



Aalto University  
School of Electrical  
Engineering

# ELEC-E8126: Robotic Manipulation

## Constrained and parallel kinematics

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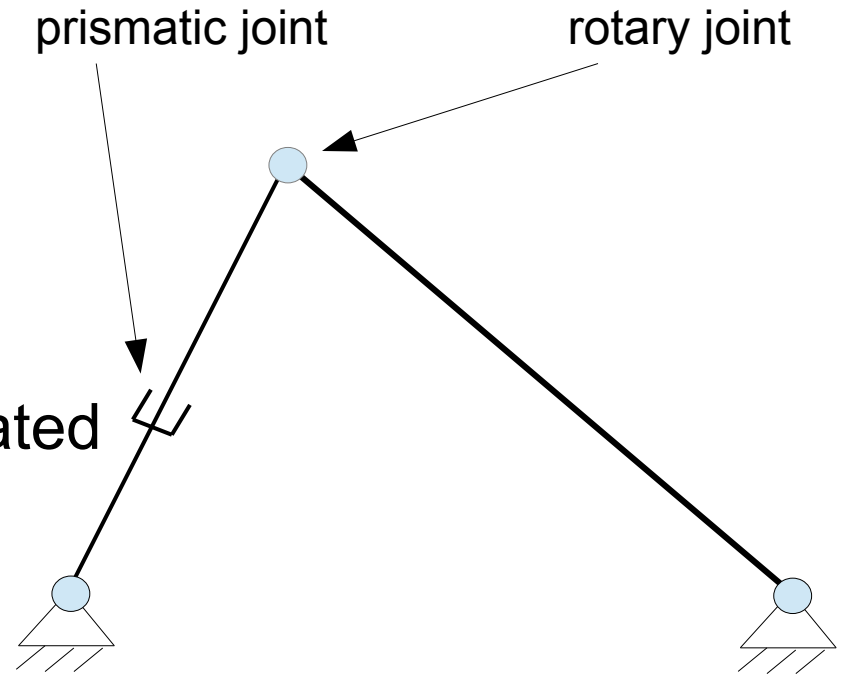
9.3.2020

# Learning goals

- Understand modeling and characteristics of closed kinematic chains.
- Understand constraints posed by closed chains in contexts of parallel robots, cooperative manipulation and dextrous manipulation.

# Terminology

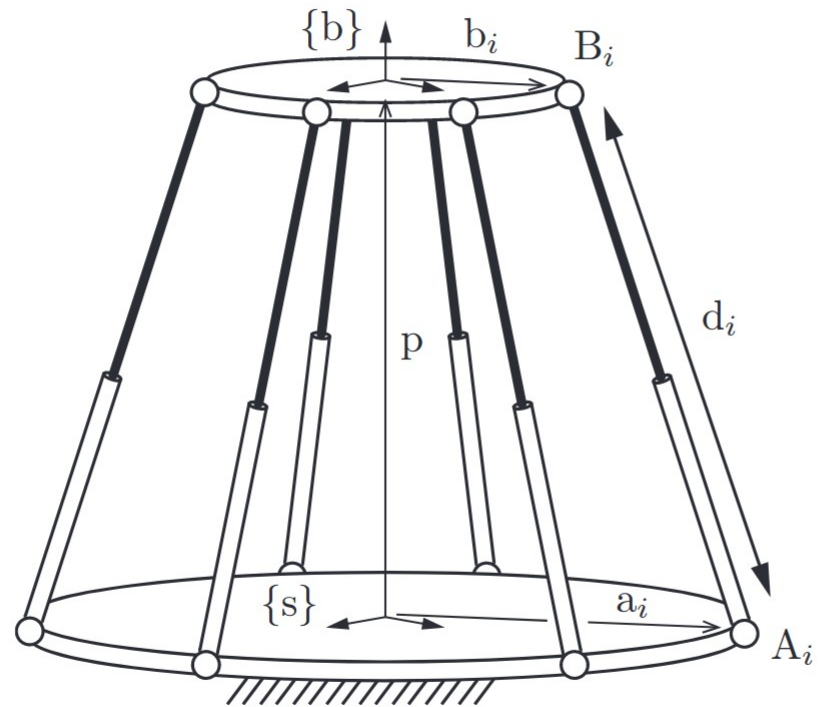
- Closed kinematic chain
- Actuated (active) vs unactuated (passive) joints
  - Why have unactuated joints?



What's the loop here?

