CHEM-E8135 Microfluidics and BioMEMS

Microfluidics 1

Basics, Laminar flow, shear and flow profiles

13.1.2021

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Outline of first 3 weeks

Today:

Microfluidics 1: Laminar flow, Hydraulic resistance, Flow profiles,

-No homework first week. But there is some background material on contact angles if they are unfamiliar from your past studies.

Next week, 20.1:

Exercise 1: *Points for participation 10:15-11:00*

Microfluidics 2: contact angle, capillary flow

The week after, 27.1:

Exercise 2: *Points for participation 10:15-11:00*

Microfluidics 3: Diffusion and adsorption

Key concepts and learning outcomes

1. **Reynolds number** and **laminar flow**.
2. **Hydraulic radius**
3. **Hydraulic resistance**, what it means and how to calculate it.
4. **Hagen-Poiseuilles Law**, what it means, how to utilize it and what is its meaning for microfluidics.
5. **Microfluidic circuit design**: how to calculate resistanecs and flows in channel networks.

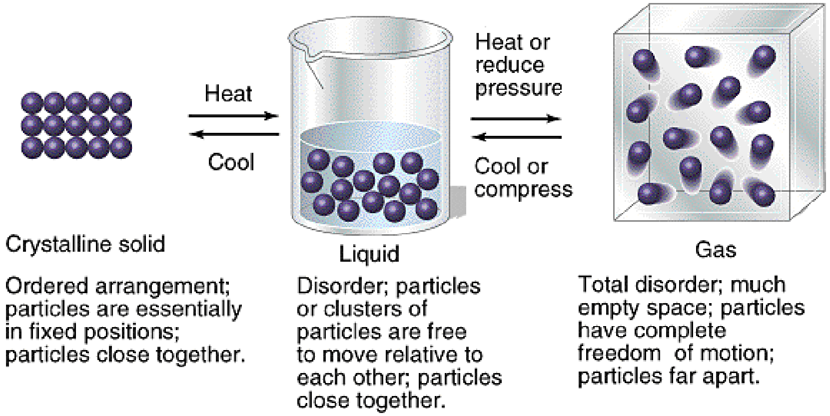
ILO1: The student understands fluid flow at the microscale, scaling laws and can analyse microfluidic circuits. Laminar flow, diffusion, Reynolds number.

Fluid, liquid and gas

Fluid:
a substance that shows continuous shear deformation in response to an applied shear force.

- Fluids are either liquids or gases (or plasmas).

Liquids: viscosity, surface tension, miscible or immiscible in other liquids, conserves volume
Gases: viscosity, NO surface tension, always mixes with other gasses, expands to fill container



Microfluidics on this course will be primarily about liquids.

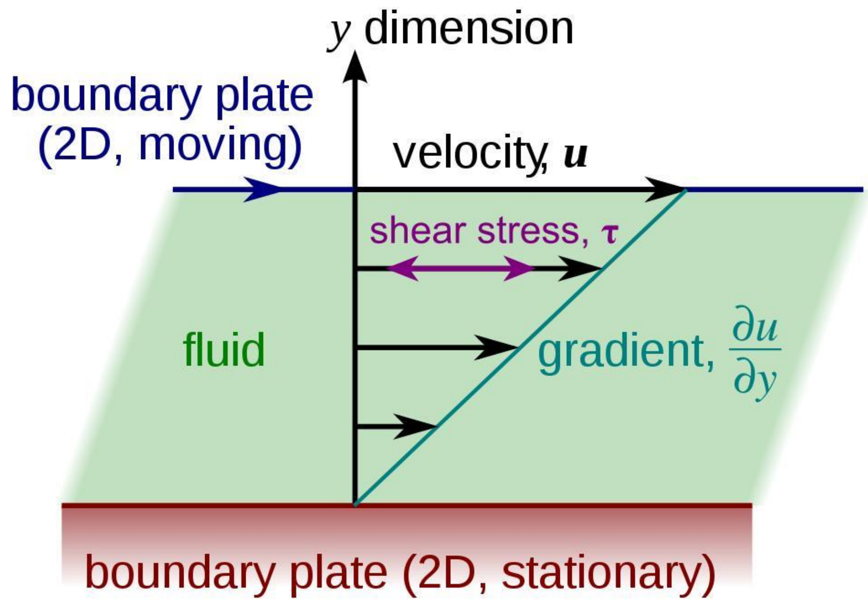
- Flow mechanics, viscosity and diffusion are equally applicable to liquids and gases.
- Surface tension effects only apply for liquids.

http://faculty.sdmiramar.edu/fgarces/zCourse/All_Year/Ch100_OL/aMy_FileLec/
04OL_LecNotes_Ch100/02_EnergyStateMatter/203_StMatter/203_StMatterIMF.htm

Shear and viscosity

- Defining quality of a fluid: continuous flow in response to an applied force.

