31E99906 Microeconomic policy

Lecture 10: Discounting

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Motivating example

Evaluating a project

- project costs C units of consumption goods today
- project payoff is B units of consumption after t units of time.
- project is small so that we can consider the total gain through marginal impacts; ignore the impact on the economy, that is.
- the project pays off if and only if

$$[-C + B\frac{1}{1+r}] \geqslant 0$$

- What is the discount factor $(1+r)^{-1}$ to be used? How does the answer depend on the time lag between costs and payoffs? Consider nuclear power investments vs. road maintenance

General principle: the return requirement should be the same as in alternative, comparable market investments. However, for long maturities, the market benchmark does not exist.

What discount rate? Asking economists:

	years	discount rate
immediate future	$1 - 5$	4
near future	$6 - 25$	3
medium future	$26 - 75$	2
distant future	$76 - 300$	1
far-distant future	$301-$	≈ 0

Table 1: Weitzman (AER, 2001) asked 2160 economists: What should be the discount rate to be used for projects with different maturities?

What discount rate? Asking markets:

Real interest rates for a set of countries since the late-nineteenth century[1]

Table 1 Annualized real returns, 1900 to 2006

	Bill	Bond (10 year)	Equity
Australia	0.6%	1.3%	7.8%
Canada	1.6%	2.0%	6.3%
Denmark	2.3%	3.0%	5.4%
France	−2.9%	−0.3%	3.7%
Italy	−3.8%	−1.8%	2.6%
Japan	−2.0%	−1.3%	4.5%
The Netherlands	0.7%	1.3%	5.4%
Sweden	1.9%	2.4%	7.9%
Switzerland	0.8%	2.1%	5.3%
United Kingdom	1.0%	1.3%	5.6%
United States	1.0%	1.9%	6.6%

[1] Gollier C. 2012. Pricing the Planet's Future: The Economics of Discounting in an Uncertain World.

What are the government guidelines?

US

- First, 7% is the average before-tax rate of return to private capital, taken as an estimate of the opportunity cost of capital. Second, 3% is the average return to 10-year government bonds, taken as an estimate of the social rate of time preference.

UK

- calibrated "Ramsey rule", giving 3.5% for near term maturities. For longer maturities, stepwise decline: 3% for 31-75 years, 1% for LONG maturities.

Norway

- 4% up to 40 years; 3% up to 75 years; 2% thereafter

Plan for the lecture

Two related steps to be take in this class

1. Consumer choice theory for consumption streams
    - Prices are given as usual in consumer theory: interest rates, asset prices
    - Ramsey rule (different from Ramsey pricing!)
2. The determinants of returns
    - What returns are consistent with observed consumption choices?
    - The answer helps when market returns are not available
    - How discounting should change when economic/other conditions change. Think of the current economic situation.
    - We develop understanding the practical discounting guidelines for projects of different types and length
    - uncertainty (a little bit of this)

Plan

- discounting and consumer theory: the link between interest rates, asset prices, and consumption
  - provides guidance on how the discount rate should be chosen in both public and private projects
  - Ramsey rule (different from Ramsey pricing!)
- determinants of equilibrium discounting
  - how discounting should change when economic/other conditions change
  - uncertainty (a little bit of this)
- helps in:
  - understanding the practical discounting guidelines for projects of different types and length
  - developing arguments for using rates that differ from the market rates

Preferences

Preferences over consumption streams

- Denote a consumption stream by $\mathrm{c} = (c_0, c_1, c_2, ...c_n)$. This is equivalent to the consumption bundle x that we saw in consumer theory (lecture 1) but the goods are now consumptions at different periods
- If we can rank consumption streams $\mathrm{c} = (c_0, c_1, c_2, ...c_n)$ then we can say that c is preferred to some other $\tilde{\mathrm{c}}$ if and only if

$$u(\mathrm{c}) \geq u(\tilde{\mathrm{c}})$$

Just like in consumer's choice theory.

Additional structure imposed on preferences

1. Time separability

$$u(c_0, c_1, c_2, ...c_n) = \sum_{t=0}^{n} u_t(c_t)$$

   Rules out habit formation, for example. Very restrictive but we need to start somewhere. We will mostly work with two periods

$$u(c_0, c_1) = u_0(c_0) + u_1(c_1)$$

2. Exponential discounting

$$u_t(c_t) = \beta^t u(c_t)$$

Thus, the same utility from consumption every period but scaled by the discount factor β^t where $\beta \leq 1$. Note that β discounts utility, not money.

