



L3
L4
L1
L2
L5



Halo orbits around L1 and L2 are particularly useful:
L1 e.g., SOHO; L2 e.g., Planck and Herschel

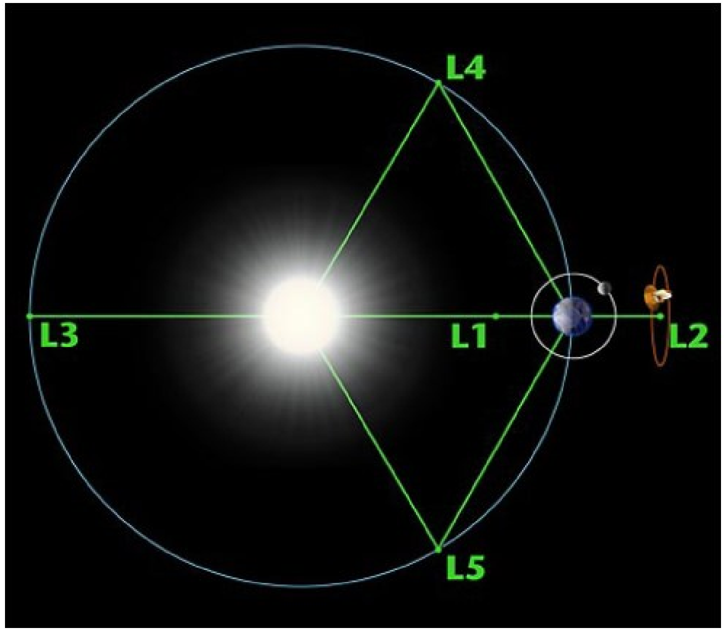
Good to know about Lagrange points



Mars

Jupiter

Troyan asteroids around the Sun–Jupiter system's L4 and L5



People often have two misconseptions:

- L1 is NOT the point where gravitational forces of the Sun and the Earth balance each other
- How can a S/C stay around L2?

These are three-body problems!!

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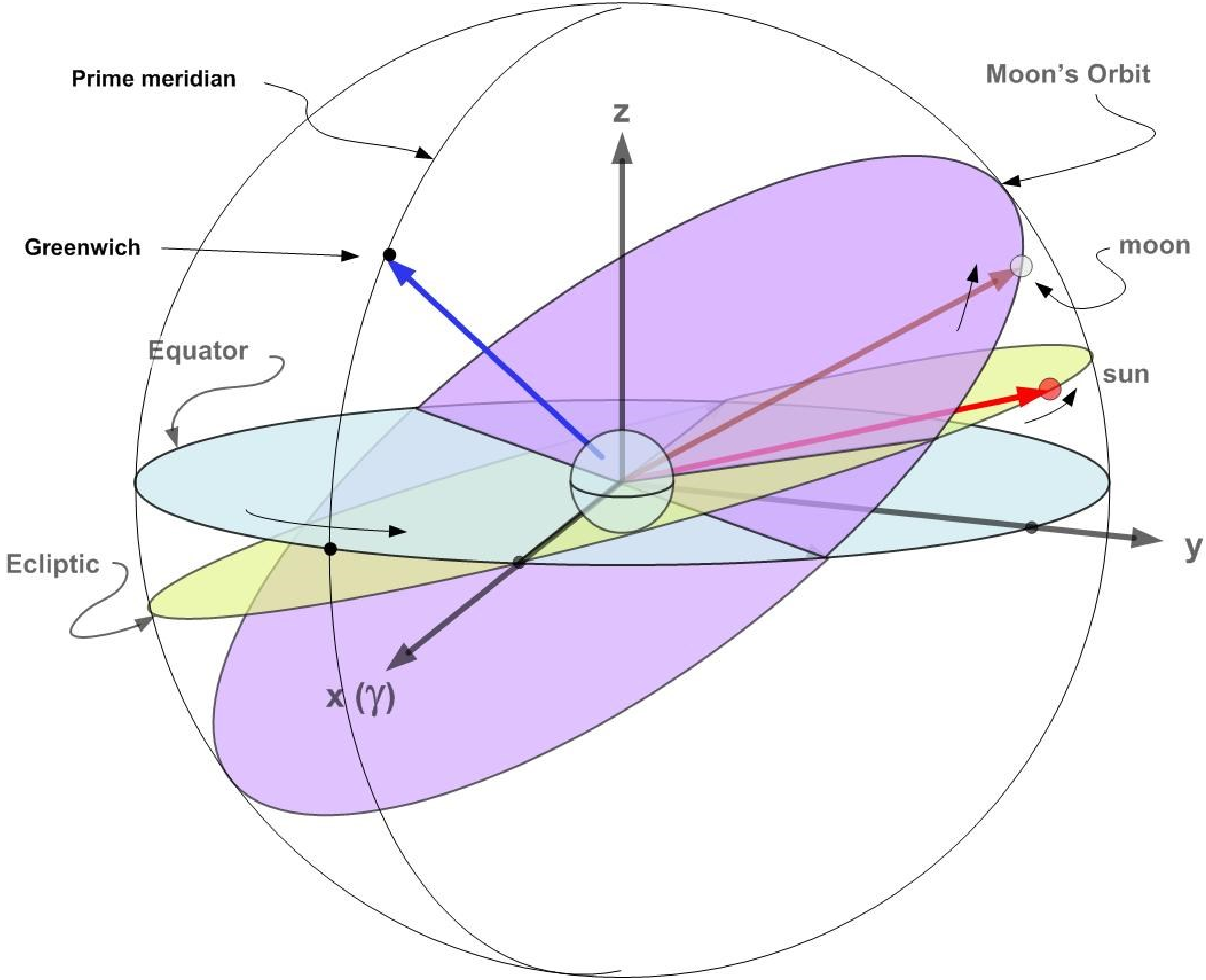
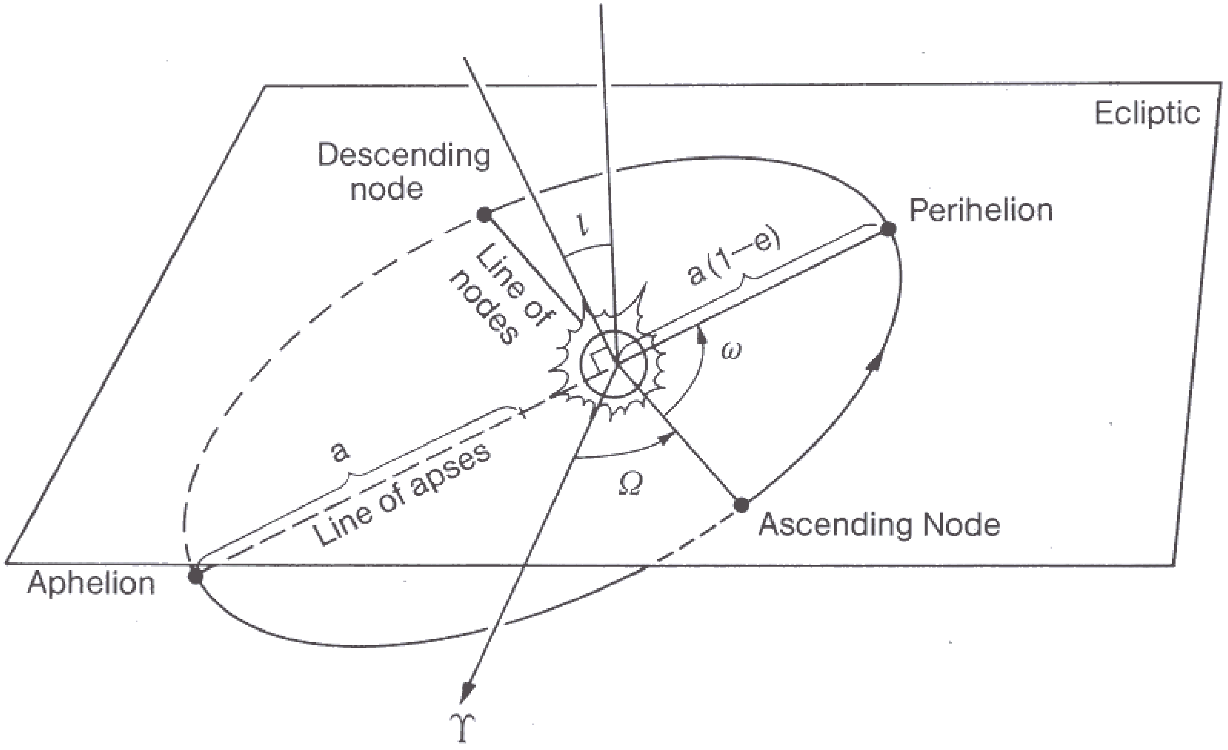
Describing orbital motion


