Matrix Computations
MS-A0001
Hakula/Mirka
Problem Sheet 2, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the “papers” can be found on the course pages.

1 Introductory Problems

INTRO 1 Solve the lower triangular system $L\,c = b$ followed by $U\,x = c$. What is A in $A\,x = b$?

$$L = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 11 \end{pmatrix}.$$

INTRO 2 Tridiagonal matrices are an important special case. Find the decomposition $A = LU$ and the symmetric one $A = LDL^T$.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}, \quad A = \begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{pmatrix}.$$

INTRO 3 Let A be a 3×3 orthogonal matrix, with elements:

$$\alpha_{11} = \tfrac{3}{7}, \quad \alpha_{12} = -\tfrac{2}{7}, \quad \alpha_{13} > 0, \quad \alpha_{21} < 0, \quad \alpha_{22} = \tfrac{6}{7}.$$

Find A and A^{-1}.

INTRO 4 Show that if for the matrix A it holds that $A^{\mathrm{T}} + A = O$ and the inverse of $(I + A)^{-1}$ exists, then the matrix $B = (I + A)^{-1}(I - A)$ is orthogonal.

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2 Homework Problems

EXERCISE 1 Find the decomposition $A = LU$, given

$$A = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

EXERCISE 2 Find all decompositions $PA = LU$ of

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & c & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

for all $c \in \mathbb{R}$.

EXERCISE 3 Choose any vector x and set $r = x^{\mathrm{T}}x$ and $u = x/\sqrt{r}$. Form $H = I - 2uu^{\mathrm{T}}$ and its inverse H^{-1}. Is H either symmetric or orthogonal or both? Further, show that $H\,H = I$ independent of x, and give a geometric interpretation of the related mapping.

EXERCISE 4 Find $M\,M^{-1}$, when

1. $M = I - uv^{\mathrm{T}}$ and $M^{-1} = I + uv^{\mathrm{T}}/(1 - v^{\mathrm{T}}u)$.
2. $M = A - uv^{\mathrm{T}}$ and $M^{-1} = A^{-1} + A^{-1}uv^{\mathrm{T}}A^{-1}/(1 - v^{\mathrm{T}}A^{-1}u)$.
3. $M = I - UV$ and $M^{-1} = I_n + U(I_m - VU)^{-1}V$.
4. $M = A - UW^{-1}V$ and $M^{-1} = A^{-1} + A^{-1}U(W - VA^{-1}U)^{-1}VA^{-1}$.

Is the notation M^{-1} justified in all cases? (Notice: These are valid and highly useful identities in real life situations!)