Probability generating functions cheat sheet

The 7 properties of probability generating functions

Random variable X where $P(X = k) = p(k)$. Further, $S = \sum_{i=1}^{N} X_i$ and X_i are identically independently distributed random variables.

$$g(z) = p(0) + p(1)z + p(2)z^2 \cdots = \sum_{k=0}^{\infty} p(k)z^k \tag{1}$$

$$p(k) = \left[\frac{1}{k!}\frac{d^k}{dz^k}g(z)\right]_{z=0} \tag{2}$$

$$\langle X^m \rangle = \left[(z\frac{d}{dz})^m g(z)\right]_{z=1} \tag{3}$$

$$g_{X_1+X_2}(z) = g_{X_1}(z) * g_{X_2}(z) \tag{4}$$

$$g_S(z) = [g_{X_i}(z)]^N \tag{5}$$

$$g_{X_1+c}(z) = g_{X_1}(z) * z^c \tag{6}$$

$$g_S(z) = g_N(g_{X_i}(z))\,, \tag{7}$$

where N is a constant in (5) and independent random variable in (7).

Newman's notation

g_0 : PGF for the degree distribution
g_1 : PGF for the excess degree distribution
h_0 : PGF for the small component size distribution (select a random node, probability that it in a component of size s that is not the giant)
h_1 : PGF for the "excess small component size distribution" (select a random node, follow a link and delete it, probability that you reach a component of size s' that is not the giant)
S : The probability that a randomly selected node belongs to the giant component
$u = h_1(1)$: Probability that following a link does not lead to the giant component

Solutions

Assumptions: The network is created with the configuration model and is sparse enough such that there are no loops

$$h_0(z) = zg_0(h_1(z)) \tag{8}$$

$$h_1(z) = zg_1(h_1(z)) \tag{9}$$

$$S = 1 - g_0(u) \tag{10}$$

$$u = g_1(u) \tag{11}$$