Problem set 1, 13.01.2021:

(1.4) A system contains n atoms, each of which can only have zero or one quanta of energy. How many ways can you arrange r quanta of energy when
(a) n = 2, r = 1;
(b) n = 20, r = 10;
(c) $n = 2 \times 10^{23}$, $r = 10^{23}$?

(2.5) Two bodies, with heat capacities C_1 and C_2 (assumed independent of temperature) and initial temperatures T_1 and T_2 respectively, are placed in thermal contact. Show that their final temperature T_f is given by

$$T_f = (C_1T_1 + C_2T_2)/(C_1 + C_2).$$

If C_1 is much larger than C_2, show that

$$T_f \approx T_1 + C_2(T_2 - T_1)/C_1.$$

(3.6) In experimental physics, it is important to repeat measurement. Assuming errors are random, show that if the error in making a single measurement of a quantity X is Δ, the error obtained after using n measurements is $\Delta/\sqrt{n}$. (Hint: after n measurements, the procedure would be to take the n results and average them. So you require the standard deviation of the quantity $Y = (X_1 + X_2 + \cdots + X_n)/n$, where $X_1, X_2, \ldots, X_n$ can be assumed to be independent, and each has standard deviation Δ.

(3.8) We define the moment generation function $M(t)$ for a random variable x by

$$M(t) = \langle e^{tx} \rangle.$$

Show that this definition implies that

$$\langle x^n \rangle = M^{(n)}(0),$$

where $M^{(n)}(t) = d^n M/dt^n$ and further that the mean $\langle x \rangle = M^{(1)}(0)$ and the variance $\sigma_x = M^{(2)}(0) - [M^{(1)}(0)]^2$.

The number of the problem refers to the textbook.

Deadline for Problem set 1: 22nd January at 10:00 a.m.
Send the solutions to bayan.karimi@aalto.fi