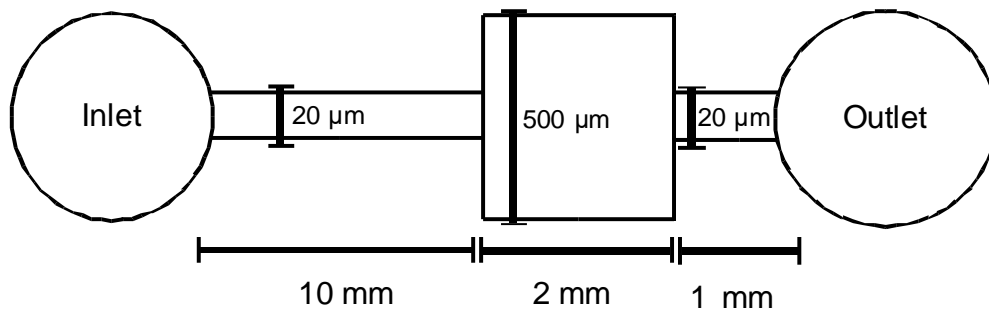


### 1. Hagen-Poiseuille's Law and a microfluidic circuit

A microfluidic chip consists of an inlet, an outlet, two 20 microns wide channel segments and a 500 microns wide reaction chamber as shown in the picture (top view). The channel depth throughout the chip is  $80\text{ }\mu\text{m}$  and the lengths and widths of the three segments have been marked to the picture. The liquid used on the chip is water at  $20^\circ\text{C}$  ( $\mu = 1\text{ mPa}\cdot\text{s}$ ).

- a) Calculate the fluidic resistance of the chip using the hydraulic radius approximation.
- b) Calculate the volumetric flow rate (in  $\mu\text{l}/\text{min}$ ) when a  $1000\text{ Pa}$  pressure difference is applied between the inlet and the outlet.

*(Bonus if you are really fast: Repeat the calculations but have the 3 elements in a parallel configuration instead of series.)*



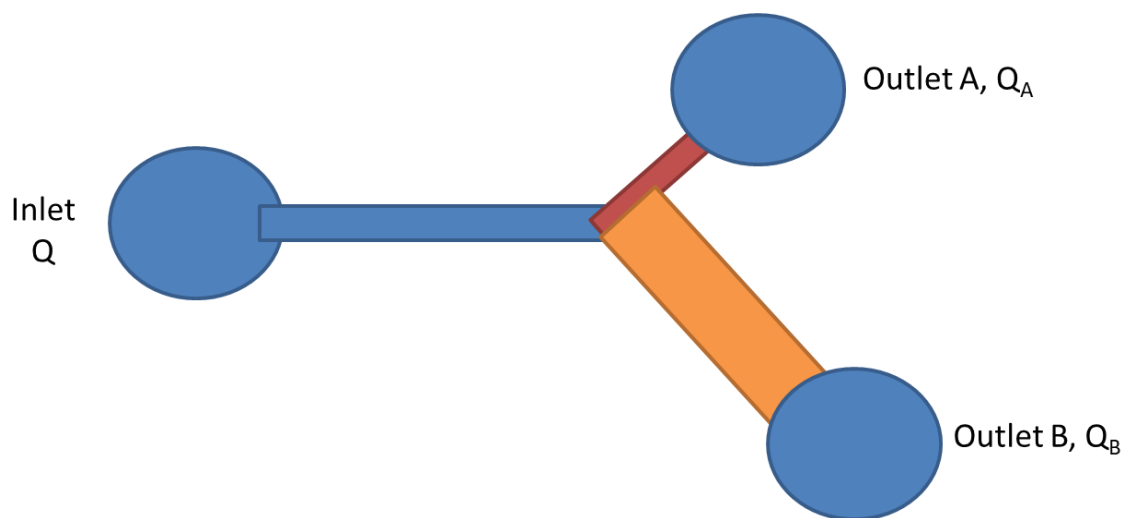
- c) The chip inlet is connected to a pressure pump with tubing that has circular cross section and inner radius of  $1\text{ mm}$ . The length of the tubing however varies from experiment to experiment between  $10\text{ cm}$  and  $30\text{ cm}$ . Does this variation have a big effect on the flow rate?

## 2. A microfluidic circuit and scaling

Liquid with volumetric flow rate  $Q$  is pumped with a syringe pump to Y shaped channel. (The value of  $Q$  does not matter for this question but let's say it is 900 nl / min)

After the shared channel, the channel splits into two. Channel A has dimensions width  $w$ , height  $h$  and length  $L$ . Channel B has dimensions width  $2w$ , height  $2h$  and length  $2L$ . Which fraction of the fluid flow goes to outlet A and outlet B respectively?

(Hint: you do not need to calculate out the hydraulic resistance. Use scaling arguments to show that the resistance in channel B is  $x$  times the resistance in channel A. The volumetric flow rate is then inversely proportional to the resistance, as per Hagen-Poiseuille's law).



## 3. Capillary pressure, bond number.

A microfluidic channel has a rectangular cross section of  $20\text{ }\mu\text{m} \times 20\text{ }\mu\text{m}$  and the advancing water contact angle of water and the chip is  $20^\circ$ . Surface tension of water in RT is about 72 mN/m.

a) What is the bond number? (For Bond number, the characteristic length scale can be the hydraulic diameter or simply the microchannel height)

b) What is the capillary pressure?

c) How high would the liquid rise if the channel was turned into vertical orientation. (1 mm of water is about 10 Pa)

**(Bonus if you are really fast:** The sidewalls and the bottom of the microchannel have contact angle  $20^\circ$  but the top wall (the bonded layer) has contact angle of  $120^\circ$  instead. What is the maximum width to height ratio of such a channel that fills spontaneously by capillary forces?)