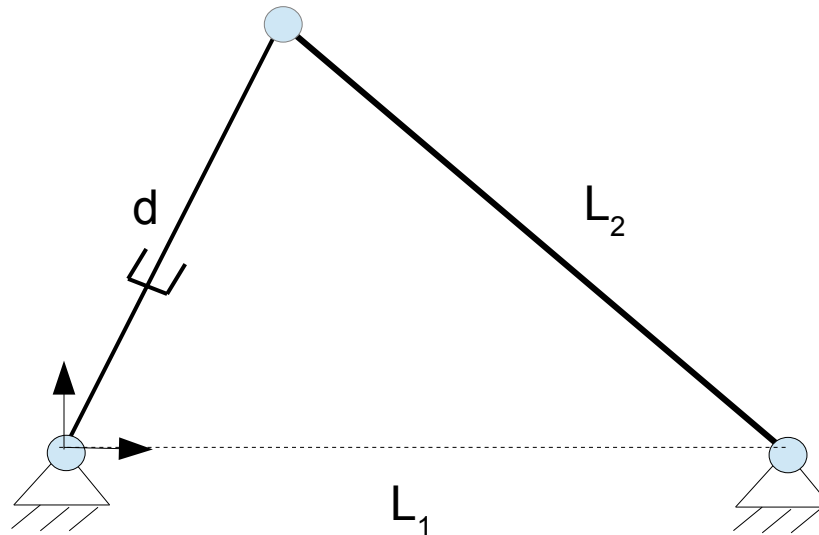
# Typical parallel robot characteristics

- Small workspace
- Accurate
- Rigid structure
- More difficult to model than serial.



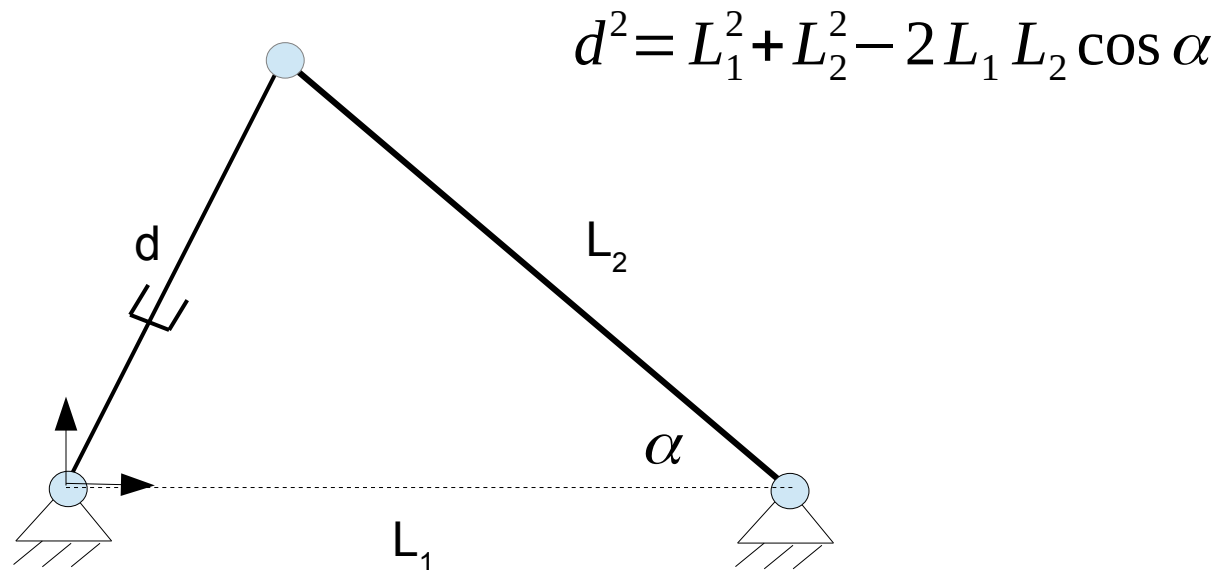
# Kinematic constraints

- What is the position of top point with respect to the length of prismatic joint  $d$ ?
  - What is the constraint equation?



