Matrix Computations
MS-A0001
Hakula/Mirka
Problem Sheet 5, 2020

NOTE[1]

The due date is published on the course pages. Homework can be submitted only digitally. Instructions on labeling the "papers" can be found on the course pages.

1 Introductory Problems

INTRO 1 Let λ be an eigenvalue of A and let $x \neq o$ be the corresponding eigenvector. Show, that matrix A^p has an eigenvalue λ^p and the corresponding eigenvector is x ($p \in \mathbb{N}$).

INTRO 2 Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}.$$

Is it possible to choose the eigenvectors so that they are orthonormal?

INTRO 3 Find the eigenvalues and eigenvectors of

$$\begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

and form an orthonormal basis using the eigenvectors.

INTRO 4 Diagonalise the matrix

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 8 \end{pmatrix}.$$

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2 Homework Problems

EXERCISE 1 Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

EXERCISE 2 The matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ -2 & 5 & -1 \end{pmatrix}$$

has eigenvectors $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^{\mathrm{T}}$, $\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}^{\mathrm{T}}$, and $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$. Let $x = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}^{\mathrm{T}}$. Find $A^{11}x$.

EXERCISE 3 Is the matrix

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & -2 & 0 & 3 \end{pmatrix}$$

diagonalisable? If so, find the similarity transform.

EXERCISE 4 Find the eigenvalues and eigenvectors of

$$\text{a)} \begin{pmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}, \quad \text{b)} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

and form an orthonormal basis using the eigenvectors.