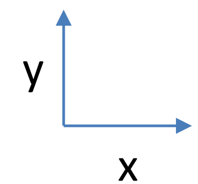
- Contrast to a solid: fixed deformation as a response to an applied force (Hookes Law)



- Shear stress **τ** = force / area
- Shear stress is directly proportional to the <u>shear rate</u>:

$$\mathbf{F_x/A = \tau = \mu\, \delta u_x/\delta y}$$

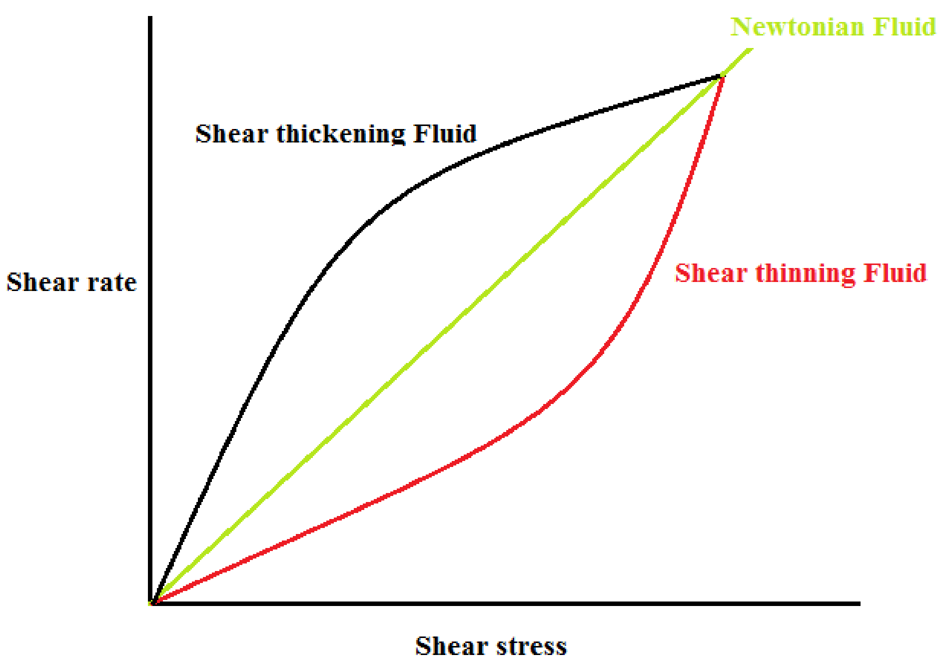
- The constant of proportionality is called viscosity, μ
- High viscosity = ”thick” liquid that flows slowly (like honey)
- Low viscosity = ”thin” liquid that flows easily (like water)



Viscosity

- Shear viscosity, or dynamic viscosity **μ, unit is Pa*s**
- Kinematic viscosity **ν (= μ/ρ), unit is m^2/s**

- **Newtonian fluid:** Viscosity independent of shear.
- Water and air are close to Newtonian.
- Blood is shear thinning.



Viscosity of water

Temperature - *t* - (°C)	Dynamic Viscosity - *μ* - (*Pa s, N s/m²) x 10⁻³*)	Kinematic Viscosity - *ν* - (*m²/s) x 10⁻⁶*)
0	1.787	1.787
5	1.519	1.519
10	1.307	1.307
20	1.002	1.004
30	0.798	0.801
40	0.653	0.658
50	0.547	0.553
60	0.467	0.475
70	0.404	0.413
80	0.355	0.365
90	0.315	0.326
100	0.282	0.29

Dimensionless numbers

- A dimensionless number compares the relative magnitudes of two different phenomena to find out which one is dominant at a given size scale.
- In microfluidics, different forces are dominant compared to a macroscopic systems:

Weak: inertial forces, gravity and other body forces

Strong: surface forces, viscous forces

- Dimensionless numbers are **approximations** that do not take into account the fine details of the physical system. Nevertheless, they are very useful since they show whether one force is orders of magnitude larger than the other, or whether both are significant.

Reynolds number: inertial forces vs viscous forces

Bond number: body forces vs surface forces (e.g. gravity vs surface tension)

Capillary number: viscous forces vs surface forces

Peclet number: convection vs diffusion

(Bond, Capillary and Peclet numbers will be discussed in later lectures.)

Reynolds number: Laminar or turbulent?

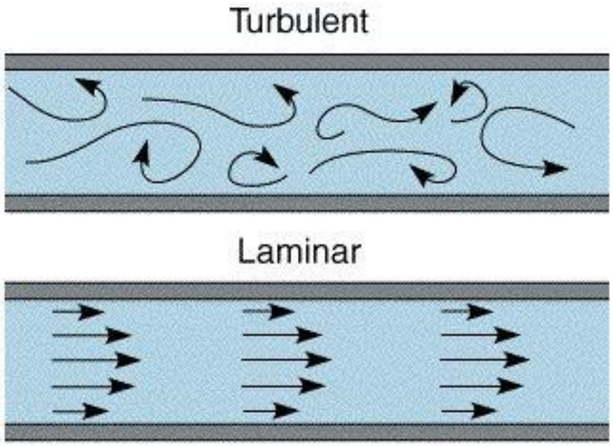
Reynolds number: inertial force / viscous force