Prime meridian
Moon's Orbit
z
Greenwich
moon
Equator
sun
y
Ecliptic
x (γ)

Orbital elements: Astronomer’s view

1. Semi major axis a
2. Eccentricity ε (or e)
3. Inclination i (or ι)
4. Right ascencion of the ascending node Ω (from vernal equinox ¡)
5. Argument of perihelion ω; or length of the perihelion $\varpi = \Omega + \omega$
6. Perihelion time τ



Orbital elements: Satellite operator's view

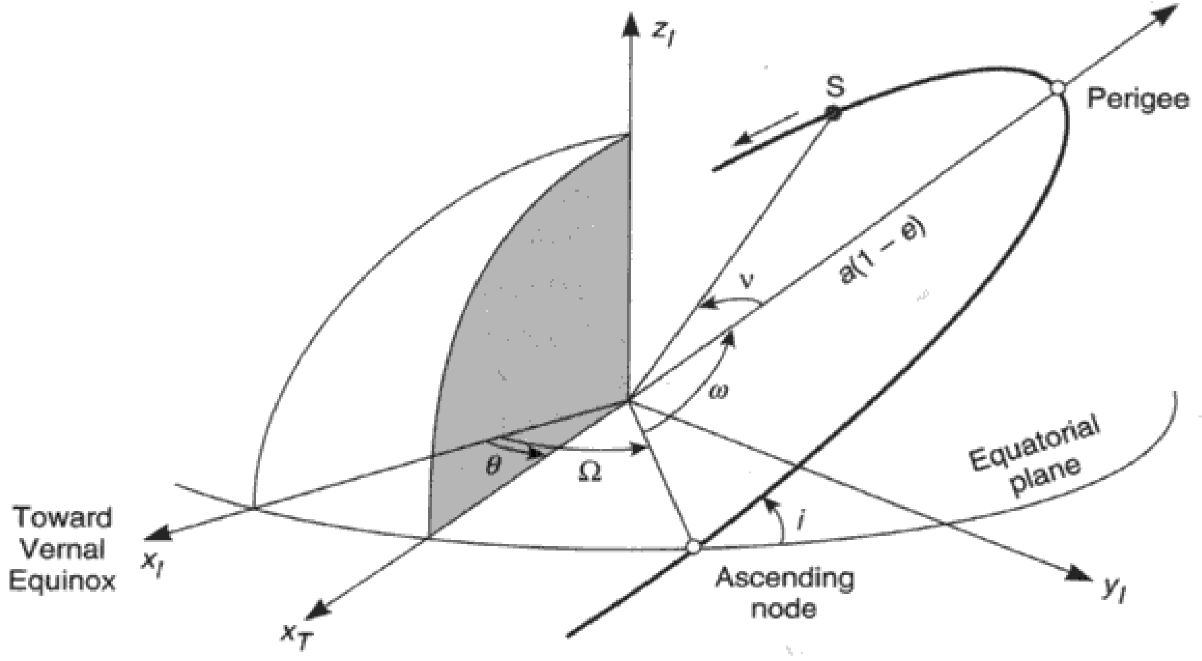


Figure 3.11 Characterization of an ideal orbit and the satellite position by Keplerian elements: $\{a, e, i, \Omega, \omega, \text{and } \nu\}$.

Also other sets of elements are used; the number of independent elements is always 6

Orbital Elements

Semi-major axis a **(size of the ellipse)**

- the longest axis of the ellipse going through the two foci.

Eccentricity e **(oblateness of the ellipse)**

- the elongation of the ellipse

Inclination of the orbit i **(orbit position relative to equator)**

- the orbit plane is tipped relative to the reference equatorial plane.

Argument of Perigee ω **(place of the perigee)**

- an angle in the orbital plane between the ascending node and peri-apsis, measured in the direction of the satellite's motion

Right ascension of the ascending node Ω **(ellipse rotation around poles)**

- the angle measured to the point where the orbit crosses the equatorial plane relative to a reference direction known as the vernal equinox.

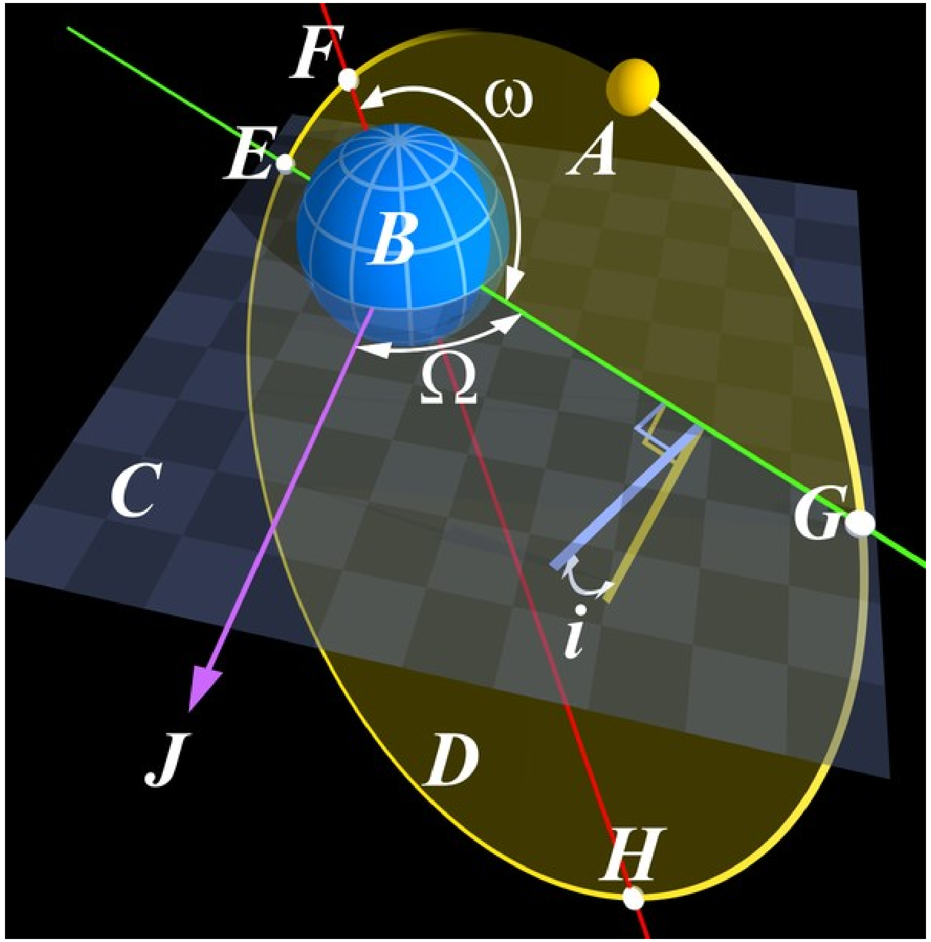
True anomaly ν **(place of the satellite on orbit)**

- the angular distance of a point in an orbit past the point of peri-apsis, measured in degrees

Parameters of Elliptical Orbit

Legend:

- A – Minor, orbiting body
- B – Major body being orbited by A
- C – Reference plane, e.g. the
- D – Orbital plane of A
- E – Descending node
- F – Periapsis
- G – Ascending node
- H – Apoapsis
- i – Inclination
- J – Reference direction; for orbits in or near the ecliptic, usually the vernial point
- Ω – Longitude of the ascending node
- ω – Argument of the periapsis
- The red line is the line of apsides; going through the periapsis (F) and apoapsis (H); this line coincides with the major axis in the elliptical shape of the orbit
- The green line is the node line; going through the ascending (G) and descending node (E); this is where the reference plane (C) intersects the orbital plane (D).



