Matrix Computations
MS-A0001
Hakula/Mirka
Problem Sheet 4, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems

Let us define a van Gogh square (the one with an ear cut off) with points

$$(0,0), (1,0), (1,3/4), (2/3,1), (0,1).$$

Form a matrix G that you can use in geometric transformations.

INTRO 1 Let

$$\hat{A}_1 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}.$$

Classify the transform. Draw in the same picture both G and its image $\hat{H}_1 = \hat{A}_1 G$.

INTRO 2 Let

$$\hat{A}_2 = \frac{1}{5}\begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}.$$

Classify the transform. Draw in the same picture both G and its image $\hat{H}_2 = \hat{A}_2 G$.

INTRO 3 Let

$$A = \begin{pmatrix} 13 & 8 & 6 \\ -13 & -8 & -4 \\ 8 & 5 & 5 \end{pmatrix}.$$

Compute $\det((-5AA^{\mathrm{T}})^7)$ using the rules for determinants.
Answer: $-5^{21} \cdot 2^{14} = -\ 7\ 812\ 500\ 000\ 000\ 000\ 000.$

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INTRO 4 Evaluate determinant

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix}.$$

This is the so-called Vandermonde determinant which is central in many applications.

2 Homework Problems

EXERCISE 1 Consider an affine mapping $F : x \mapsto \hat{A}_1 x + b_1$, where $\hat{A}_1$ as above and $b_1 = \left(\sqrt{3} - \frac{1}{2}, 4 - \frac{3\sqrt{3}}{2}\right)^T$. Classify the transform (or mapping) precisely, i.e., determine at least one fixed point x_1. Draw in the same picture both G and its image $H_1 = \hat{A}_1 G + B_1$, where $B_1 = (b_1\ b_1 \dots b_1)$.

EXERCISE 2 Consider an affine mapping $F : x \mapsto \hat{A}_2 x + b_2$, where $\hat{A}_2$ as above and $b_2 = \left(-\frac{4}{5}, -\frac{8}{5}\right)^T$. Classify the transform (or mapping) precisely, i.e., determine at least one fixed point x_2. Draw in the same picture both G and its image $H_2 = \hat{A}_2 G + B_2$, where $B_2 = (b_2\ b_2 \dots b_2)$.

EXERCISE 3 Under which conditions on the parameters α and β the following determinants are equal to zero?

$$\text{a) } \begin{vmatrix} \alpha+\beta & \alpha+2\beta & \alpha+3\beta \\ \alpha+3\beta & \alpha+\beta & \alpha+2\beta \\ \alpha+2\beta & \alpha+3\beta & \alpha+\beta \end{vmatrix}, \quad \text{b) } \begin{vmatrix} 2 & \alpha & 2 \\ 1 & \alpha^2 & \alpha \\ \alpha & \alpha^3 & 1 \end{vmatrix}.$$

EXERCISE 4 A tetrahedron T is defined by its corner points $(-1, -2, 4)$, $(5, -1, 0)$, $(2, -3, 6)$, $(1, -1, 1)$. Find the volume of T.