Two period version

In what follows, we use the two-period model

$$u(c_0, c_1) = u(c_0) + \beta u(c_1)$$

But the interpretation of periods can be flexible

- real time between periods 0 and 1 is T could be years
- length of time period alters the discount factor: pure time discount over T discrete periods is β^T if β is annual; the continuous time discount rate is given by $\beta^T = e^{-\delta T}$. For example, $\delta = .05$ means 5% rate and $\beta = 0.95$
- similarly gross return $(1 + \bar{r})$ implies a continuous time return r through $1 + \bar{r} = e^r$. Below, I will use just one r for both discrete and continuous time.

Key concept: consumption smoothing

Consumer prefers to smooth consumption over time, and the preferences or tastes should reflect this. This comes back to "MRS" of consumption over time. In two periods,

$$MRS = \frac{u'(c_0)}{\beta u'(c_1)}.$$

Suppose $u(c_t)$ is concave, so marginal utility is decreasing in c_t. Further, we may think that consumption depends on wealth, $c_t = w_t$. The consumer has wealth w_0 in the first period and is expecting to have so much more wealth w_1 in the period second that

$$MRS = \frac{u'(w_0)}{\beta u'(w_1)} > 1.$$

This consumer would not like to save from $t = 0$ to $t = 1$, unless compensated by a sufficient return. Without return, the "poor" first period consumer has to sacrify more the "rich" next period consumer gains: $u'(w_0) > \beta u'(w_1)$.

We can measure the degree of this resistance by asking what is the minimum return that would induce some saving? It is given by

$$\frac{u'(w_0)}{\beta u'(w_1)} = 1 + r.$$

Obviously, r depends on w_0, w_1, β, and $u(\cdot)$. To focus on resistance coming from pure wealth effect (decreasing marginal utility) and not from impatience, set $\beta = 1$ (no discounting) for a moment. By the first-order Taylor approximation we have

$$u'(w_0) - u'(w_1) \simeq u''(w_1)(w_0 - w_1)$$

which implies that r can be written as

$$r \simeq \frac{w_1 - w_0}{w_1}\left\{\frac{-w_1 u''(w_1)}{u'(w_1)}\right\}.$$

The required compensation depends thus on

1. the "growth rate" $\frac{w_1 - w_0}{w_1}$
2. elasticity of marginal utility $\gamma = \frac{-w_1 u''(w_1)}{u'(w_1)}$, defined to be positive.

Variable γ has many names depending the context. Here, it measures the concavity of the utility: more concave utility translates into more desire for consumption smoothing.

The functional form

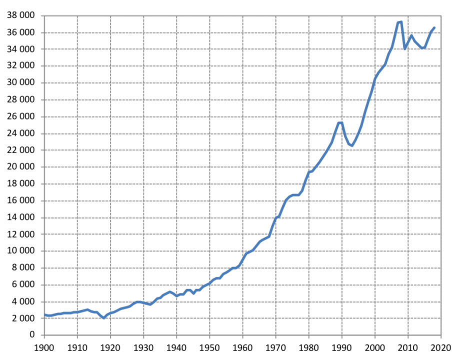
$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

is convenient because $\gamma = \frac{-cu''(c)}{u'(c)}$ is constant (a parameter of $u(c)$). When $\gamma \to 0$, the person does not have preferences for the time profile of consumption (excluding discounting). When γ becomes very large, preferences imply extreme aversion to volatility/changes in consumption levels. When $\gamma \to 1$, the utility becomes $u(c) = \ln(c)$.

Illustration

GDP in Finland (2010 prices) over the period 1900-2018. Thought experiment: consider consumer standing in 1900

- How important to you is the wealth in 2018? $\rightarrow$ pure impatience, β
- What are the incentives to save for the benefit of individuals that are 15 times wealthier (as measured by GDP)? $\rightarrow$ consumption smoothing, γ
- labor productivity has increased by factor 25 $\rightarrow$ return r



Consumer choice theory

Consumer's choice for given returns for savings

We start using the above framework for linking the prices of assets and their returns to consumer's choices. Let p_t be the current price, m_{t+1} discount factor, and z_{t+1} is payoff (e.g., price+dividend). This will allow us to see how the consumer discounts future payoffs. Think that the consumer takes prices of assets and their payoffs as given. We want to show the following, over two periods, $t = 0, 1$:

