Microfluidics and BioMEMS, Class Exercise 1, 20.1.2021

1. Microchannel volume

Calculate the total volumes of the following typical microfluidic channel dimensions.

a) width = 100 µm, height = 100 µm, length = 2 cm
b) width = 10 µm, height = 10 µm, length = 2 mm

2. Microchannel flow rate

A syringe pump is used to pump **0.25 µl / min** of liquid through the two channels as in **1a and 1b.** What are the corresponding average linear velocities (average linear velocity = Q/A) and the maximum linear velocities in the channels?

The maximum velocity of the parabolic flow profile is in the center of the channel and it is related to the average velocity simply as $v_{ave} = \frac{1}{2} * v_{max}$.

3. Pressure in microchannels

Calculate the *hydraulic resistance* of the channels with dimensions given in 1a and 1b. Use the *hydraulic radius* approximation. The dynamic viscosity for water at room temperature is about 1 mPa*s.

You have a pressure pump and want to pump with a flow rate of of 0.25 µl / min. Which pressure should you use?

4. Laminar or turbulent?

The channel dimensions are as in 1a, and the volumetric flow rate is still **0.25 µl / min**. Calculate the Reynolds number. Is the flow laminar or turbulent? The liquid being pumped is water, with viscosity µ = 1 mPa s (milli pascal seconds) at around room temperature. The density of water is 1000 kg / m^3.

Bonus exercise: Try to design a microfluidic channel dimensions and flow rate so that the flow becomes turbulent. Note that if you only increase the flow rate in a small channel, you will increase the pressure as well. You can calculate this pressure by Hagen Poiseuille’s law. Absolute maximum pressure in e.g. PDMS-glass chip is 300kPa, after which the bonding breaks. Likely the interconnections leak at much lower pressures.

1. Microchannel volume

Calculate the total volumes of the following typical microfluidic channel dimensions.

a) width = 100 µm, height = 100 µm, length = 2 cm
b) width = 10 µm, height = 10 µm, length = 2 mm

You can brute force the units: convert everything to SI base units and work from there. Often there are easier ways though.

Hint: Choose good base units. Either units you have learned to use in a given situation or if unknown, then units without big powers of 10.

For a microchannels and volumes, mm is often convenient since mm^3 = µl

a) 0. 1 mm * 0. 1 mm * 20 mm = 0.2 mm^3 = **0.2 µl = 200 nl**

Same calculation in µm instead: ($µm^3$ = 1 fl)

100µm * 100 µm * 20 000µm = 200 000 000 $µm^3$ = 200 000 000 fl = 0.2 µl

These are typical microchannel dimensions.

b) 0. 01 mm * 0. 01 mm * 2 mm = 0.0002 mm^3 = 0.2 nl **= 200 pl**

(Note that all dimensions are 1/10 of a so the volume is 1/1000)

These are also typical microchannel dimensions.

Often good units:

Dimensions: nm, µm, mm

Volume: pl, nl, µl

Pressure: Pa, kPa

Volumetric flow rate: µl/s, µl/min, ml/min, nl/s

Linear velocities: mm/s, µm/s, mm/min

2. Microchannel flow rate

A syringe pump is used to pump 0.25 µl / min of liquid through the two channels as in **1a and 1b.** What are the corresponding average linear velocities? (average linear velocity = Q/A).

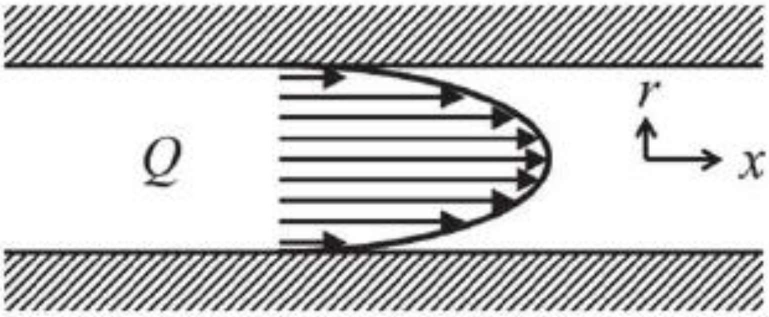
Since you already have the channel dimensions in mm:s, it is best to also have the volume as mm^3. 0.25µl / min = 0.25 mm^3 / min.

$$a)\ v = \frac{0{,}25\ mm^3/min}{0.1\ mm*0.1mm} = 25\frac{mm}{min} \approx 0.4\frac{mm}{s} = 400\frac{\mu m}{s}$$

$$b)\ v = \frac{0{,}25\ mm^3/min}{0.01\ mm*0.01mm} = 2500\frac{mm}{min} \approx 40\frac{mm}{s}$$

We calculated the v_{ave}, for which Q= v_{ave} * A

There is a very simple relation between v_{ave} and the maximum velocity of the parabolic flow profile: v_{ave} = ½ * $_{vmax}$.



Conclusion, with set channel dimensions and a set volumetric flow rate, it is very easy to calculate the average velocity, the maximum velocity and the parabolic velocity profile.

This is possible because of the well-behaved (relatively) simple physics of laminar flow.

The above is strictly true for channels with circular cross section. It is approximately true also for rectangular cross section. The velocity at all walls is still 0 and the flow is overall parabolic, but not exactly due to the corners. In these cases there is usually no analytical expression for the flow profile so we just approximate with e.g. the hydraulic radius approximation. The concept of average linear velocity is still valid.

3. Pressure in microchannels

Calculate the hydraulic resistance of the channels with dimensions given in 1a and 1b. Use the hydraulic radius approximation.