Inertial force: $F = \rho\ V\ a = \rho\ L^3\ L/t^2 = \rho v^2 L^2 = \rho\ v^2 A$

Viscous force: $F = \mu\ \delta u/\delta y\ A = \mu v/L\ A$

$$Re = \frac{\rho v L}{\mu}$$

ρ = density
v = velocity
L = *characteristic length scale*
μ = dynamic viscosity



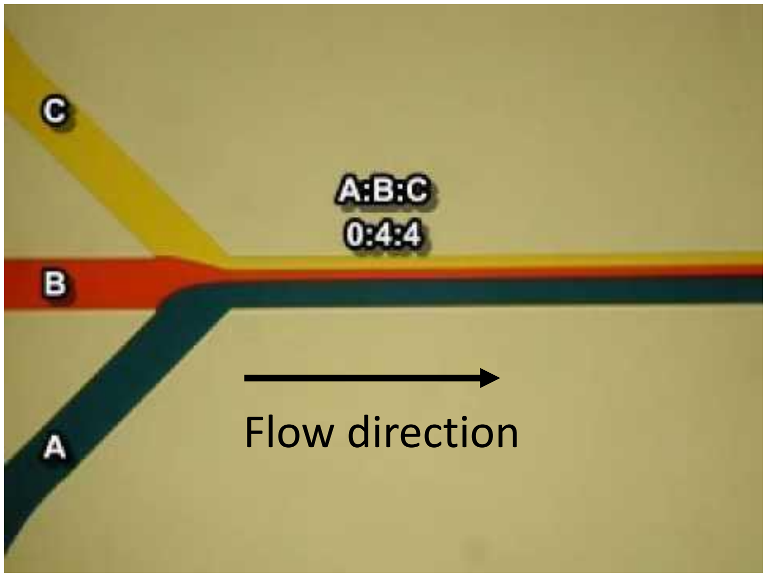
Re < ≈ 10, the flow is laminar

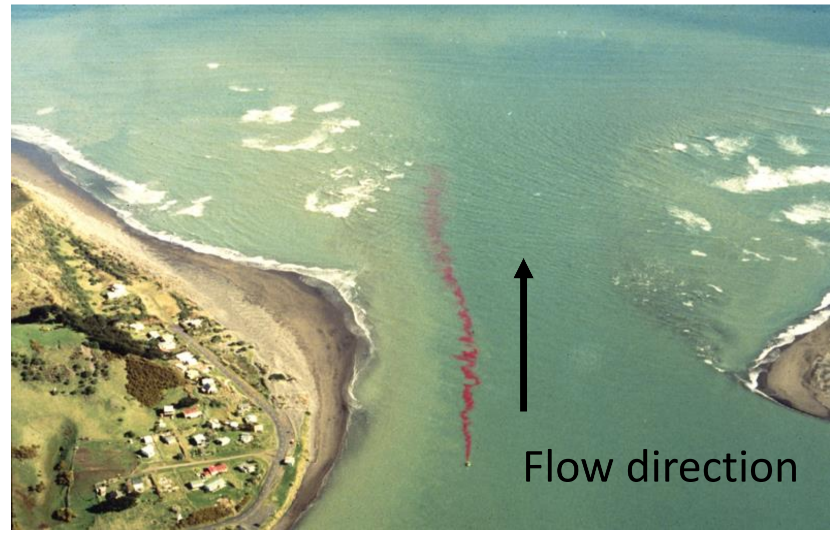
Re > ≈ 2000, the flow is turbulent

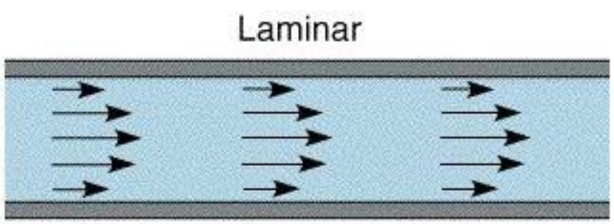
Note the exact threshold varies from source to source, they are rules of thumb**.**

Microfluidics is almost always laminar.

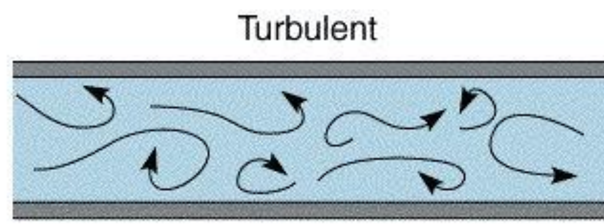
Laminar and turbulent







$$\mathrm{Re} = \frac{\rho v L}{\mu}$$



Micro: small L (and v)

Macro: large L (and v)

https://www.youtube.com/watch?v=tvdz5gHrOUY

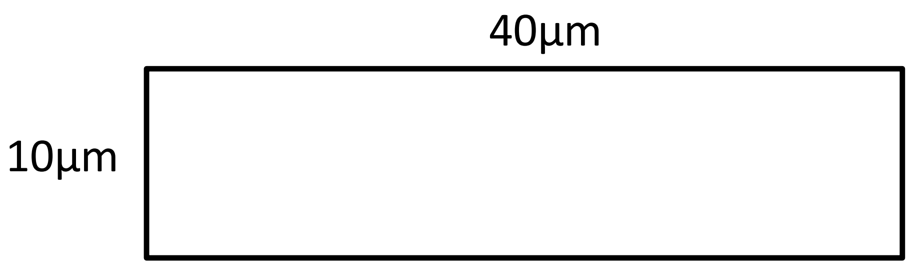
http://www.ifh.uni-karlsruhe.de/lehre/envflu_i/related_resources/photos.htm

Hydraulic radius and diameter

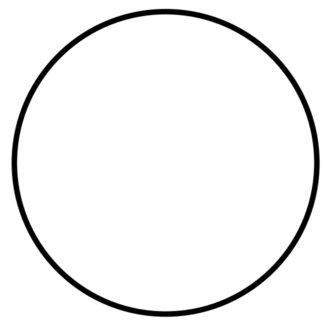
- What is the “characteristic length scale” of a system? It was needed for e.g. Reynolds number and other dimensionless numbers.
- Hydraulic radius, $r_H = 2A/p$

  (2 x area / perimeter of the channel cross section)
- For Reynolds number, use Hydraulic diameter $d_H = 2r_H$

- Example, consider these two microchannel cross sections:



Rectangle 10 x 40 μm
r_H = 8 μm, **d_H = 16 μm**



Circle, diameter 16 μm
r_H = 8 μm, **d_H = 16 μm**

To a rough first order approximation through Reynolds number, the laminar/turbulent properties of these two channels are thus the same.