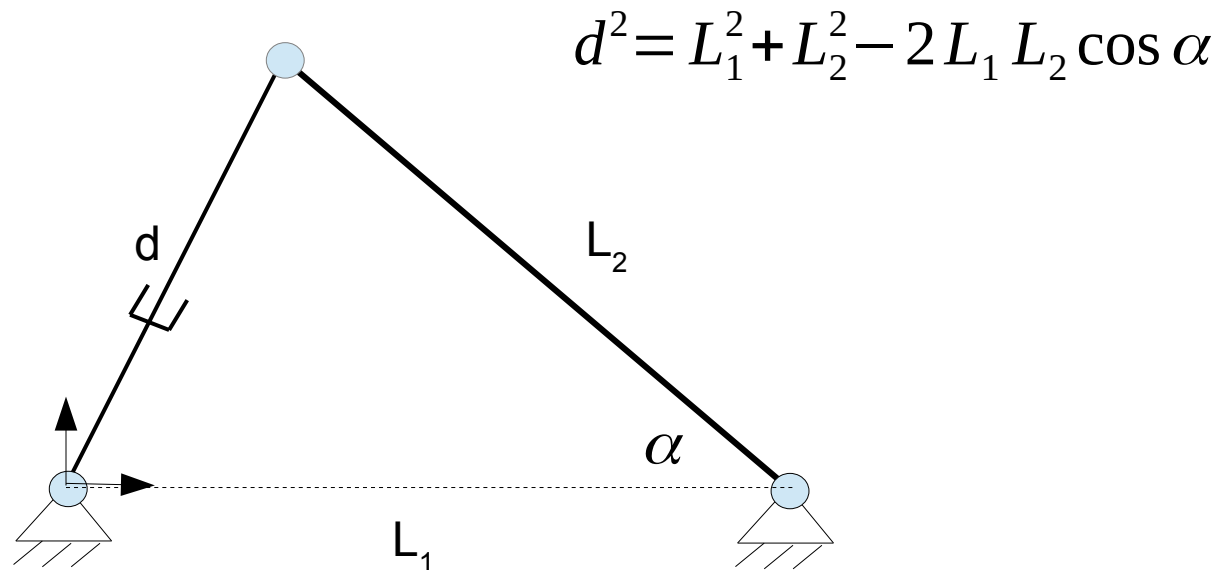
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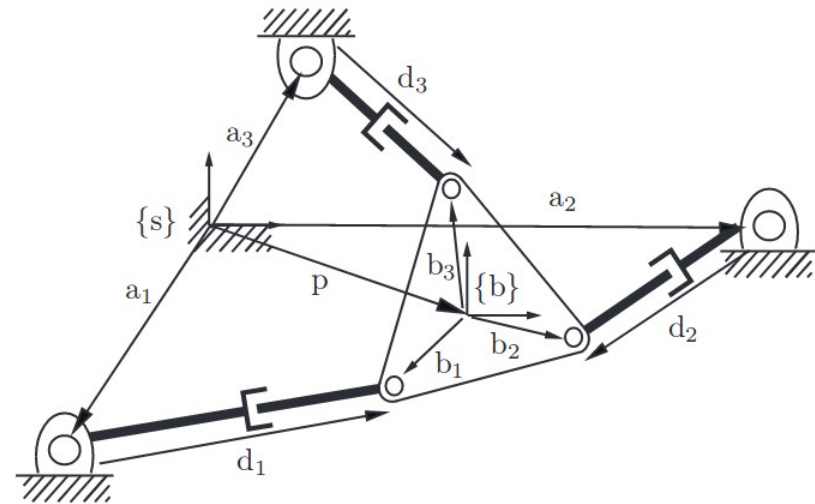


How to now solve the position of the top point?

e.g.  $L_2 \cos \alpha = (L_1^2 + L_2^2 - d^2) / (2L_1)$

# 3x RPR – mechanism and kinematics

- Planar mechanism with 3 RPR structures.
- Prismatic joints actuated, revolute joints passive.
- Constraint equations



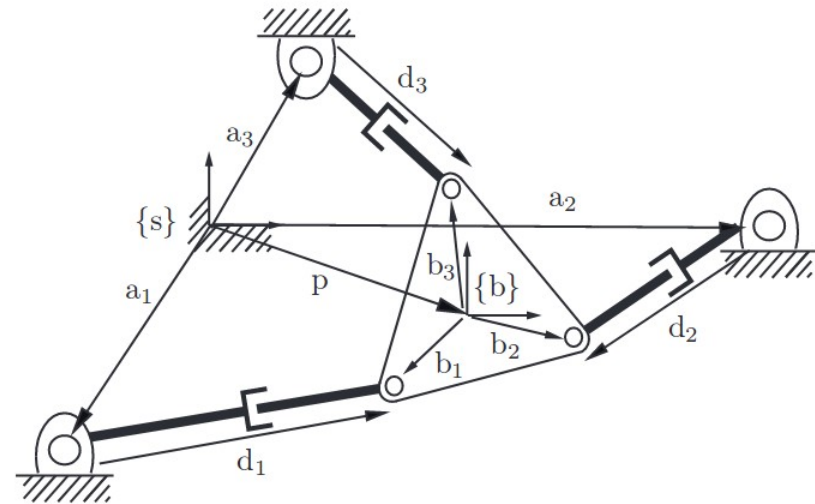
$$d_i^2 = (p_x + b_{ix} \cos \phi - b_{iy} \sin \phi - a_{ix})^2 + (p_y + b_{ix} \sin \phi + b_{iy} \cos \phi - a_{iy})^2$$

What do these provide us?



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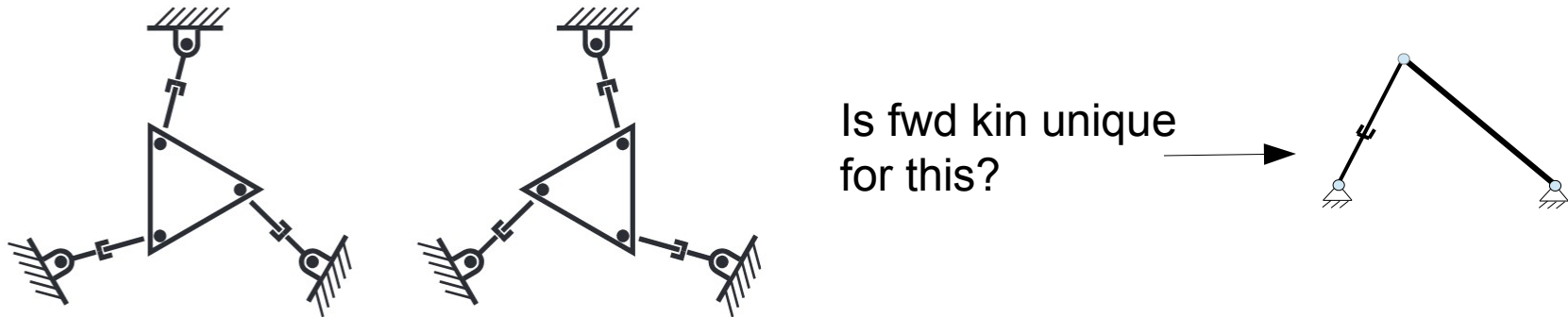