[Source: Wikipedia]

Source: www.wikipedia.org

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Artificial satellites around the Earth

Satellite oribit classification

LEO: low Earth orbit

- lowest stable orbit at about 180 km
- often polar oribits
- Earth observation and military satellites

MEO: medium altitudes

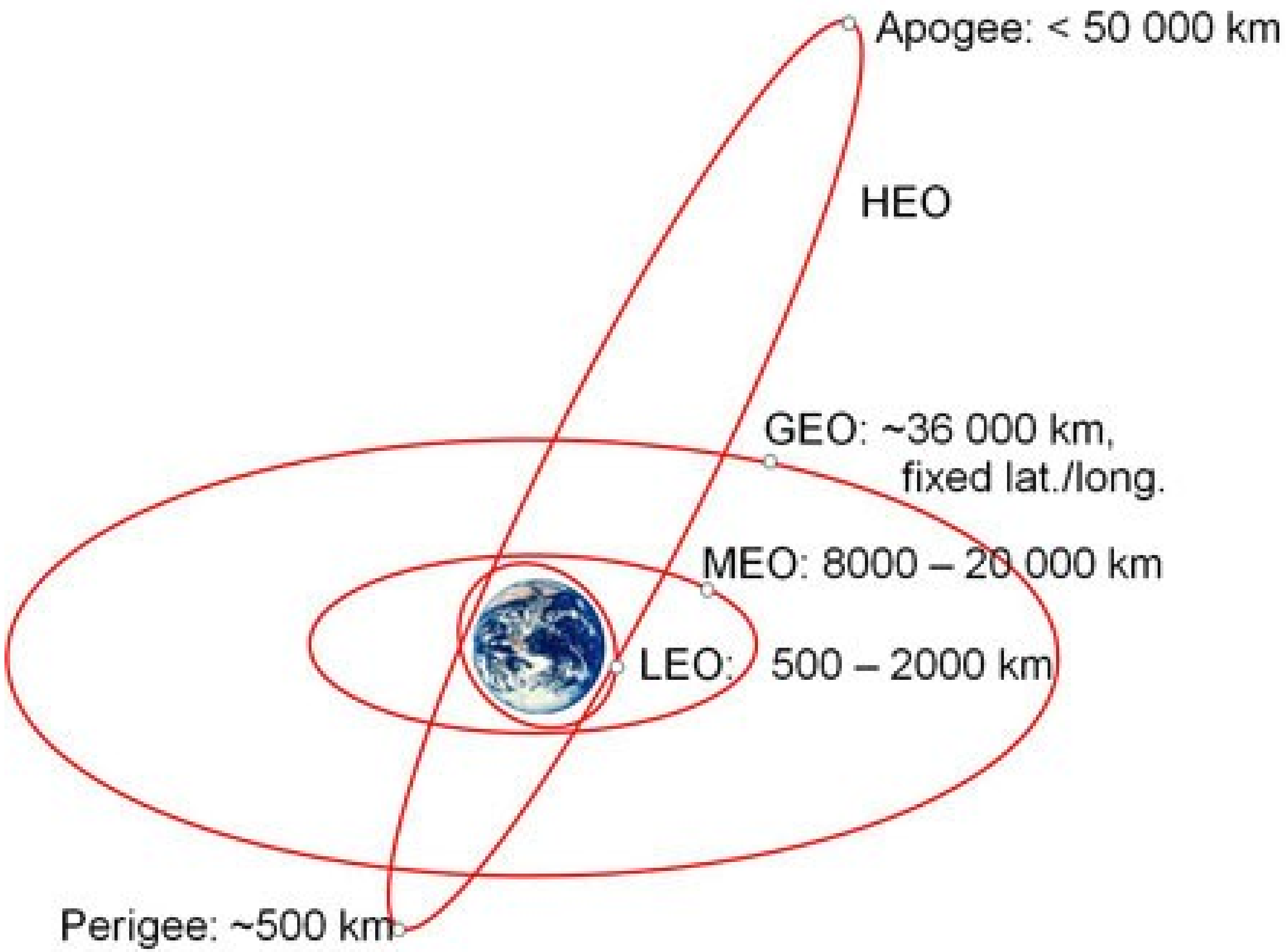
- e.g. GPS and Galileo

GEO: geostationary orbit

- Altitude: 35786 km, geocentric distance 6.6 RE

HEO: highly elliptical orbit

- Apogee above GEO
- some scientific S/C
- Molnyia



Types of Orbits

Altitude classifications

- ***Low Earth orbit (LEO):*** Geocentric orbits with altitudes up to 2,000 km
- ***Medium Earth orbit (MEO)***: Geocentric orbits ranging in altitude from 2,000 km to just below geosynchronous orbit at 35,786 km.
- ***High Earth orbit***: Geocentric orbits above the altitude of geosynchronous orbit 35,786 km

Centric classifications

- ***Galactocentric orbit***: An orbit about the center of a galaxy. The Sun follows this type of orbit
- ***Heliocentric orbit***: An orbit around the Sun. In our Solar System, all planets, comets, and asteroids are in such orbits, as are many artificial satellites and pieces of space debris. Moons by contrast are not in a heliocentric orbit but rather orbit their parent planet.
- ***Geocentric orbit***: An orbit around the planet Earth, such as that of the Moon or of artificial satellites.

Types of Orbits

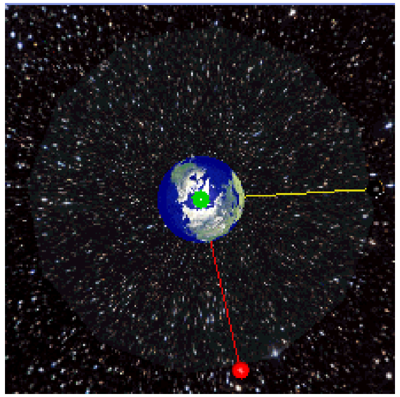
Eccentricity classifications

- There are two types of orbits: ***closed (periodic) orbits***, and ***open (escape) orbits***. Circular and elliptical orbits are closed. Parabolic and hyperbolic orbits are open.
- ***Circular orbit***: An orbit that has an eccentricity of 0 and whose path traces a circle.
- ***Elliptic orbit***: An orbit with an eccentricity greater than 0 and less than 1 whose orbit traces the path of an ellipse.
- ***Parabolic orbit*:** An orbit with the eccentricity equal to 1. Such an orbit also has a velocity equal to the escape velocity and therefore will escape the gravitational pull of the planet. If the speed of a parabolic orbit is increased it will become a ***hyperbolic orbit***.

Inclination classifications

- ***Inclined orbit***: An orbit whose inclination in reference to the equatorial plane is not 0.
- ***Non-inclined orbit***: An orbit whose inclination is equal to zero with respect to some plane of reference.

Types of Orbits



Synchronicity classifications

Synchronous orbit:

An orbit whose period is a rational multiple of the average rotational period of the body being orbited and in the same direction of rotation as that body. This means the track of the satellite, as seen from the central body, will repeat exactly after a fixed number of orbits.

