

Descriptive Statistics II

Matti Sarvimäki

Principles of Empirical Analysis
Lecture 3

Course outline and learning objectives

- Data and measurement
 - ① introduction, data
 - ② today: descriptive statistics
 - ③ **more descriptive statistics**
- Experimental methods
 - ① causality and research designs
 - ② statistical significance
 - ③ statistical power
 - ④ noncompliance
- Quasi-experimental methods
 - ① observational data and quasi-experiments
 - ② difference-in-difference (DiD)
 - ③ regression discontinuity design (RDD)
 - ④ regression and matching
- Structural methods
- Today's learning objectives. After this lecture you should understand
 - ① the meaning of central concepts for conditional descriptive statistics
 - ② how to characterize the conditional distributions
 - ③ how to characterize distributions of more than one variable more generally
- Outline
 - conditional descriptive statistics
 - correlation
 - example: income distribution

Conditional descriptive statistics

- Conditional descriptives are statistics of a variables *conditional* on another variables
 - e.g. conditional expectation

$$\mathbb{E}[Y|X = x]$$

i.e. expectation of random variable Y when another random variable X takes value x

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- empirical counterpart: conditional sample average
- Conditional descriptive statistics build on the joint distribution of two or more variables

Summary for variables: earn
by categories of: edul

| edul | mean | N |
|--------------|--------------|-------------|
| Less/unknown | 15527 | 1807 |
| Secodary | 22076 | 2720 |
| Bachelor | 32644 | 1080 |
| Master | 42292 | 346 |
| Lis./PhD | 57950 | 20 |
| Total | 23297 | 5973 |

Source: FLEED teaching data
tabstat earn, by(edul) stat(mean N)
alternatively try: tabulate edul, sum(earn)
(see the full code at course website)

- A simple, yet efficient way to display (small) data of two variables is **cross tabulation**
 - 1 the no. rows = no. values that Y can take
 - 2 the no. columns = no. values that X can take
 - 3 the cells report no. observations with value (y, x)

| edul | woman | | Total |
|--------------|-------|-------|-------|
| | 0 | 1 | |
| Less/unknown | 1,128 | 894 | 2,022 |
| Secondary | 1,430 | 1,313 | 2,743 |
| Bachelor | 439 | 651 | 1,090 |
| Master | 181 | 185 | 366 |
| Lis./PhD | 17 | 6 | 23 |
| Total | 3,195 | 3,049 | 6,244 |

Source: FLEED teaching data
tabulate edul woman

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| | 0 | 1 |
| Less/unknown | 18.07 | 14.32 |
| Secodary | 22.90 | 21.03 |
| Bachelor | 7.03 | 10.43 |
| Master | 2.90 | 2.96 |
| Lis./PhD | 0.27 | 0.10 |
| | | 100.00 |

Source: FLEED teaching data
tabulate edul woman, cell nofreq

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- Alternatively, cross tabulation cells may report the share of observations with value (y, x)
- This is the empirical counterpart of the **joint density function**

$$f_{XY}(x, y) = \mathbb{P}(X = x, Y = y)$$

i.e. the probability that random variable X takes the value x *and* that random value Y takes the value y

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Marginal distribution

- The marginal distribution of Y is defined as

$$f_Y(y) = \sum_{x \in X} f_{XY}(x, y)$$

- This is just probability of X when not taking the value of Y into account

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| Secondary | 22.90 | 21.03 | 43.93 |
| Bachelor | 7.03 | 10.43 | 17.46 |
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Marginal distribution

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- This is just probability of Y when not taking the value of X into account
- Similarly, the marginal distribution of X is

$$f_X(x) = \sum_{y \in Y} f_{XY}(x, y)$$

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- The conditional distribution of Y is defined as

$$f_{Y|X}(y|x) = \frac{f_{XY}(x, y)}{f_X(x)}$$

i.e. the probability that Y takes value y conditional that X takes value x

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i.e. the probability that Y takes value y conditional that X takes value x

- Example: Probability that a working age woman living in Finland in 2010 had a bachelor degree
 - $\hat{P}(X = w, Y = b) = .1043$
 - $\hat{P}(X = w) = .4883$
 - $\hat{P}(Y = b|X = w) = \frac{.1043}{.4883} \approx .213$
 - where the "hats" indicate that we are using **estimates** of the population probabilities $\mathbb{P}(\cdot)$

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- Let's get back to conditional expectation. When Y is continuous^a, the **conditional expectation function** (CEF) is

$$\mathbb{E}[Y|X = x] = \int t f_{Y|X}(t|X = x) d(t)$$

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^aDiscrete version: $\mathbb{E}[Y|X = x] = \sum t f_{Y|X}(t|X = x)$