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What do these provide us?

# Forward kinematics

- Fwd kinematics of parallel kinematics often non-unique.



- Different type of singularity compared to serial mechanisms. But in which way?

# Jacobian and constraint Jacobian

- Jacobian can be obtained also from inverse kinematics.

– Why/how?  $\theta = f_{ik}(\mathbf{x})$        $\dot{\mathbf{x}} = J(\theta) \dot{\theta}_a = \frac{\partial f_{fk}(\theta)}{\partial \theta} \dot{\theta}_a \leftarrow$  active

# Jacobian and constraint Jacobian

- Jacobian can be obtained also from inverse kinematics.
  - Why/how?  $\theta = f_{ik}(x)$        $\dot{x} = J(\theta) \dot{\theta}_a = \frac{\partial f_{fk}(\theta)}{\partial \theta} \dot{\theta}_a$  ← active
- Constraint Jacobian – Jacobian of the set of constraint equations.

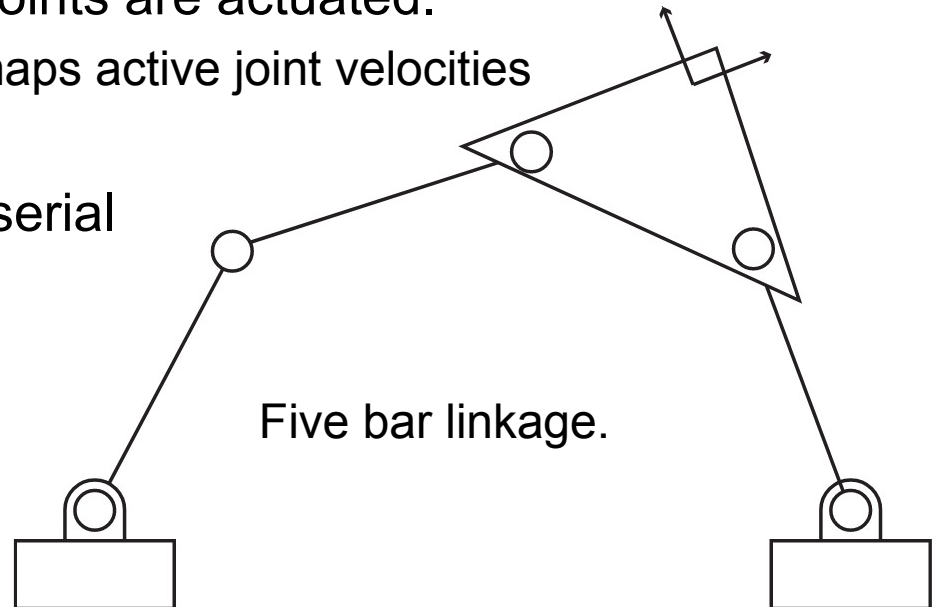
$$h(\theta) = 0$$

$$H(\theta) \dot{\theta} = \begin{bmatrix} H_a(\theta) & H_p(\theta) \end{bmatrix} \begin{bmatrix} \dot{\theta}_a \\ \dot{\theta}_p \end{bmatrix} = 0$$

active
passive

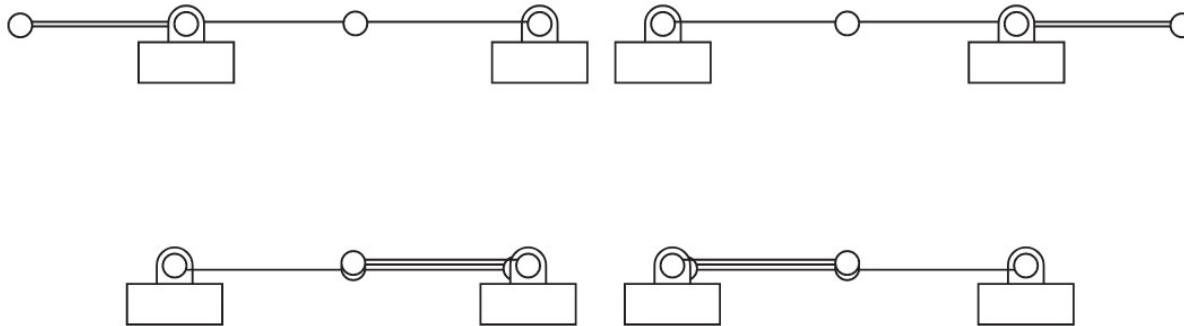
# Singularities of parallel mechanisms

- End-effector singularities
  - Jacobian loses rank  $rank(J) < n$ .
  - Do not depend on which joints are actuated.
    - Even though Jacobian maps active joint velocities to Cartesian velocities.
  - Similar to singularities of serial robots.