Any questions up to this point?

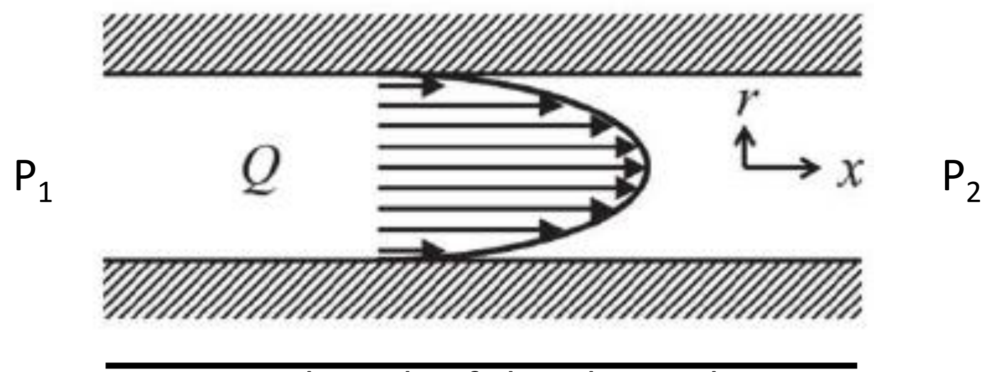
Next, we move to looking at pressure driven laminar flow in a microchannel, which is the main content of this lecture.

We will continue to practice this in exercises and design tasks.

Pressure driven laminar flow: Poiseuille flow

- Assumptions: Newtonian and non compressible fluid, laminar flow, cylindrical channel

- Velocity profile inside a microchannel is parabolic.
- Ideally, the velocity at channel walls is 0. (called *no-slip boundary condition*)

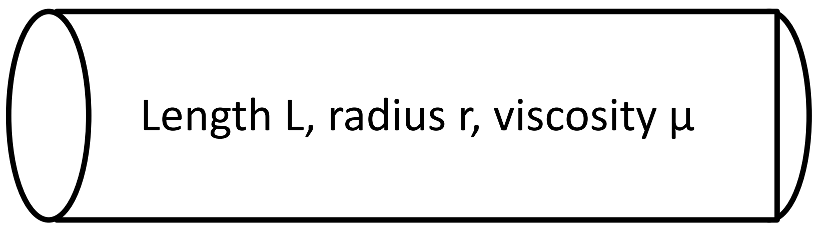


Parabolic velocity profile: $$u = -\frac{\Delta P}{L}\frac{1}{4\mu}(R^2 - r^2)$$

ΔP= pressure difference = P_1-P_2
L = length of the channel (**note, here NOT the characteristic dimension)**
R = the radius of the channel

Hagen-Poiseuille's Law

If a pressure difference of ΔP is applied over a **cylindrical** channel, what is the volumetric flow rate Q (μl/min)?



$$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

R_H = hydraulic resistance, which determines how much volumetric flow Q goes through a channel with pressure ΔP

If using a pressure pump: You set the pressure with the pump so calculate R_H, use Hagen-Poiseuille to calculate the flow rate

If using a syringe pump: You set the flow rate with the pump so calculate R_H and use Hagen-Poiseuille to calculate the back pressure

Flow rate and channel size

Hagen-Poiseuille's law: $$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

Notice the L^1 and inverse r^4 dependency of hydraulic resistance. Why inverse r^4 dependency?

The flow profile is parabolic, but we can still think of the volumetric flow as: $Q=A*V_{ave}$

One r^2 comes from the area of the cross section of the channel. $A \sim r^2$

The other r^2 comes from the average velocity of parabolic flow profile. $v \sim r^2$

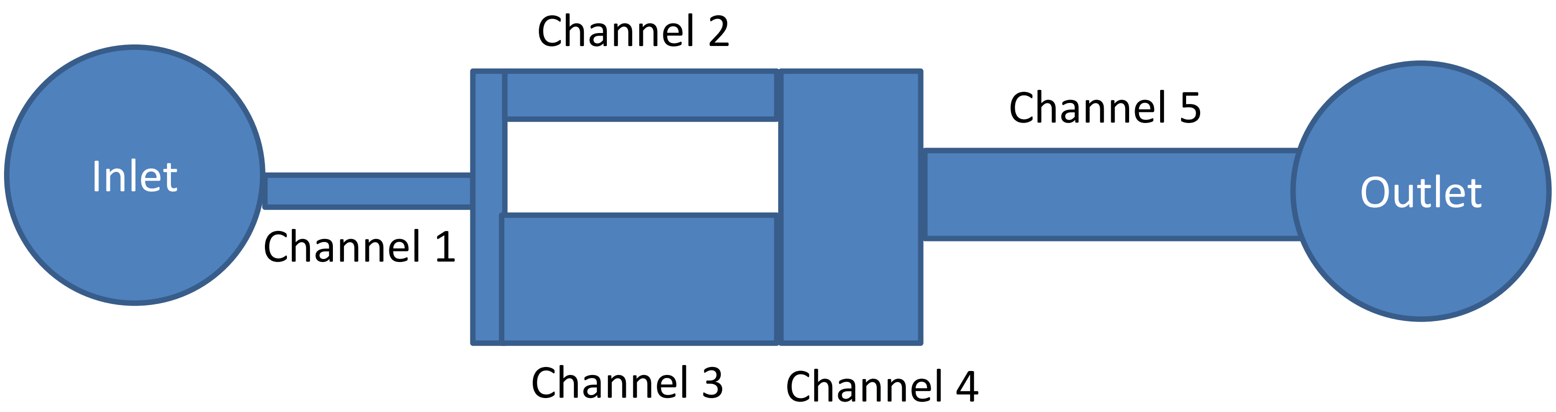
If you *halve* the dimension of the channel, the average velocity is ¼ and the cross-sectional area is also ¼ of the original. Therefore, with the same pressure, the volumetric flow rate is 1/16 of the original.

