1. (**Just for fun**) (4p)

   (a) In the lectures, when we analyzed the equilibrium for the z-pinch, we assumed that the current density was constant across the plasma cross section. This is not very realistic. The current is typically driven by an (axial) electric field. Knowing that the temperature in the center of the plasma is higher than at the edge, find a justification for the current density to be be peaked at the center. (2p)

   (b) Without prior knowledge of the shape of the current density profile, a simple parabolic form would be the first guess: $j(r) = j_0(1 - r^2/a^2)$. More freedom for the profile shape is obtained by a 'generalized' parabolic form: $j(r) = j_0[(1 - r^2/a^2)]^\alpha$, where $\alpha > 0$. Plot such a profile for at least three different values of α, including a case with $\alpha < 1$. Discuss the different profile shapes and which form you would consider most 'realistic'. (2p)

2. (**On amperes and teslas**) (6p)

   (a) Assume now that the current density is of the form $j(r) = j_0[(1 - r^2/a^2)]^\alpha$. Calculate the radial dependence of the plasma current, $I_\mathrm{p}(r)$. (2p)

   (b) Derive also the expression for the poloidal field $B_\theta(r)$ induced by the plasma current both in the region $r < a$ and in the region $r > a$. (4p)

3. (**Staying safe**) (6p)

   In the lectures, when discussing the screw-pinch, a new quantity called the *safety factor* was introduced: $q \equiv rB_z/RB_\theta$.

   (a) Using the expression for $B_\theta(r)$, calculated in the previous problem (also given in lecture slides), calculate the safety factor value at the edge of the plasma, $r = a$, assuming $B_z =$ constant. (2p)

   (b) Calculate the value of the safety factor also at the magnetic axis, $q(r = 0)$, with the same assumption. (4p)

4. (**Shearing the magnetic field**) (4p)

   In the lectures, the concept of *magnetic field shear*, $s \equiv (r/q)dq/dr$, was introduced. Assuming a screw-pinch geometry with $B_z =$ constant, show that the *magnetic field shear* can be expressed in terms of the plasma current as $s = 2 - rI'(r)/I(r)$ (prime denotes derivative).

5. (**Food for thought: Getting lost**)

   So far, we have only been concerned about the loss of the plasma, i.e., particles. From the fusion point-of-view, even more crucial is the loss of *energy*, since the fusion reaction cross section very sensitively depends on temperature. Write about what loss mechanisms you think there could be for energy.