You have a pressure pump and want to pump with a flow rate of of 0.25 µl / min. Which pressure should you use?

Hagen-Poiseuille equation: P=R*Q. It connects the pressure (P) across the microchannel, the microchannel dimensions (which determine R, the hydraulic resistance) and the volumetric flow rate (Q).

STEP 1:

First, let’s calculate the hydraulic radii of the channels a and b:

$$r_{HA} = \frac{2A}{P} = \frac{2wh}{2w+2h} = \frac{wh}{w+h} = \frac{100\mu m * 100\mu m}{100\mu m + 100\mu m} \approx 50\mu m$$

Similarly, for channel b the hydraulic radius is 5µm.

STEP 2:

Next, lets calculate the hydraulic resistances.

$$R_{HA} = \frac{8\mu L}{\pi r_H^4} = \frac{8 \cdot 1mPas \cdot 20mm}{\pi (0.05mm)^4} \approx 8000\ Pas/mm^3$$

and for channel B:

$$R_{HB} = \frac{8\mu L}{\pi r_H^4} = \frac{8 \cdot 1mPas \cdot 2mm}{\pi (0.005mm)^4} \approx 8000\,000\ Pas/mm^3$$

STEP 3:

And finally we use Hagen-Poiseuille’s law to calculate the needed pressure to achieve the desired flow rate:

$$P_A = 8000 \frac{Pa * s}{mm^3} * 0.25 \frac{mm^3}{60s} \approx 33\ Pa$$

$$P_B = 8000\ \frac{kPa * s}{mm^3} * 0.25\frac{mm^3}{60s} \approx 33\ kPa$$

Since a) and b) were a relatively typical big and small microchannel, and 0.25µl/min is a relatively normal flow rate, it follows that pressures in tens of pascals to 10s of kilopascals are common in microfluidics.

Bonus: why is the hydraulic resistance of b) 1000 times higher than the resistance of a?

The hydraulic resistance for a circular cross section channel is calculated as: $R_H = \frac{8\mu L}{\pi r^4}$, where µ is the viscosity, L is the length of the channel and r is the radius of the channel. So we see that it scales inversely with the fourth power of the cross sectional dimension and scales linearly with the length of the channel.

The cross sectional dimension of b) is 1/10 of the characteristic dimension in a), which means a factor of 10 000 (from r^4) higher hydraulic resitance from that. However, the length of b) is also 1/10, which gives a factor of 0.1 (from L). Multiplying together we get a factor of 1000.

The hydrostatic pressure of 1mm of water is about 10 Pa. In case of a), instead of pump we could thus use a water column with height 3.3mm, which could be reasonable. In case of b) it is probably not practical since 3.3m of water would be needed.

4. Laminar or turbulent?

The channel dimensions are as in 1a, and the volumetric flow rate is still **0.25 µl / min**. Calculate the Reynolds number. Is the flow laminar or turbulent? The liquid being pumped is water, with viscosity µ = 1 mPa s (milli pascal seconds) at around room temperature. The density of water is 1000 kg / m^3.

Let's construct the Reynolds number. Understanding is better than remembering. Reynolds number was the ratio of inertial forces to viscous forces. The parameters that affect these are:

L = characteristic length scale

v =characteristic velocity

µ = dynamic viscosity

ρ = density

If you can remember that those four parameters appear in the formula, it is easy to reason which ones go to the numerator and which go to the denominator. The ones at the numerator are the ones that mean that the larger they are, the more prominent are inertial forces. They are the density, the velocity and the dimension. The viscosity on the other hand goes to the denominator as higher viscosity leads to higher magnitude of viscous force. (Note, for a more thorough derivation, see the lecture slides and the additional reading material

$$\mathrm{Re} = \frac{\rho v L}{\mu}$$

Some comments on the terms:

L: For a microchannel the most proper "characteristic length scale" would be the hydraulic diameter, which is twice the hydraulic radius. However, you could use any cross sectional dimension and it would lead to very similar Reynolds number. But choosing the length of the microchannel would be an error. Unfortunately the symbol L is commonly used in Hagen-Poiseuille to mark the length of the microchannel, so be wary that you don't mix the two concepts.

v: For the characteristic velocity, use the average linear velocity, Q/A. (You can also use the maximum velocity for the Reynolds number, remember that it is an order of magnitude estimation)

ρ: Density, simply use the density of the liquid.

μ: For the form presented above and in the lecture slides, you need to use the dynamic viscosity μ, in the units pascal seconds. There is also a form of Reynolds number that uses the kinematic viscosity which is μ/ρ.

Water viscosity around 1 mPas at RT

Units of viscosity:

$$\mu = 1mPas = 10^{-3}\frac{N}{m^2}s = 10^{-3}\frac{kgm/s^2}{m^2}s = 10^{-3}\frac{kg}{ms}$$

$$\mathrm{Re} = \frac{\rho v L}{\mu}$$

We calculate the hydraulic diameter:

$$L = 2R_h = \frac{4A}{P} = \frac{4*100\,\mu m*100\,\mu m}{4*100\,\mu m} = 100\,\mu m = 0.1mm$$

The average linear velocity we already calculated in 2a) to be ≈ 0.4 mm/s.

$$Re = \frac{\rho v L}{\mu} = \frac{\frac{1000kg}{m^3}*\frac{0.4mm}{s}*0.1mm}{1mPas}$$

$$Re = \frac{1000*0.4*10^{-3}*0.1*10^{-3}}{10^{-3}}*\frac{kg*m*m}{m^3*s*kg*m*s^{-2}*m^{-2}*s}$$

$$= 0.04$$

< 1 so the flow is definitely laminar.