The smaller the channel, the more pressure it takes to drive liquid through it.

Fluidic circuits 1

Hagen Poiseuille's law allows us to calculate the flow in a single channel. But what if is a channel network?

In the below example, a pressure drives liquid from inlet to outlet.

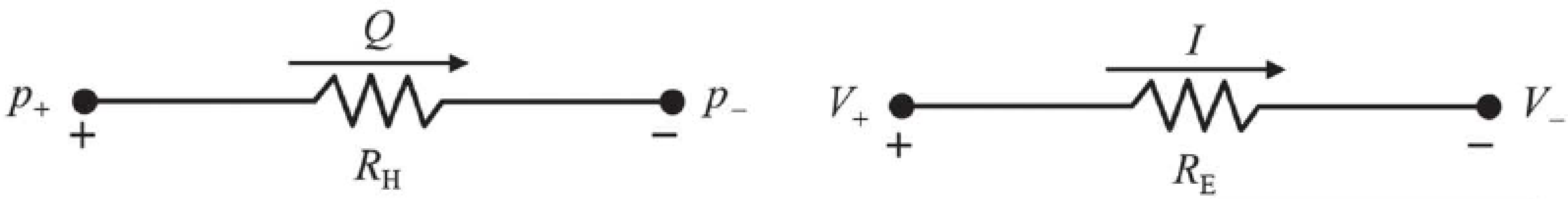


There are 5 different channel elements.

Channels 2 and 3 are in parallel.

Channels 1, 2/3, 4 and 5 are in a series.

Fluidic circuits 2



Hagen-Poiseuille's law: $\Delta p = p_+ - p_- = Q R_H$

Ohm's law: $V = V_+ - V_- = I R_E$

- Analogous to electric circuits
- Hydraulic resistances in series and parallel sum exactly as electrical resistors
- volume is conserved and pressure drop over a loop is 0. (Analogous to Kirchoffs laws)

Series: $$R_{total} = R_1 + R_2 + \ldots + R_N$$

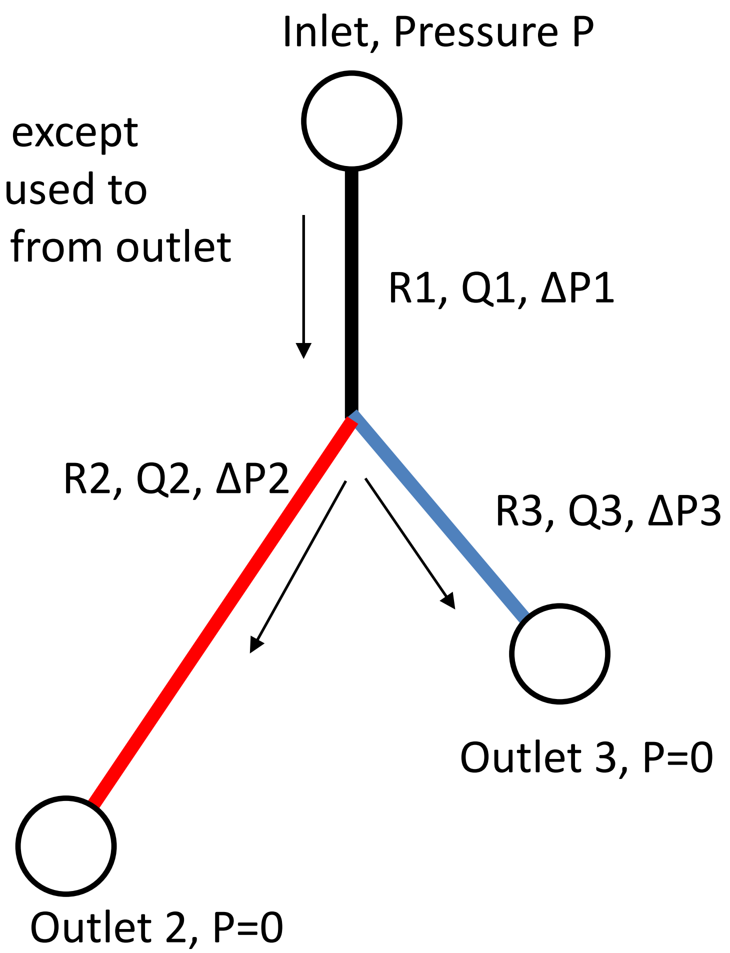
Parallel: $$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots. + \frac{1}{R_N}$$

Fluidic circuit:

1. calculate R of each component,
2. calculate R_{total},
3. insert R_{total} into Hagen-Poiseuille

An example:

The red and blue channels are otherwise the same except the red channel is twice as long. A syringe pump is used to Pump 3 µl / min of water. How much of this comes from outlet 2 and how much from outlet 3?