- ***Geosynchronous orbit (GSO):*** An orbit around the Earth with a period equal to one sidereal day, which is Earth's average rotational period of 23 hours, 56 minutes, 4.091 seconds. For a nearly circular orbit, this implies an altitude of approximately 35,786 km. The orbit's inclination and eccentricity may not necessarily be zero.
- **Geostationary orbit (GEO):** A circular geosynchronous orbit with an inclination of zero. To an observer on the ground this satellite appears as a fixed point in the sky

Two Line Elements (TLE)

```
ISS (ZARYA)
1 25544U 98067A   14260.21469767  .00018739  00000-0  33124-3 0   989
2 25544  51.6477   3.4590 0002230 112.1112 330.1591 15.50366882905605
TIANGONG 1
1 37820U 11053A   14260.09847079  .00037692  00000-0  34424-3 0  3932
2 37820  42.7690 118.2614 0007048 254.3650 242.2970 15.67126632170471
FLOCK 1-12
1 39528U 98067DT  14260.08687349  .00182287  00000-0  90427-3 0  6018
2 39528  51.6355 341.6772 0005790 353.2487   6.8436 15.80898745 33599
FLOCK 1-18
1 39556U 98067DZ  14260.09390708  .00171334  00000-0  89242-3 0  5596
2 39556  51.6403 343.2669 0005652 358.9003   1.1985 15.79867661 31675
FLOCK 1-20
```

<u>North American Aerospace Defense Command</u>

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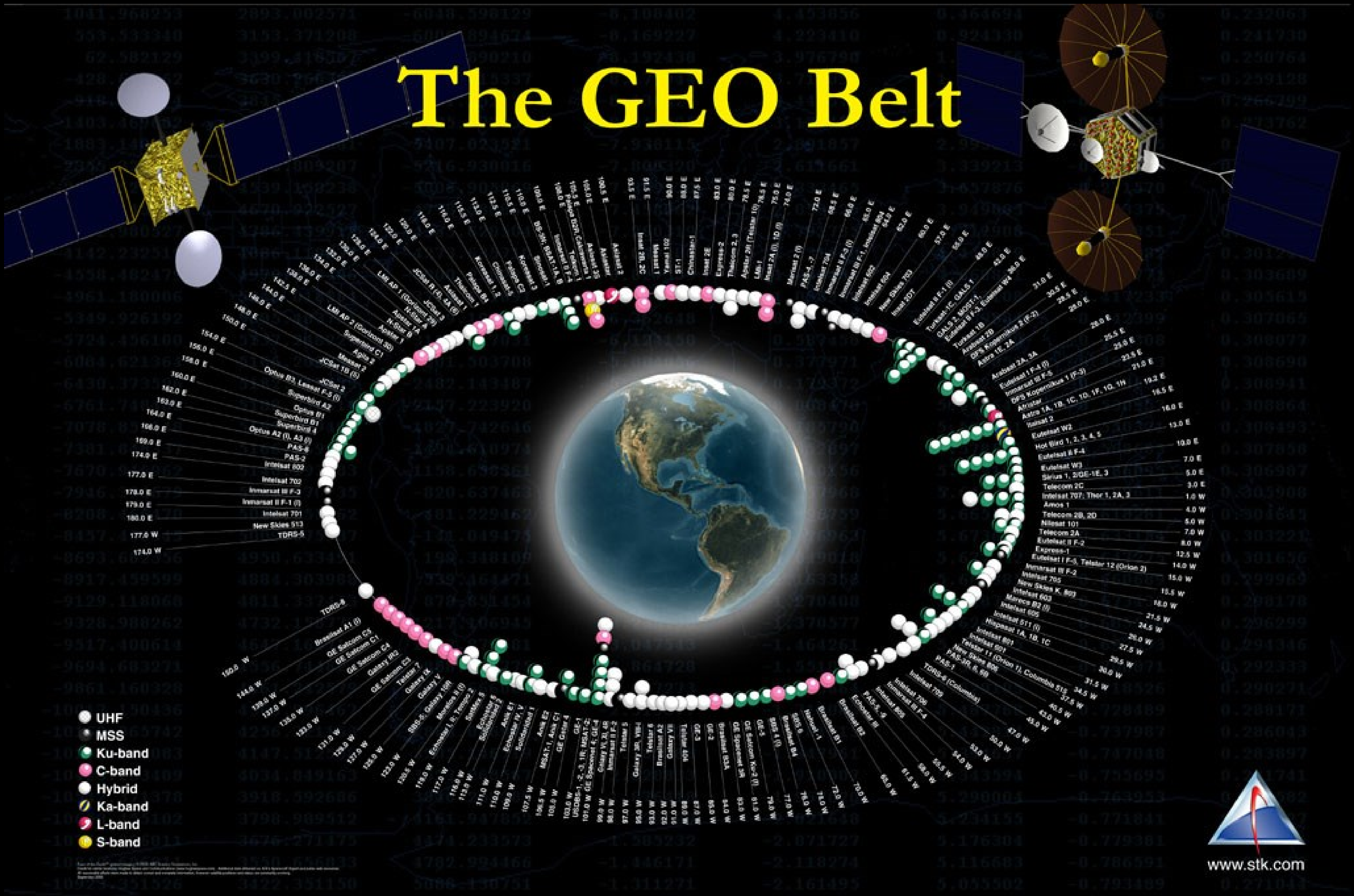
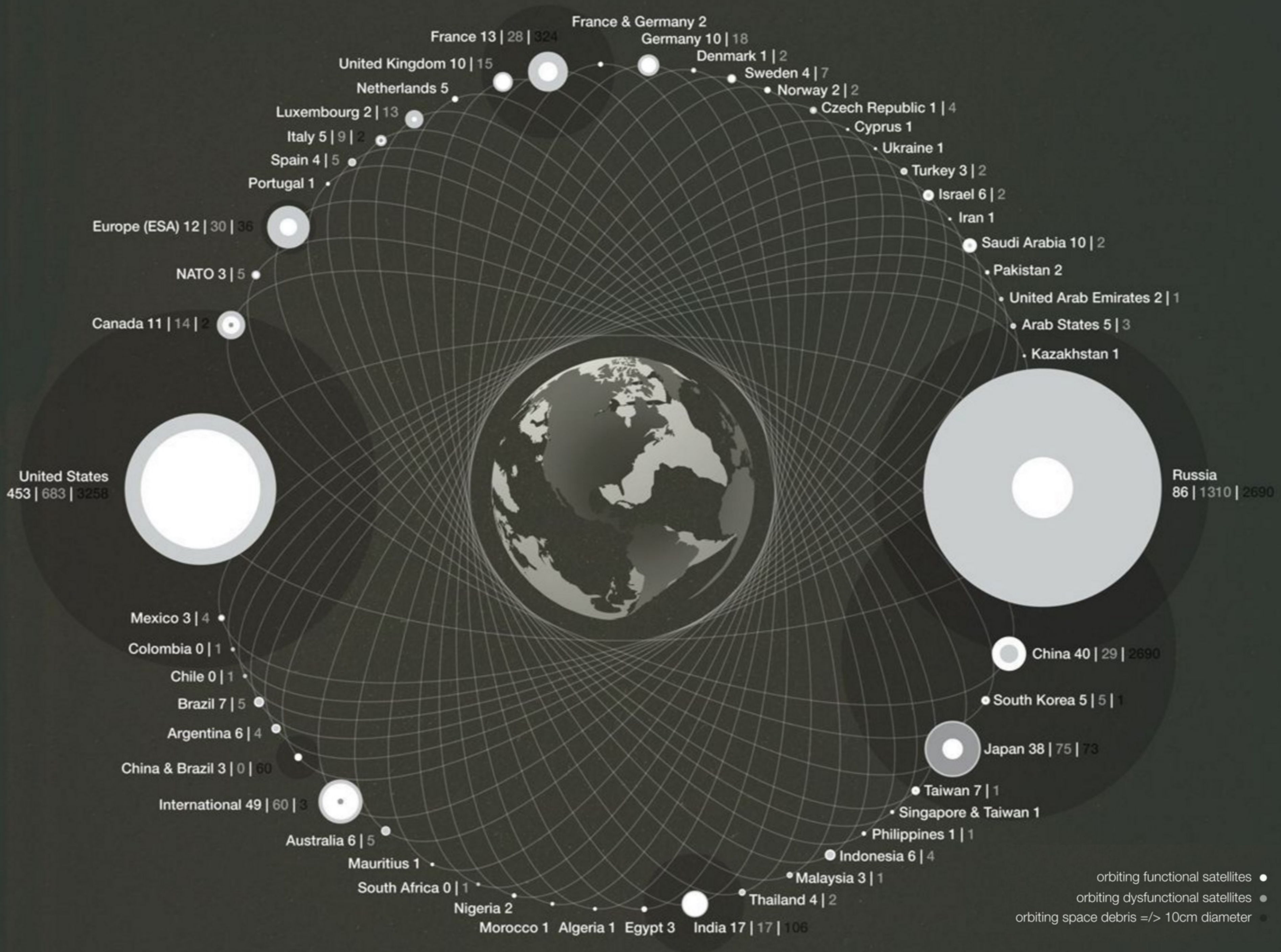
SPACE
JUNK
'Urgent need'
to remove space debris