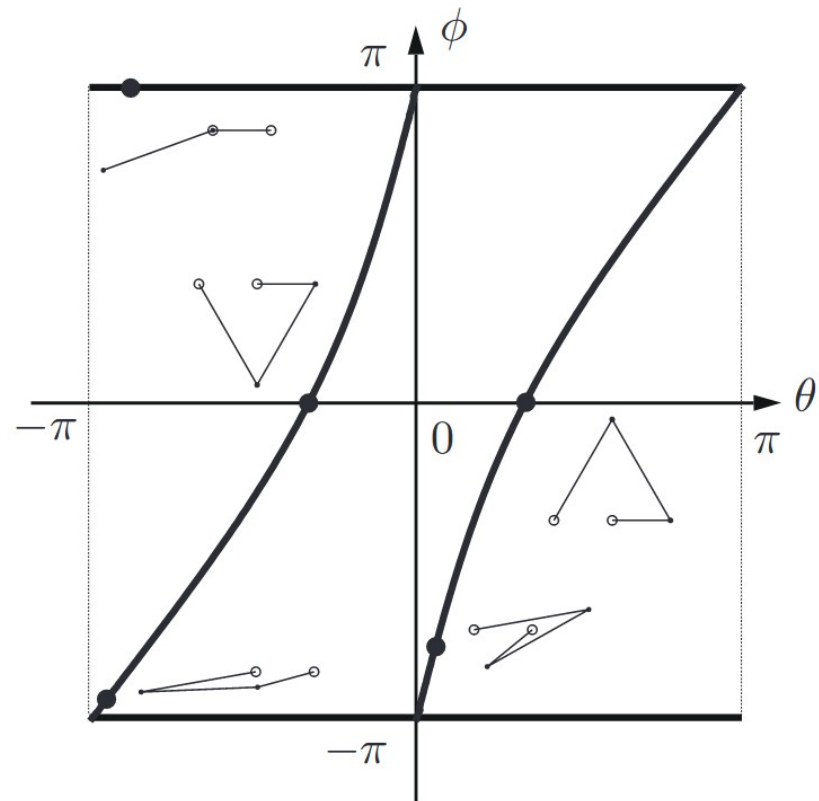
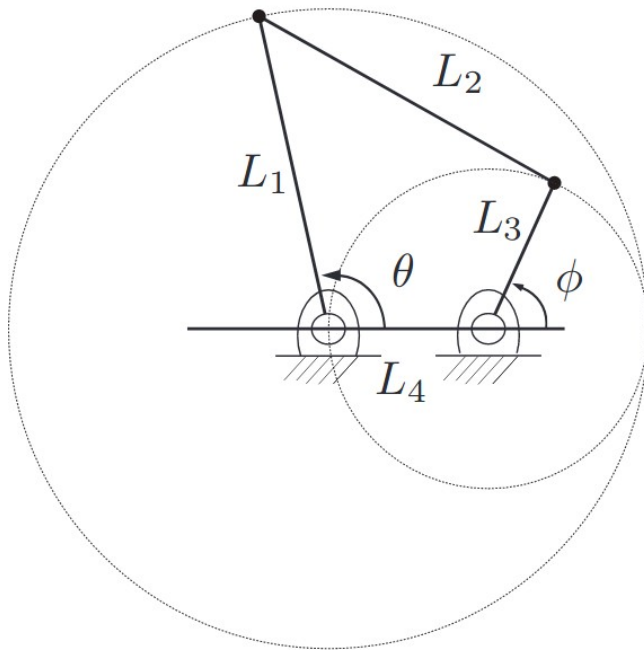
# Singularities of parallel mechanisms

- Configuration space singularities
  - Constraint Jacobian loses rank  $rank(H) < p$ .
  - Do not depend on which joints are actuated.
  - Branching points/regions in full configuration space.



# Configuration space singularity example

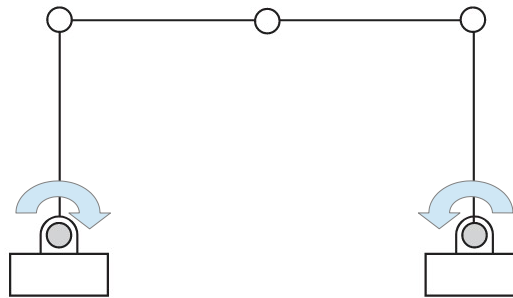
- Four bar linkage



# Singularities of parallel mechanisms

- Actuator singularities

- Constraint Jacobian of passive joints loses rank  $rank(H_p) < p$
- Changing the set of actuated joints will eliminate the singularity.
  - But new one(s) may be created.