1. Conservation of volume:

$Q_1 = Q_2 + Q_3$

2.Pressure drop across both paths is the same:

$P = \Delta P_1 + \Delta P_2 = \Delta P_1 + \Delta P_3$

Ratio of flow in channels 2 and 3:

$\Delta P_2 = \Delta P_3$ -> *apply Hagen Poiseuille's law*

$R_2 Q_2 = R_3 Q_3$

$Q_2 / Q_3 = R_3 / R_2$

Hagen-Poiseuille: $\Delta P = R_H Q$

Example, continued

The red and blue channels are otherwise the same except the red channel is twice as long. A syringe pump is used to Pump 3 µl / min of water. How much of this comes from outlet 2 and how much from outlet 3?

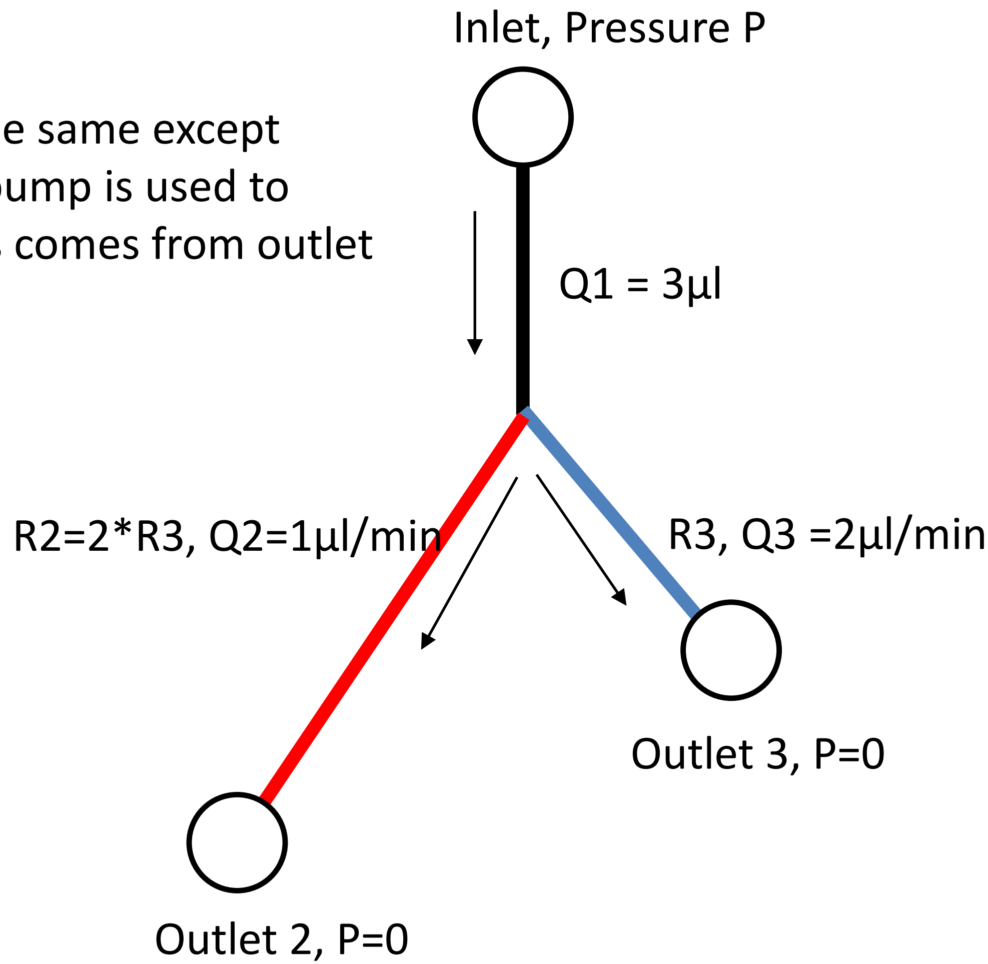
$R_2 = 2*R_3$ (Flow resistance of a cylinder)