Result 1

$$p_0 = \frac{\beta u'(c_1)}{u'(c_0)} z_1.$$

You see that the consumer effectively the discounts the future payoff by

$$m_1 = \frac{\beta u'(c_1)}{u'(c_0)}.$$

To derive the previous page results, consider consumer with income in both periods and who can buy assets to save for the future:

- income is y_t in period $t = 0, 1$
- consumer can buy with price p_t at t a (certain) payoff next period $t + 1$. Call this payoff z_{t+1}. If the asset is stock, then the payoff is

$$z_{t+1} = p_{t+1} + d_{t+1},$$

  next period value plus dividend. For one-period bond, the payoff is

$$z_{t+1} = 1.$$

  Thus, it pays back one unit of consumption

Let s denote the amount of the payoff bought, and write the consumer's budget constraints for both periods

$$\begin{aligned} c_0 &= y_0 - p_0 s \\ c_1 &= y_1 + z_1 s \end{aligned}$$

- Sum up the budget constraints

$$c_0 + c_1 = y_0 + y_1 + (z_1 - p_0)s$$

- Using $s = (y_0 - c_0)/p_0$ and rearranging

$$\begin{aligned} c_0 + c_1 &= (y_0 + y_1) + \frac{z_1 - p_0}{p_0}(y_0 - c_0) \\ &= (y_0 + y_1) + r(y_0 - c_0) \end{aligned}$$

where $r = \frac{z_1 - p_0}{p_0}$ is the rate of return for the asset.

We can now write the consumer's saving and consumption problem as

$$\max_{c_0,c_1}\{u(c_0)+\beta u(c_1)\}$$

$$s.t.$$

$$(1+r)y_0+y_1=(1+r)c_0+c_1$$

From the first-order conditions:

$$\frac{u'(c_0)}{\beta u'(c_1)}=(1+r).$$

If we call the left-hand side the marginal rate of substitution, MRS, we see that MRS equals the gross returns on savings, which is the relative price of consumptions between the two periods. A familiar result from the consumer theory!

Recall the definition of r and rewrite the previous as

$$\begin{aligned}\frac{u'(c_0)}{\beta u'(c_1)} &= (1 + \frac{z_1 - p_0}{p_0}) \\ p_0 &= \frac{\beta u'(c_1)}{u'(c_0)} z_1,\end{aligned}$$

which is the result we wanted. Note

- $p_0 u'(c_0)$ is the current cost in "utils" if an additional unit of the asset is bought
- $\beta u'(c_1) z_1$ is the increase in future utils from having payoff z_1

Rewriting the previous:

$$p_0 = \frac{1}{1+r} z_1$$

The payoff z_1 thus sells at discount as $\frac{1}{1+r} < 1$. This follows from definitions but the MRS for consumption choices should produce this same discount for risky assets. We could build upon this framework if we considered risky assets and their pricing

The determinants of returns

Consumption-based determinants of risk-free interest rates

$\rightarrow$We now turn the question around: *What is the interest rate that is consistent with consumption choices?*
This question follows if we do not obtain the interest rate from the market, as we discussed in the beginning of the lecture. We found earlier that "MRS=gross return"

$$\frac{u'(c_0)}{\beta u'(c_1)} = (1 + r)$$

We may add uncertainty about future consumption c_1. This can arise, e.g., if y_1 is uncertain:

$$\tilde{y}_1 = y_1 + \tilde{x}$$

where $\tilde{x}$ is random income component.

Think $t = 0$ today, and $t = 1$ "future" come after period of lenght T. The consumption choice is otherwise the same:

$$\max_{\{c_0, c_1\}} u(c_0, c_1) = u(c_0) + e^{-\delta T} E u(\tilde{c}_1)$$

$$\tilde{c}_1 = \tilde{y}_1 + (y_0 - c_0) e^{rT}$$

where we use time discount rate δ as defined earlier. r is now the rate of return for savings

The first-order condition is

$$u'(c_0) = e^{(r-\delta)T} E u'(\tilde{c}_1) \tag{1}$$

To see the connection to "MRS=gross return", rewrite as

$$\frac{u'(c_0)}{e^{-\delta T} E u'(\tilde{c}_1)} = e^{rT} \Leftrightarrow \frac{u'(c_0)}{\beta^T E u'(\tilde{c}_1)} = (1+r)^T.$$

Note that investment in asset giving a sure return r can be thought of as an investment in a zero-coupon bond maturing at T. Condition (1) determines the demand for such bonds given return, or alternatively, return for given consumptions. Take logs of condition above and rewrite

$$r = \delta + \frac{1}{T} \ln\left(\frac{u'(c_0)}{Eu'(\tilde{c}_1)}\right) \tag{2}$$

δ is the pure rate of preference for the present. The equilibrium rate deviates from δ for two reasons.