The GEO Belt
UHF
MSS
Ku-band
C-band
Hybrid
Ka-band
L-band
S-band
www.stk.com


France & Germany 2
France 13 | 28 | 324
Germany 10 | 18
United Kingdom 10 | 15
Denmark 1 | 2
Sweden 4 | 7
Netherlands 5
Norway 2 | 2
Luxembourg 2 | 13
Czech Republic 1 | 4
Italy 5 | 9 | 2
Cyprus 1
Ukraine 1
Spain 4 | 5
Turkey 3 | 2
Portugal 1
Israel 6 | 2
Europe (ESA) 12 | 30 | 36
Iran 1
Saudi Arabia 10 | 2
NATO 3 | 5
Pakistan 2
United Arab Emirates 2 | 1
Canada 11 | 14 | 2
Arab States 5 | 3
Kazakhstan 1
United States
453 | 683 | 3258
Russia
86 | 1310 | 2690
Mexico 3 | 4
Colombia 0 | 1
China 40 | 29 | 2690
Chile 0 | 1
South Korea 5 | 5 | 1
Brazil 7 | 5
Argentina 6 | 4
Japan 38 | 75 | 73
China & Brazil 3 | 0 | 60
Taiwan 7 | 1
International 49 | 60 | 3
Singapore & Taiwan 1
Philippines 1 | 1
Australia 6 | 5
Indonesia 6 | 4
Mauritius 1
Malaysia 3 | 1
South Africa 0 | 1
Thailand 4 | 2
Nigeria 2
Morocco 1
Algeria 1
Egypt 3
India 17 | 17 | 106
orbiting functional satellites
orbiting dysfunctional satellites
orbiting space debris =/> 10cm diameter

WASTE IN SPACE

Currently, a thick band of levitating space junk—composed primarily of broken satellite pieces and discarded rocket boosters—skirts the Earth. Two or three times a day, a satellite circling our planet narrowly misses a torrent of the orbital debris. This phenomenon has jeopardized not only current space travelers, but future missions as well.

WHAT IS SPACE DEBRIS?

Nonfunctional, human-made materials in orbit caused by everything from spent booster stages to satellite collisions and explosions.

73%

of tracked debris reside in low-Earth orbit (LEO), 1,200 miles above our planet's surface.

HOW MUCH SPACE JUNK IS UP THERE?

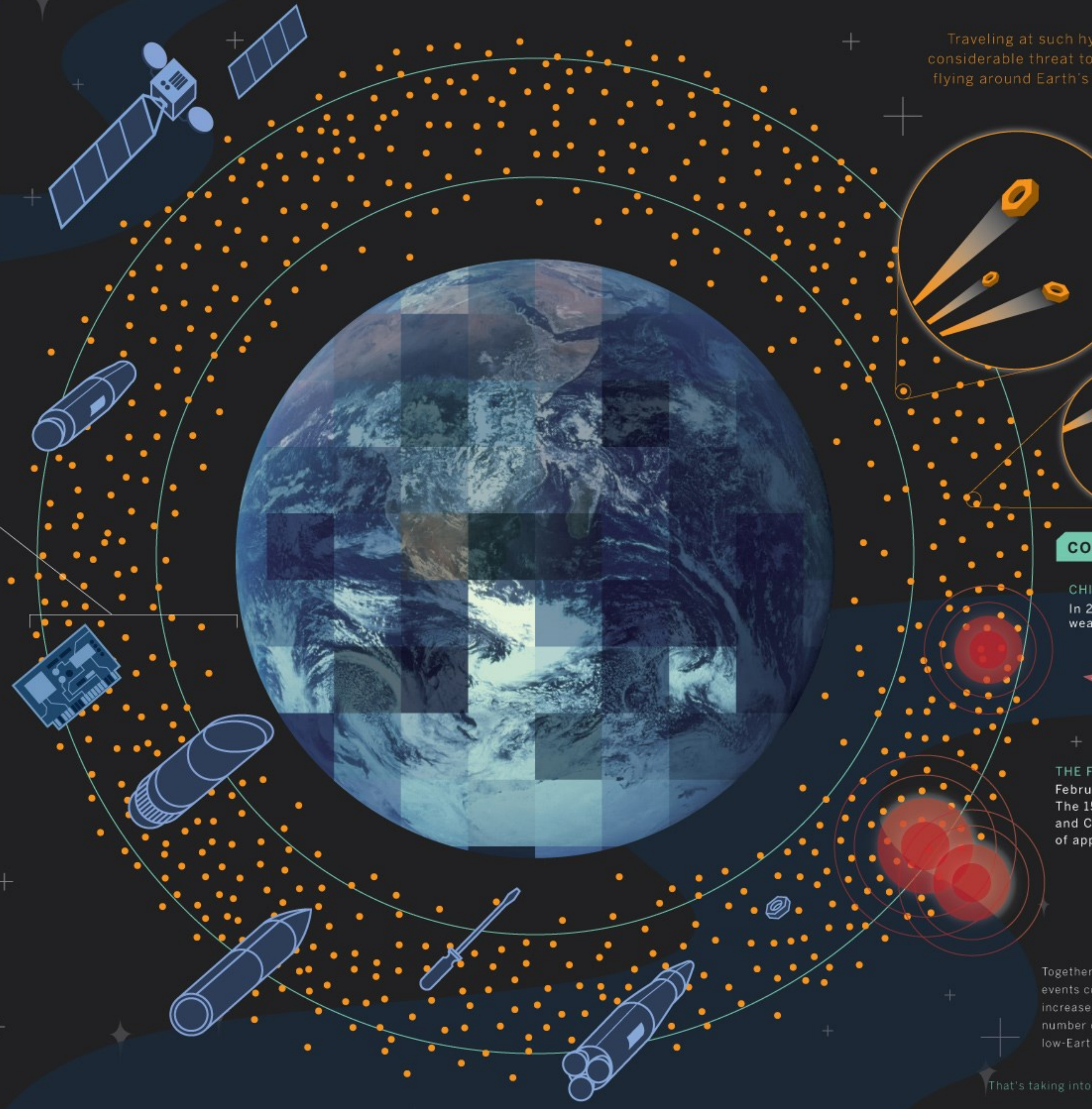
The amount of space debris larger than four inches in diameter in Earth's orbit being tracked by the U.S. Space Surveillance Network:

More than **21,000** objects = 4 inches

500,000 objects

Estimated amount larger than one centimeter in diameter—or the size of a marble.