What happens?

Can you avoid by changing actuated joints? How?

What happens if rotation is opposite?



# Closed chains and manipulation

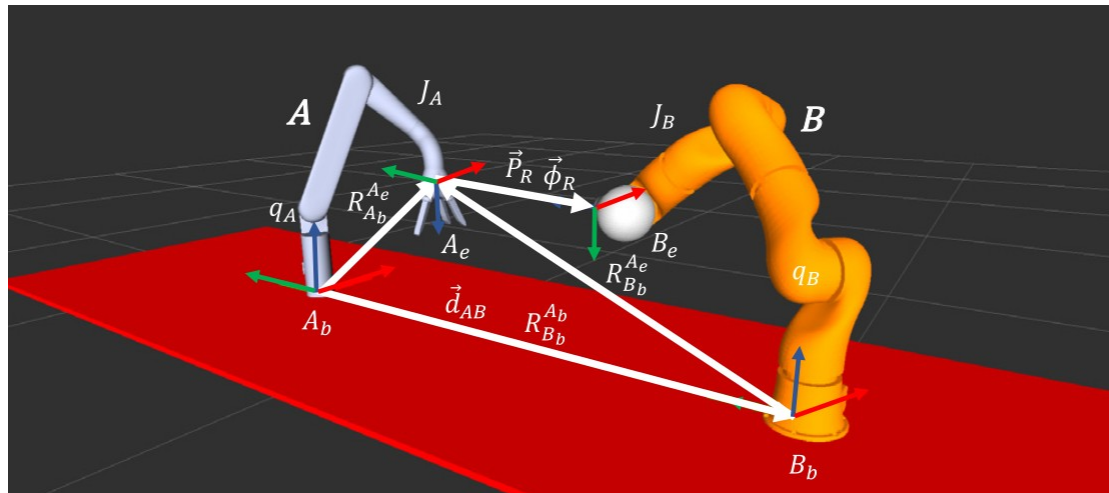
- Are there situations where manipulation creates closed chains?

# Closed chains and manipulation

- Are there situations where manipulation creates closed chains?
- Cooperative (dual-arm) manipulation
- Dextrous (in-hand) manipulation
- Let's take a quick look at these.

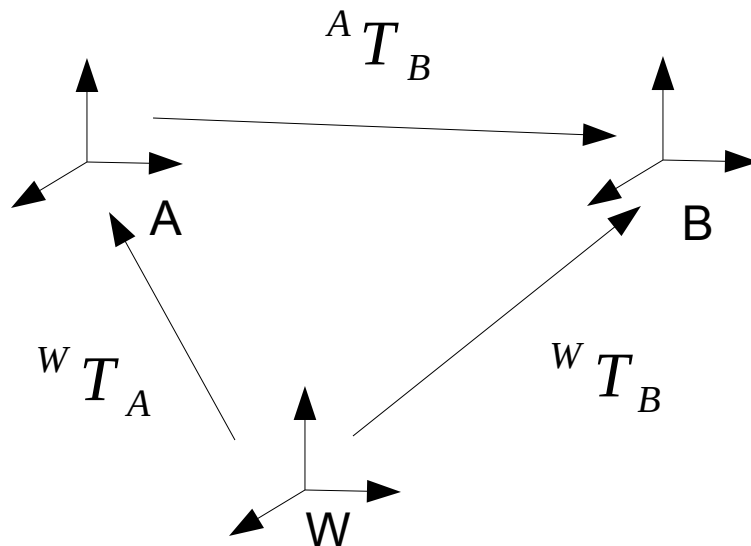
# Cooperative manipulation

- Cooperative manipulation: Multiple arms manipulate a tightly grasped (rigid) object.
- What kind of constraints exist?



# Kinematic constraints

- Chain remains closed (and rigid) →
  - Robot velocities need to match in object frame.
  - Alternatively, relative pose remains constant.



Closed kinematic chain?

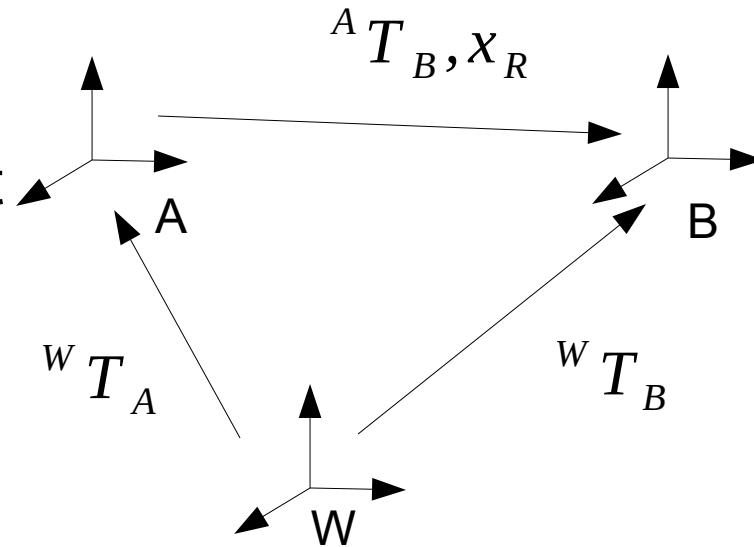
# Relative Jacobian

- Jacobian of relative pose is called relative Jacobian

$$\dot{x}_R = J_R \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix} \quad \text{Size?}$$