1. Income effect: if $E\tilde{c}_1 > c_0$. Higher expected income tends to reduce incentives to save and increase rate r. The income effect is measured by the index of relative risk aversion:

$$A(c)c = \frac{-cu''(c)}{u'(c)}$$

Recall that for CRRA utility

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

we have

$$A(c)c = \gamma.$$

2. Precautionary effect: uncertainty about future income level (large deviations from the expected level can occur) makes the investor cautious. If $u'(c)$ is convex, then $Eu'(\tilde{c}_1) > u'(E\tilde{c}_1)$, so that uncertainty makes the investor value marginal utility more. Such an investor is called prudent. Prudence tends to increase savings and reduce the equilibrium rate r. Prudence is measured by

$$P(c) = \frac{-cu'''(c)}{u''(c)}.$$

For CRRA, we have

$$P(c) = 1 + \gamma.$$

To make these effects explicit, we can take the second-order Taylor approximation of (2)

$$r \simeq \delta + \frac{1}{T}\{A(c_0)c_0\frac{E\tilde{c}_1 - c_0}{c_0} - A(c_0)c_0P(c_0)var(\frac{\tilde{c}_1}{c_0})\}$$

or for the CRRA case

$$r \simeq \delta + \frac{1}{T}\{\gamma\frac{E\tilde{c}_1 - c_0}{c_0}\} - \gamma(1+\gamma)var(\frac{\tilde{c}_1}{c_0})\}$$

Observe: the expected growth rate $E\tilde{g} = \frac{E\tilde{c}_1 - c_0}{c_0}$ multiplied by the resistance to consumption changes measured by γ captures the income effect. The greater is γ, the higher is the required rate of return on savings, given $E\tilde{g}$.

Some numbers

Asumed utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $T = 1$, and no uncertainty:

$$\Rightarrow r = \rho + \gamma[\frac{c_1 - c_1}{c_0}]$$

- Above equation: "Ramsey rule"
- Weitzman (2007, Journal of Economic Literature):$\rho = \frac{c_1 - c_1}{c_1} = .02$ and also $\gamma = 2$. Leads to $r = .06$
- some other numbers: $\rho = .0001$, $\frac{c_1 - c_1}{c_1} = .013$ and $\gamma = 1$. Leads to $r = .013$. Huge difference!

Calibration of the Ramsey rule

Source: *Link*

Table 2 Calibration of the discount rate based on the Ramsey equation (Equation 3)

Author	Inequality aversion γ	Growth rate G	Discount rate γg
Stern (1977)	2		
Cline (1992)	1.5	1%	1.5%
IPCC (1995)	1.5–2	1.6–8%	2.4–16%
Arrow (1999)	2	2%	4%
HM Treasury (2003)	1	2%	2%
Lebègue (2005)	2	2%	4%
Arrow (2007)	2–3		
Dasgupta (2007)	2–4		
Stern (2007)	1	1.3%	1.3%
Weitzman (2007a)	2	2%	4%
Nordhaus (2008)	2	2%	4%
Pindyck (2013)	1–3		

Some of the authors add a rate of impatience δ to the Ramsey rule so that the last column is only a partial representation of what these authors recommend for the discount rate. Blank cells denote that data were

Summary

Lessons:

- we have learned how the markets price assets and thereby define returns
- the returns tell us how the society discounts future
- when making public/private investment decisions we can, in principle, use those same return requirements

Complications:

- the market does not provide a benchmark return for maturities over 20-30 years. But many projects have longer horizons
- we are forced to work out the return requirement for long projects from the first principles
- To recover the return, we look at the consumer's optimal choice

Evaluating a project

- the previous is just the familiar rule:

$$[-C + B\frac{1}{1+r}] \geqslant 0$$

where $\frac{1}{1+r} = \beta\frac{u'(c_1)}{u'(c_0)}$

- We have now recovered the discount factor from the primitives