$Q_2/Q_3 = R_3/R_2$, so $Q_2 = 0.5 * Q_3$

$Q_1 = Q_2 + Q_3 = 1.5 * Q_3 = 3$ µl /min

$Q_2 = 1$ µl /min

$Q_3 = 2$ µl /min



Hydraulic resistance: $\frac{8\mu L}{\pi r^4} = R_H$

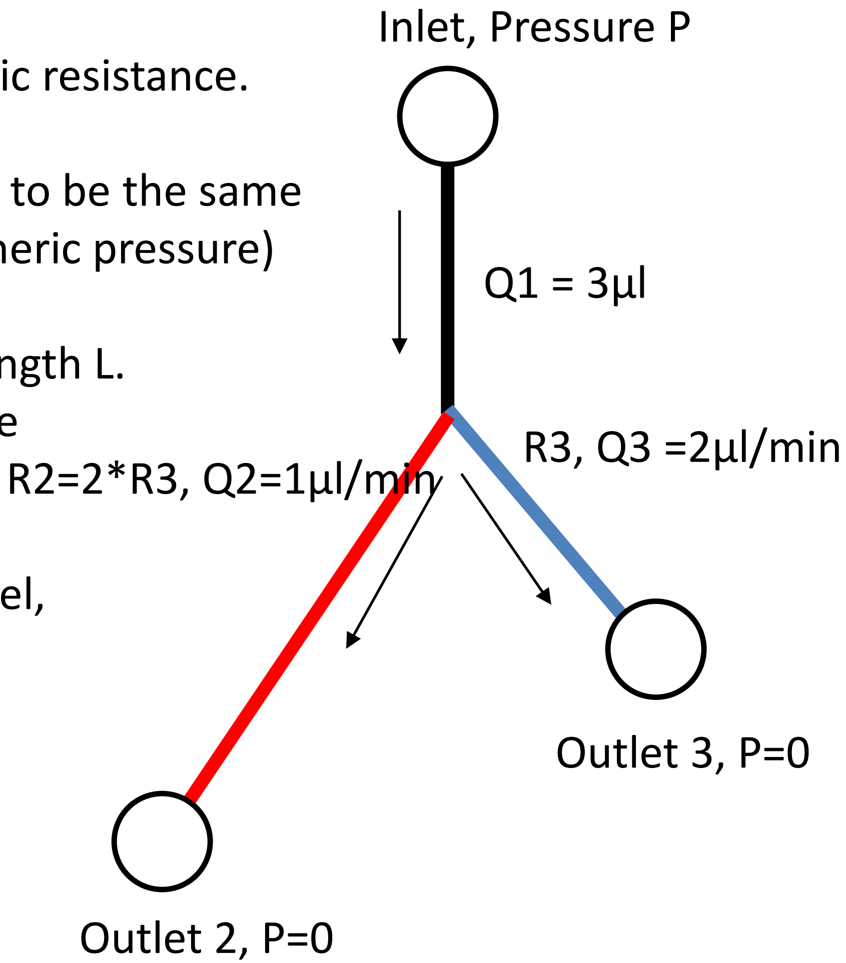
Example, summarizd

Pressure drop equals the flow rate times the hydraulic resistance.

Pressure drop in the red and the blue channels need to be the same (since both start from same pressure and in atmospheric pressure)

Flow resistance is directly proportional to channel length L.
-> Twice as long channel (otherwise indentical), twice as high hydraulic resistance

Since the resistance is twice as high in the red channel, the flow rate needs to be half.



$$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

Non-circular cross section

- Hagen-Poiseuille's Law is correct only for channels with circular cross section
- **In microfluidics, circular cross sections are not typical. How to calculate hydraulic resistance?**
- Very rough approximation, use hydraulic radius: $R_H \approx \dfrac{8\mu L}{\pi r_H^4}$
- Literature is full of all kinds of more accurate approximations for rectangular channels. For example, when channel height < width:

$$R_H \approx \frac{12\mu L}{wh^3(1-0.630h/w)}$$

- Scales most strongly with the smallest dimension of the channel!

Task:

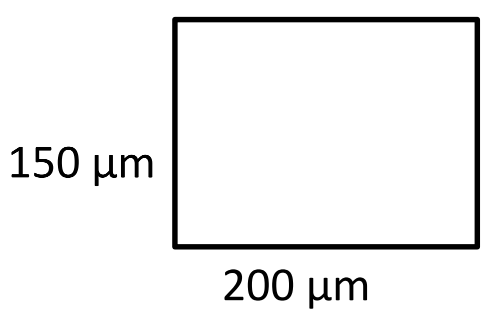
There are two channels, each 2 cm long. The cross sections are shown below. Both are connected to a pressure pump outputting 100 kPa. Which of the channels has higher volumetric flow rate Q through it?

Hagen Poiseuilles Law: $$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

Hydraulic radius: $r_H = 2A/p$

(A is cross sectional area, and p cross sectional perimeter)

Channel A



Channel B

Answer:

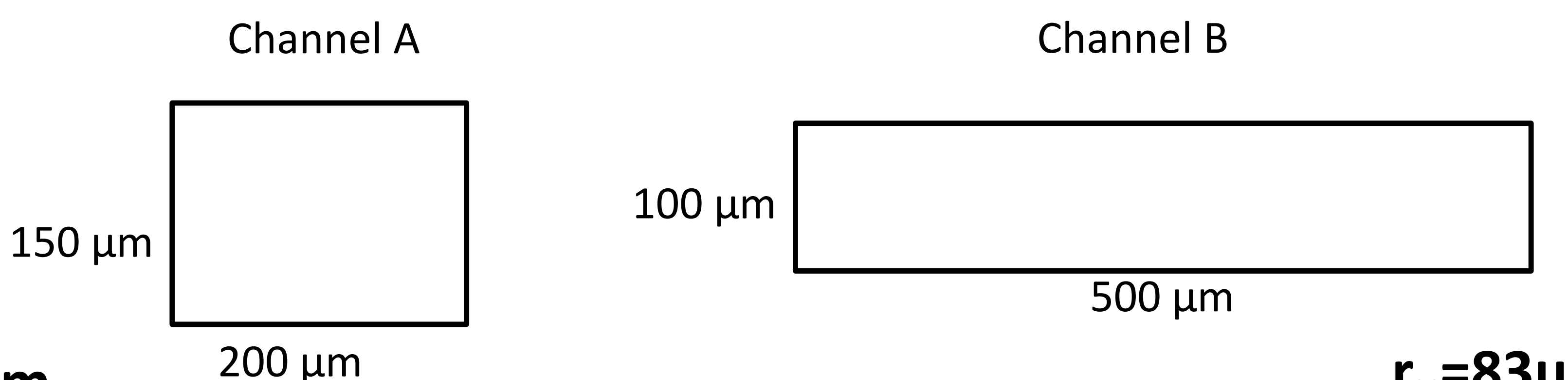
Hagen Poiseuilles Law: $$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

Hydraulic radius: $r_H = 2A/p$

The flow rate is inversely proportional to r_H^4.