There are another tens of millions of paint chip-like pieces that measure smaller than a centimeter.



WHY IT'S A SERIOUS PROBLEM

Traveling at such hyper velocities, any particle of space junk presents a considerable threat to spaceflight for any nation. And with more hardware flying around Earth's orbit, the potential of collisions between spacecraft and large orbital trash only continues to grow.

FASTER THAN THE SPEED OF SOUND

The speed of sound travels at approximately **768 mph** on a normal day.

In order to remain in orbit, the fragments in space have to move along at least **20 times that speed**, and can go up to almost

18,000 mph.

TOO CLOSE FOR COMFORT

LESS THAN 5 MILES

About 1,000 times a day, satellites and debris pass less than 5 miles from each other. Considering how expansive space is, this distance is striking.

COLLISIONS & EXPLOSIONS INCREASE DEBRIS

CHINA'S ANTI-SATELLITE MISSION

In 2007, China intentionally destroyed one of their weather satellites in space, and the event led to a

900-piece cloud of debris.

THE FIRST MAJOR IMPACT

February 10, 2009:
The 15,000 mph collision of the private Iridium 33 satellite and Cosmos 2251, a Russian military spacecraft, left a trail of approximately 2,000 pieces of low-Earth orbit debris.

Together, these two events combined increased the number of debris in low-Earth orbit by

more than **60%**

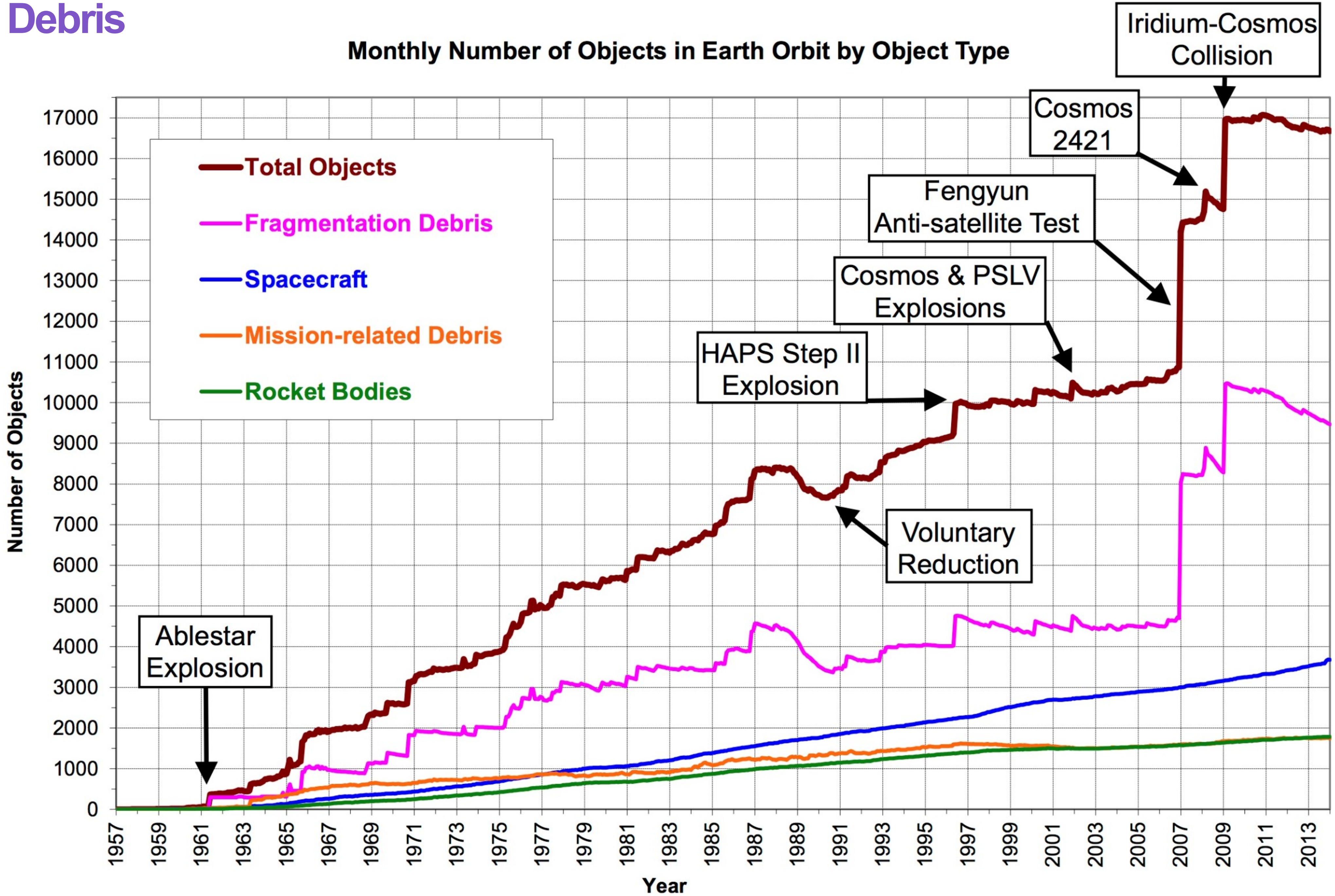
EARTH

That's taking into account everything that has accumulated over the past 50 years.

A COLLABORATION BETWEEN GOOD AND COLUMN FIVE

SOURCES: NASA, WALL STREET JOURNAL, U.S. SPACE SURVEILLANCE NETWORK, SPACE.COM, UNIVERSETODAY.COM, NATIONAL GEOGRAPHIC, ESA DEBRIS OFFICE

Space Debris



Assignment

- Draw Earth, and a satellite orbiting the Earth.
- Draw the orbit of the satellite.
- Mark rotation direction of the Earth.
- Change your picture with your neighbour. Discuss.

- Draw the Sun and the Earth in summer and winter.
- Draw the same satellite orbiting the Earth.
- Change your picture with your neighbour. Discuss.

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Additional notes

Geostationary orbit (GEO)

24-hour orbit

- satellite always on the same longitude
- good for, e.g., telecommunications and broadcasting
- good global coverage

Orbit in the equatorial plane

- poor coverage at high latitudes

GEO is becoming crowded

Periapsis & Apoapsis

The point on the orbit nearest the occupied focus is called the periapsis

The point farthest from the occupied focus is called the apoapsis.

If the central body is the Earth, these points are also referred to as the perigee and apogee, respectively. Similarly, if the central body is the sun, the names change to perihelion and aphelion.

