Worksheet 4

MS-E1621, Algebraic Statistics

October 14, 2020

Group members: Write your names here.
The 2×2 independence model is

$$\mathcal{M}_{X \perp\!\!\!\perp Y} = \{p = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \in \Delta_3 : p_{ij} = \alpha_i \beta_j, (\alpha, \beta) \in \Delta_1 \times \Delta_1\}.$$

Let the matrix of counts be

$$u = \begin{pmatrix} 19 & 141 \\ 17 & 149 \end{pmatrix}.$$

1. Write down the likelihood function $L(\theta|D) = \prod p_\theta(j)^{u_j}$.

2. Write down the log-likelihood function $l(\theta|D) = \log L(\theta|D)$.

3. The maximum likelihood estimate $\hat{\theta}$ is the maximizer of the log-likelihood function:

   $$\hat{\theta} = \text{argmax}_{\theta \in \Theta} l(\theta|D).$$

   How would you find the maximum likelihood estimate for the model $\mathcal{M}_{X \perp\!\!\!\perp Y}$ and the matrix of counts u using the log-likelihood function? You do not have to explicitly write down the solution, but discuss what are the steps.

4. Alternatively, the model $\mathcal{M}_{X \perp\!\!\!\perp Y}$ can be given implicitly as

   $$\mathcal{M}_{X \perp\!\!\!\perp Y} = \{P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \in \Delta_3 : p_{11}p_{22} - p_{12}p_{21} = 0\}.$$

   How would you find the maximum likelihood estimate for the implicit model?