Hence the channel with larger r_H has higher flow.

Channel A has larger r_H and hence with the same pressure, more volumetric flow.



r_H=86µm **r_H=83µm**

Microfluidic components

A very quick primer on basic components used in microfluidics:

1. Channels
2. Reservoirs
3. Pressure sources (often off chip)
4. Mixers
5. Filters
6. Valves

There will be a separate lecture on microfluidic components.

Flow actuation (pumping)

The most common flow types microfluidics:

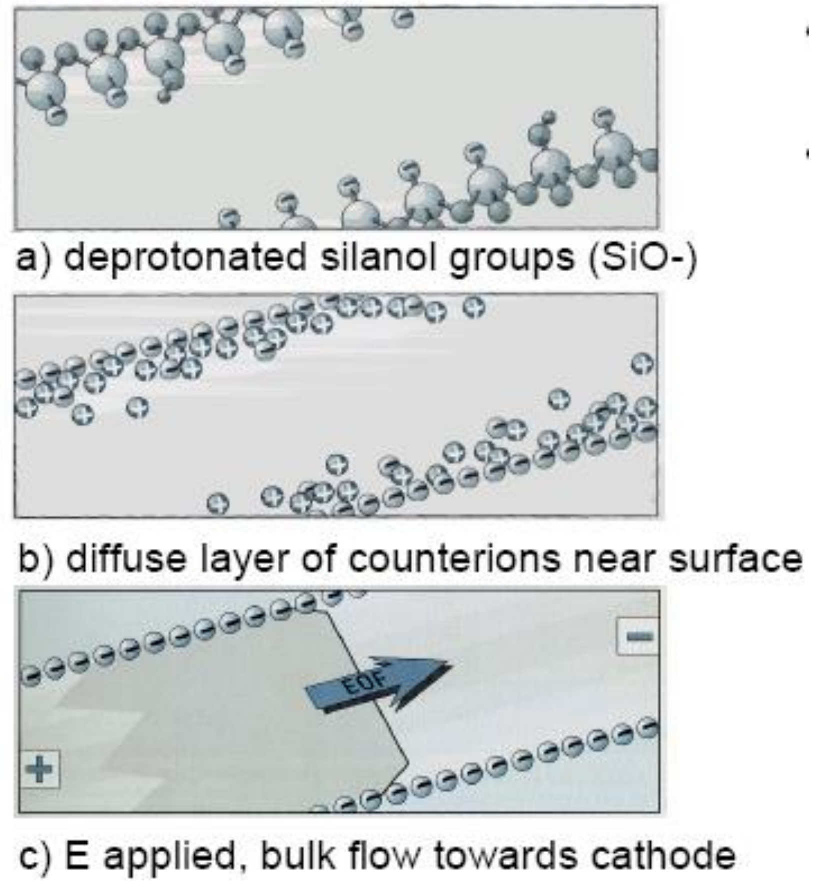
1. Pressure driven flow *(this lecture)*
2. Capillary flow *(next lecture)*
3. Electro-osmotic flow *(later lectures)*

Pressure drive flow:

- With set flow rate (syringe pump) or
- With set pressure (pressure pump)
- In both cases, use Hagen-Poiseuille's law.

Other alternatives:
centrifugal force, electrowetting, etc..

- Methods differ in flow profile, possible flow rates, required equipment and interconnections
- Different methods suited for different applications, sometimes combinations are used



Electro-osmotic flow

Typical flow rates, velocities, pressures

Typical flow rates:

Volume flow rates (Q) of micropumps are in the range of 1 nl/min to 10 ml/min (1 nl = 10^{-9} l = 10^{-12} m^3)

Typical linear velocities:

Volumetric flow rate Q = A *v [$m^3/s = m^2*m/s$]
Linear flow rate v= Q/A

If channel cross section is 100 µm*100 µm (10^{-4} m)
Q = 1 nl/min = 10^{-12} m^3/60 s = 16.7*10^{-15} m^3/s
v = 16.7*10^{-15} m^3/s /(10^{-4}*10^{-4} m^2)=1.67*10^{-6}m/s ≈ 2 µm/s

Q= 1 µl/min ➔ 2 mm/s
Q= 1 ml/min ➔ 2 m/s

Typical pressures:

Pressures can range from few Pa to many MPa

Review

- Fluids, shear, viscosity
- Reynolds number, laminar flow, flow profiles, consequences for microfluidics
- Basic physics of pressure driven laminar flow: flow resistance calculation, Hagen-Poiseuille's Law, fluidic circuits.

Reading material

- For Microfluidics 1, the extra reading material is:

Squires and Quake, Microfluidics: Fluid physics at the nanoliter scale, Rev. Mod. Phys. 77, 2005. Pages 977-995.

- Link: http://dx.doi.org/10.1103/RevModPhys.77.977

Also, if you feel unfamiliar with the concept of contact angle, there is some reading material on that in preparation for next weeks lectures in MyCourses.