Some links, used in lecture

Gravity

https://www.youtube.com/watch?v=MTY1Kje0yLg

https://www.youtube.com/watch?v=a3OQ7ek7t68

Orbit elements made easy

http://www.amsat.org/amsat/keps/kepmodel.html

Satellite orbits

https://www.youtube.com/watch?v=4K5FyNbV0nA

https://www.youtube.com/watch?v=Hcm7oQwpZfg

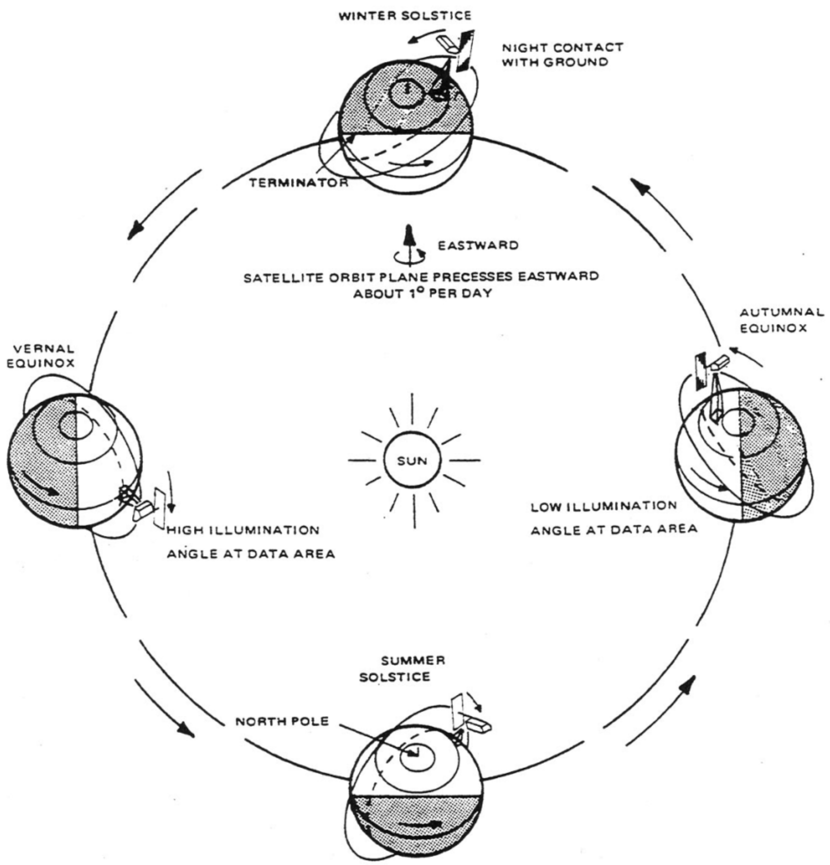
https://www.youtube.com/watch?v=uZc0YJjyWGM

Sun synchronous orbits

Combined effect of the precession of the orbit and the motion around the Sun

Same geographic location (e.g. equator) passed always at the same local time

- useful in many Earth-observation applications



Useful relations

- Angular Momentum

$$\vec{\boldsymbol{h}} = \vec{r} \times \vec{\boldsymbol{v}}$$

- Node Line Vector

$$\vec{\boldsymbol{N}} = \widehat{\boldsymbol{K}} \times \vec{\boldsymbol{h}}$$

$\vec{N}$ is line of nodes;

$\vec{K}$ is reference axis

- Argument of the periapsis

$$\omega = \cos^{-1} \frac{\vec{\boldsymbol{N}}.\vec{\boldsymbol{e}}}{|N||e|}$$

- Inclination

$$i = \cos^{-1} \frac{h_z}{h}$$

- True anomaly

$$\theta = \cos^{-1} \frac{\vec{\boldsymbol{e}}.\vec{\boldsymbol{r}}}{|e||r|}$$

- Right Ascention of the ascending node

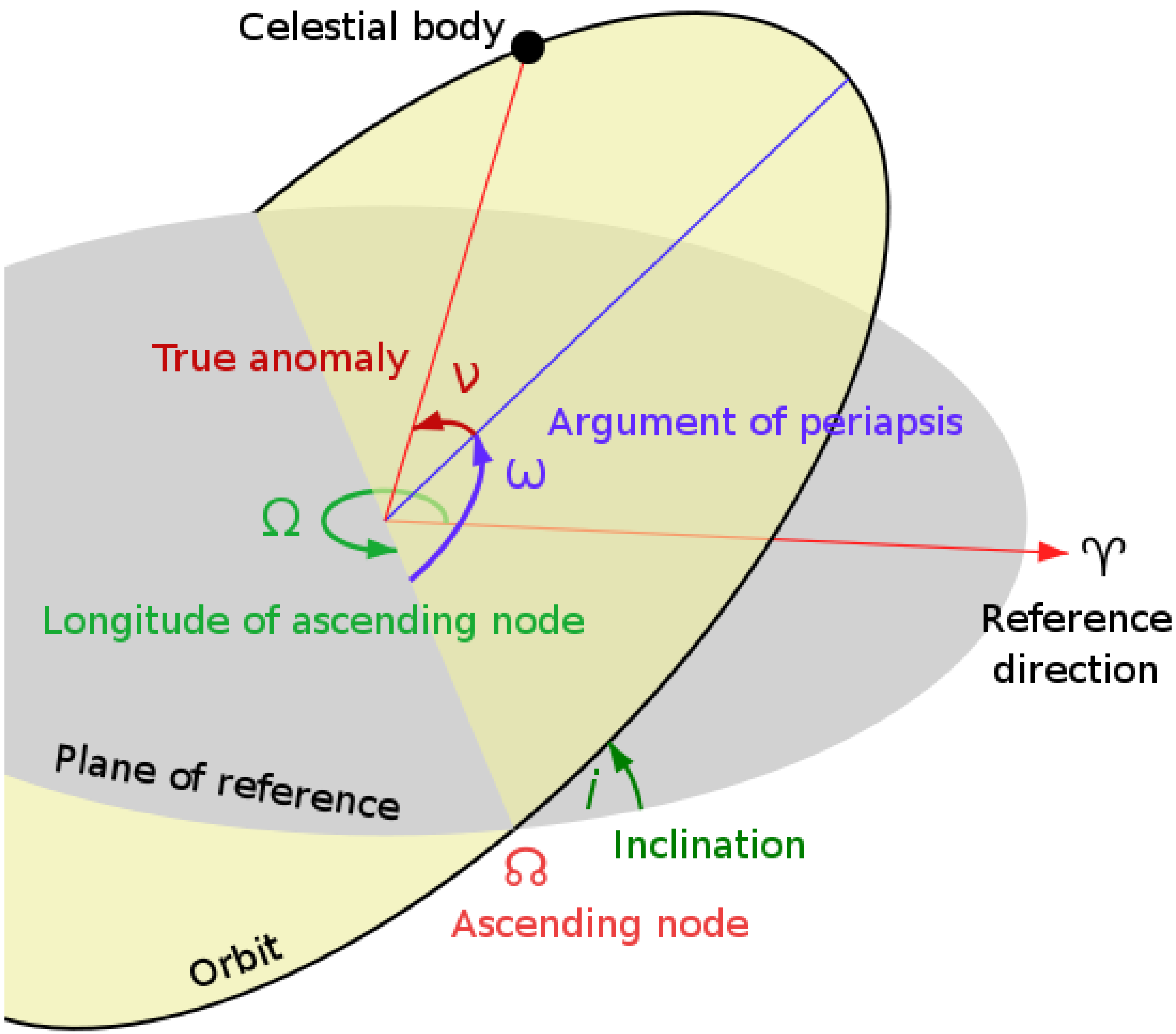
$$\Omega = \cos^{-1} \frac{\vec{\boldsymbol{I}}.\vec{\boldsymbol{N}}}{|N|}$$

