Matrix Computations
MS-A0001
Hakula/Mirka
Problem Sheet 3, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems

INTRO 1 Solve $Ax = b$, when

$$A = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 2 & 6 \\ 2 & 8 & 12 & 8 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

INTRO 2 Solve $Ax = b$, when

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 3 & -1 & 1 & -2 \\ 2 & -2 & -1 & -1 \\ 1 & -3 & -3 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 1 \\ -2 \\ -5 \end{pmatrix}.$$

INTRO 3 Show, that vectors $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^{\mathrm{T}}$, $\begin{pmatrix} 2 & 3 & 1 \end{pmatrix}^{\mathrm{T}}$ are $\begin{pmatrix} 3 & 1 & 2 \end{pmatrix}^{\mathrm{T}}$ form a basis of $\mathbb{R}^3$. Find the coordinates of $\begin{pmatrix} 3 & 2 & 1 \end{pmatrix}^{\mathrm{T}}$ in this basis.

INTRO 4 Show, that if the vectors $a_1, \ldots, a_p$ are linearly dependent, then so are the vectors $\lambda_1 a_1, \ldots, \lambda_p a_p$. Conversely, if $a_1, \ldots, a_p$ are linearly independent, show that so are $\lambda_1 a_1, \ldots, \lambda_p a_p$ if and only if the product $\lambda_1 \ldots \lambda_p$ is $\neq 0$.

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2 Homework Problems

EXERCISE 1 Solve $Ax = b$, when

$$A = \begin{pmatrix} 1 & 3 & 5 & -2 \\ 3 & -2 & -7 & 5 \\ 2 & 1 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 11 \\ 0 \\ 3 \end{pmatrix}.$$

EXERCISE 2 Let

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & 1 \\ 5 & 4 & \alpha \\ 3 & 2 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} 7 \\ -1 \\ 8 \\ \beta \end{pmatrix}.$$

Find the solutions of $Ax = b$ for all α, $\beta \in \mathbb{R}$.

EXERCISE 3 Under which conditions on α, β, γ, δ two vectors $\alpha x + \beta y$ and $\gamma x + \delta y$ are linearly independent, if the vectors x and y a) are, b) are not linearly independent?

EXERCISE 4 Let $a_1, \ldots, a_p$ be linearly independent. Are the following sets linearly independent or not:

a) $a_1,\ a_2 + a_1,\ a_3 + a_2,\ \ldots,\ a_p + a_{p-1},$

b) $a_1 - a_p,\ a_2 - a_1,\ a_3 - a_2,\ \ldots,\ a_p - a_{p-1},$

c) $a_1,\ \ldots,\ a_{j-1},\ a_j + \lambda a_k,\ a_{j+1},\ \ldots,\ a_p,$ where $k \neq j$.