- Can be calculated using individual robot Jacobians.
- What is the loop closure constraint given the relative Jacobian?

$$J_R \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{bmatrix} = J_R \dot{\theta} = 0$$



$$\dot{\theta} = J_R^+ K (x_R - x_R^*)$$

# Relative Jacobian and coordinated motion control

- Let's define a (hybrid) velocity controller using relative Jacobian.

$$\dot{\theta} = \underbrace{J_R^+ K_R (x_R - x_R^*)}_{\text{relative pose fb}} + \overbrace{\left( I - J_R^+ J_R \right) \left[ J_A \quad 0 \right]^+ \left( \dot{x}_A^* + K_P (x_A - x_A^*) \right)}^{P_R}_{\text{pose fb}}$$

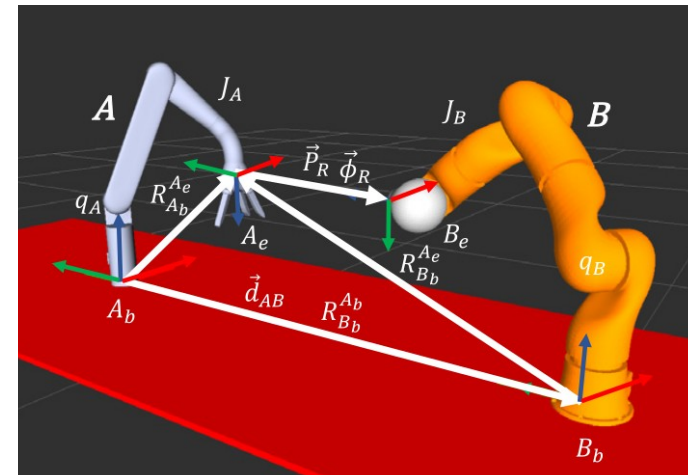
Check position+force control lecture!

# Dynamics of cooperative manipulation

- What is the total wrench applied on the object?

$${}^0 F = {}^0 F_A + {}^0 F_B = G_A F_A + G_B F_B = G \begin{bmatrix} F_A \\ F_B \end{bmatrix}$$

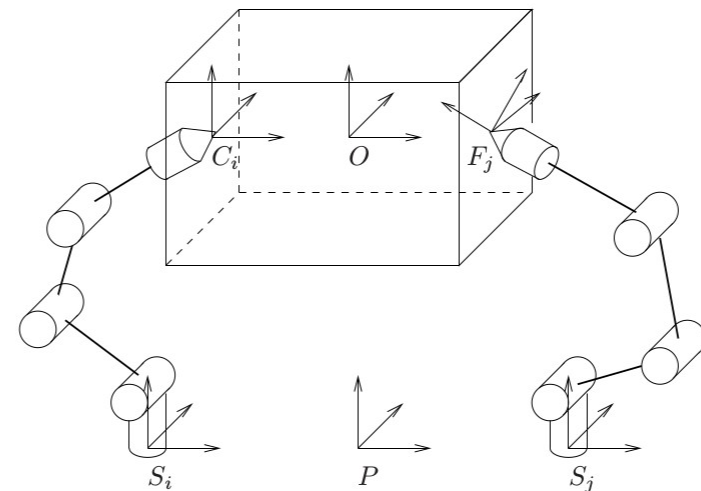
$$F = G^+ {}^0 F + V_S F_I$$



- External wrench  ${}^0 F$
- Internal forces in  $V = \text{null}(G)$ 
  - How do we want this to behave?
  - Would internal forces be useful in some case?

# Dextrous manipulation

- To manipulate grasped objects in hand,
  - finger motions need to be coordinated for grasp to remain stable.
  - object needs to be manipulable.
- What does this mean beyond force closure?





# Coordinated motion in grasping

- Finger motions have to correspond to object motion at contacts

$$J \underbrace{\dot{\theta}}_{\text{finger joint vels}} = G^T \underbrace{V_o}_{\text{object twists}}$$

Why is this more than the relative Jacobian?

- Each contact only in friction constrained directions

$$H \hat{J} \dot{\theta} = H \hat{G}^T V_o$$

Selection matrix to choose constrained directions

- What is then the constraint for fingers to be able to generate any twist  $V_o$ ?