$\vec{I}$ is reference axis

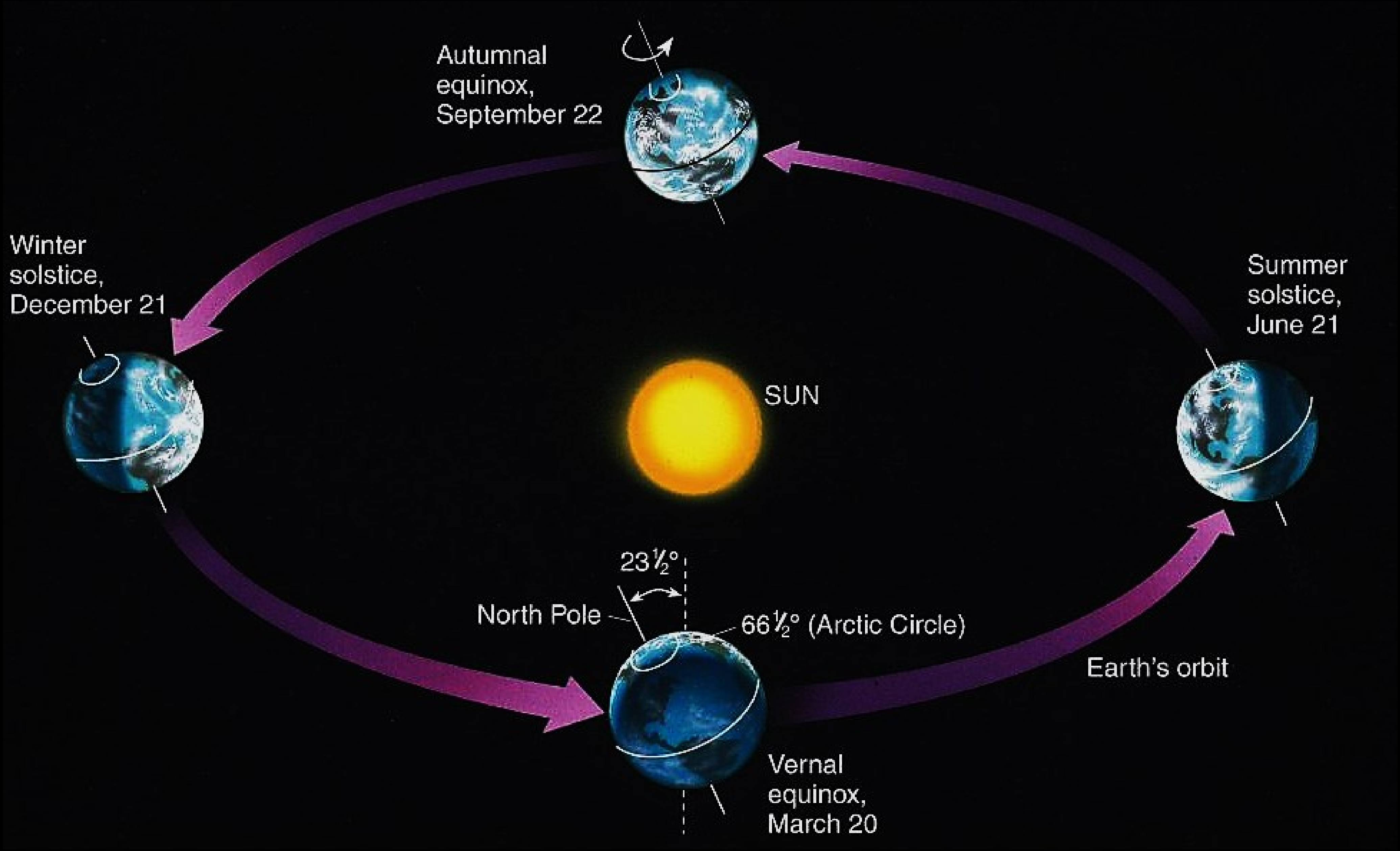
- Eccentricity

$$\vec{\boldsymbol{e}} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{\boldsymbol{r}} - r v_r \vec{\boldsymbol{v}}\right];$$

$$v_r = \frac{\vec{\boldsymbol{r}}.\vec{\boldsymbol{v}}}{r}$$



Celestial body
True anomaly
ν
Argument of periapsis
ω
Ω
γ
Reference
direction
Longitude of ascending node
Plane of reference
i
Inclination
☊
Ascending node
Orbit



Autumnal equinox, September 22
Winter solstice, December 21
Summer solstice, June 21
SUN
23½°
North Pole
66½° (Arctic Circle)
Earth's orbit
Vernal equinox, March 20

Line of Nodes

Ascending Node

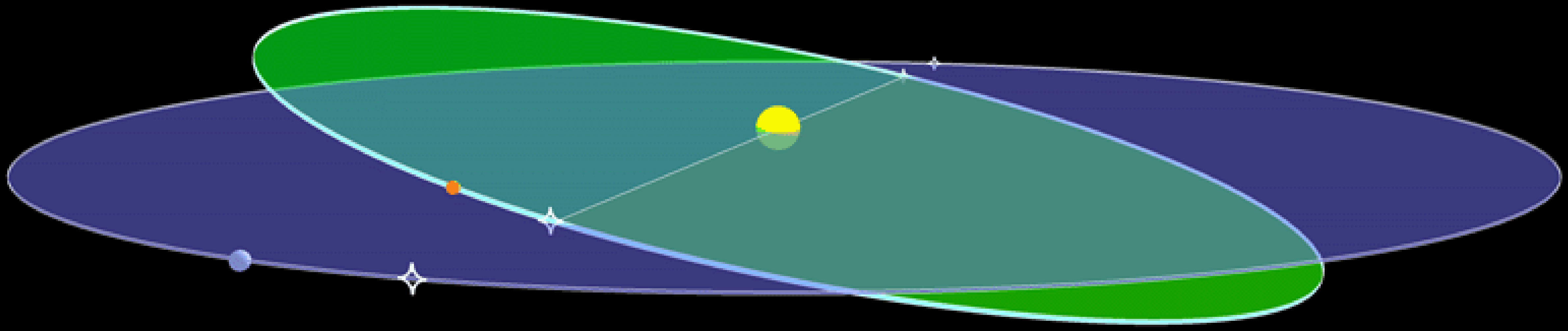
- For geocentric and heliocentric orbits, the **ascending node** (or **north node**) is where the orbiting object moves north through the plane of reference.

Descending node

- For geocentric and heliocentric orbits, the **descending node** (or **south node**) is where it moves south through the plane of reference.

In the case of objects outside the Solar System, the ascending node is the node where the orbiting secondary passes away from the observer, and the descending node is the node where it moves towards the observer.

Line of Nodes



[Source: Wikipedia]

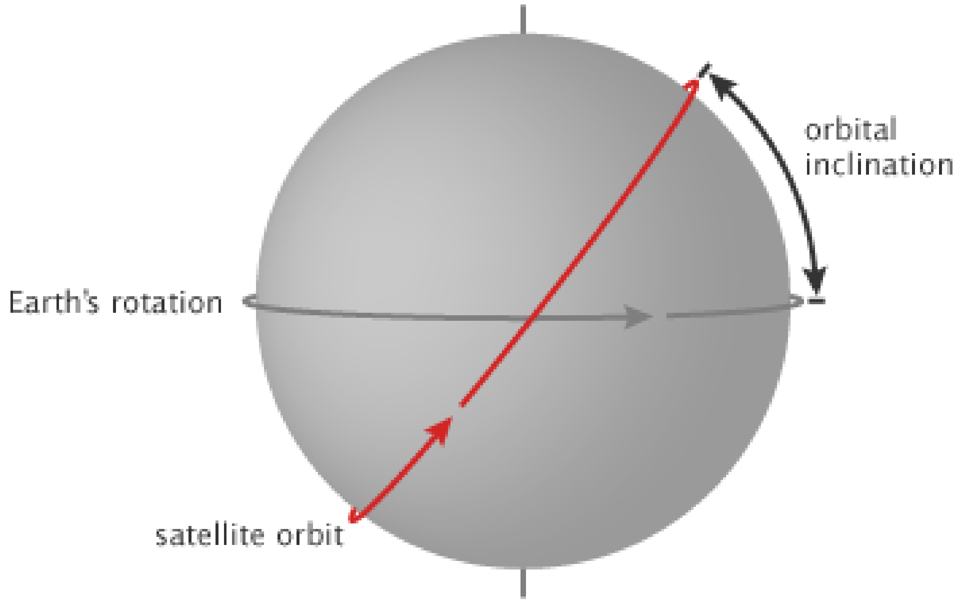
Inclination Angle

Prograde Orbit

- A spacecraft or other body moves in the same direction as the planet rotates.
- An orbit inclination between 0 and 90 degrees will generate a prograde orbit.

Retrograde Orbit

- A spacecraft or other body moves in the opposite direction to the planet's rotation
- An orbit inclination between 90 and 180 degrees is generate a retrograde orbit.



[Source: NASA]

- An object with an inclination of 90 degrees is said to be neither prograde nor retrograde, but it is instead called a polar orbit. (since it does not have an east-west directional component)