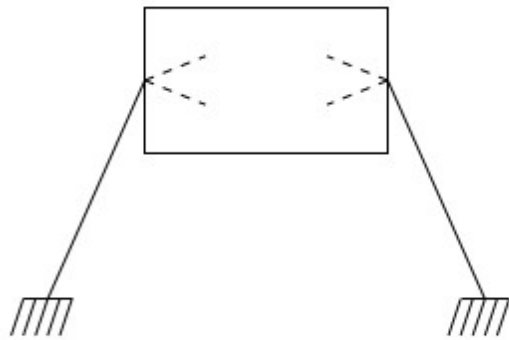
$$\text{rank}(G) = 6$$

$$\text{rank}(GJ) = 6$$

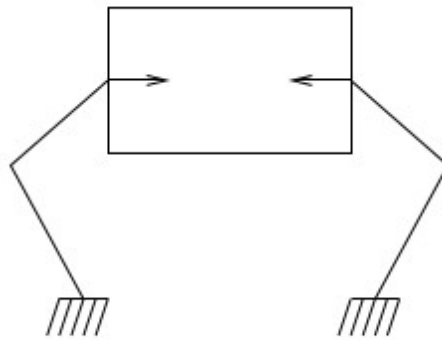
force closure (almost)

Finger motions can create any object motion.

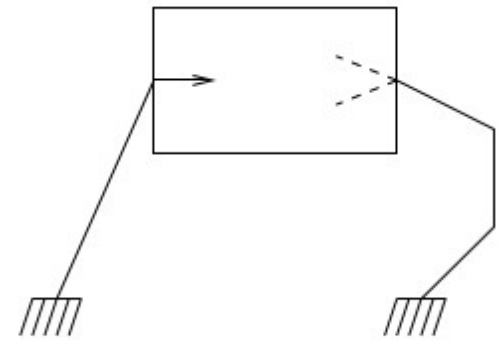
# Manipulability vs force closure



force closure  
not manipulable



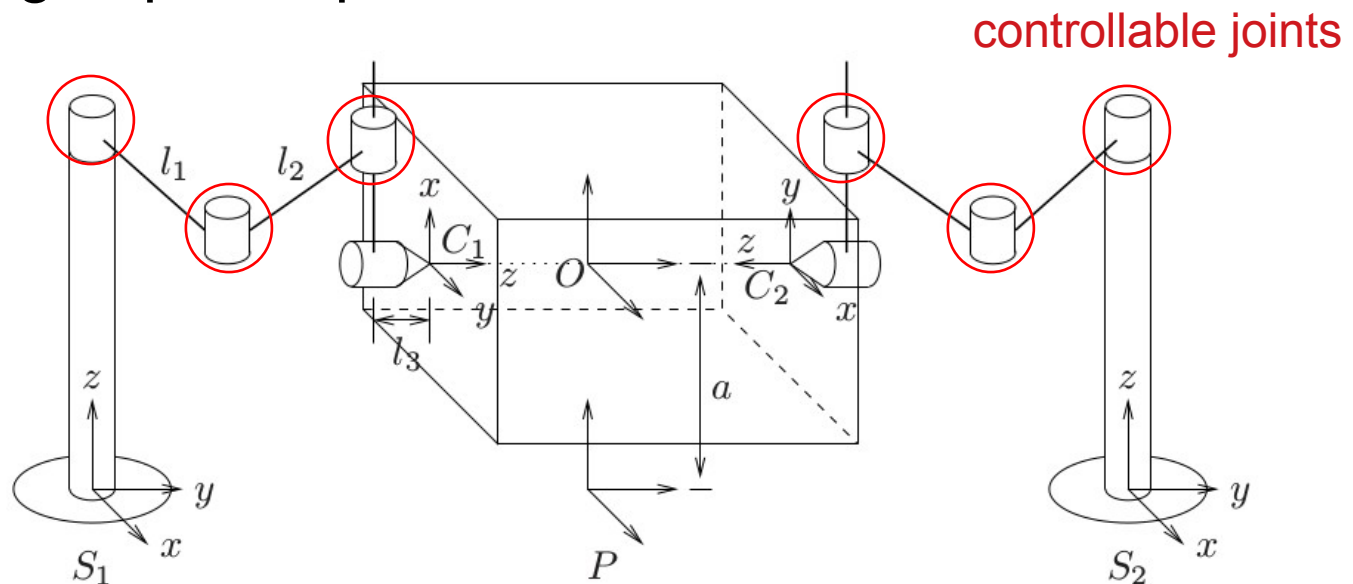
not force-closure  
manipulable



not force-closure  
not manipulable

# Manipulability example: 2x SCARA

- Assume soft fingers.
- Is the grasp force closure?
- Is the grasp manipulable?



# Summary

- Parallel robots have typically both actuated and unactuated joints in closed chains.
  - Inverse kinematics for typical parallel robots are often unique.
  - Forward kinematics often yields multiple solutions.
- Closed chains also appear in cooperative and dextrous manipulation.

# Next time: Redundancy

- Readings:
  - Chiaverini et al., “Redundant robots”, in Springer Handbook of Robotics, 2<sup>nd</sup> ed., ch. 10-10.2.2.
    - Freely available through library webpage [lib.aalto.fi](http://lib.aalto.fi). Log-in first and then search for “Springer Handbook of Robotics”.