- Let's get back to conditional expectation. When Y is continuous^a, the **conditional expectation function** (CEF) is

$$\mathbb{E}[Y|X = x] = \int tf_{Y|X}(t|X = x)d(t)$$

i.e. **population average of Y holding X fixed**

- in other words: weighted average of Y , where the weight for of each value of Y is the share of sub-population (for whom $X = x$) with this value of Y

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- in other words: weighted average of Y , where the weight for of each value of Y is the share of sub-population (for whom $X = x$) with this value of Y
- X can also be a vector, i.e. can include many conditioning variables

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Conditional expectation

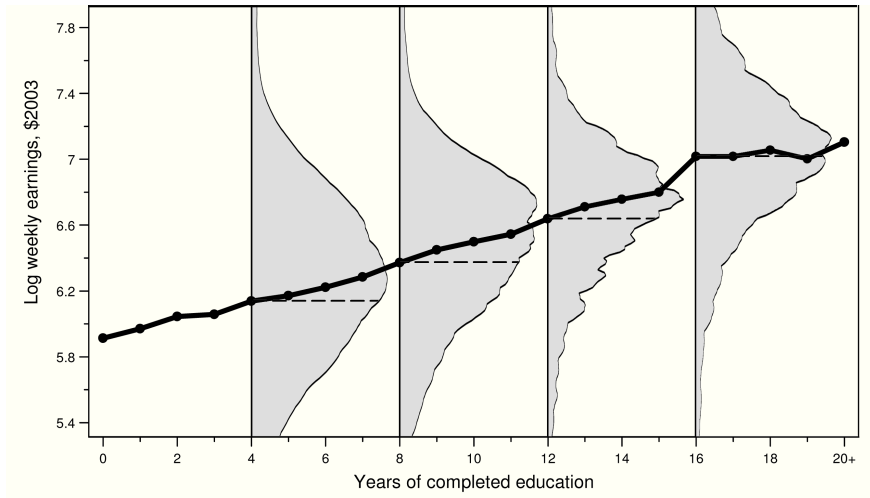


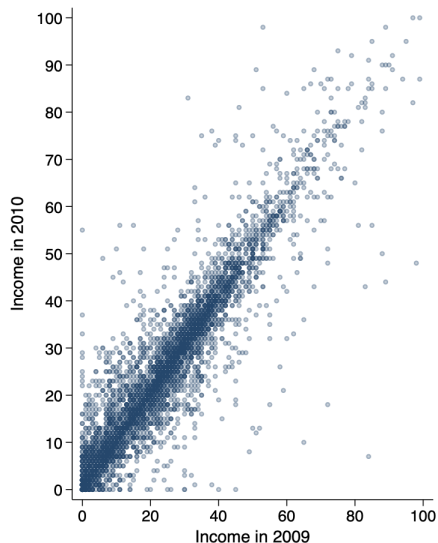
Figure 3.1.1: Raw data and the CEF of average log weekly wages given schooling. The sample includes white men aged 40-49 in the 1980 IPUMS 5 percent file.

Source: Angrist and Pischke (2009).

- Conditional expectation is a powerful way to detect how variables are associated with each other

Scatter plot

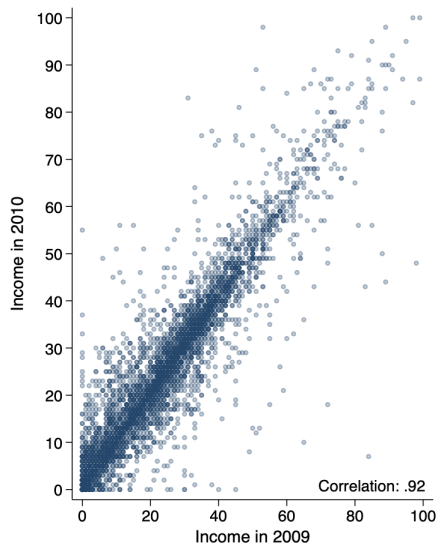
- Conditional expectation is a powerful way to detect how variables are associated with each other
- An alternative approach is to show all observations and plot two variables against each other
- Example: persistence of income over time
 - **scatter plot**: each dot in this graph shows each individual's income in 2009 and 2010



Source: FLEED teaching data
`scatter earn earn.t1, mcolor(navy%25) msize(vsmall)`

Scatter plot

- Conditional expectation is a powerful way to detect how variables are associated with each other
- An alternative approach is to show all observations and plot two variables against each other
- Example: persistence of income over time
 - **scatter plot**: each dot in this graph shows each individual's income in 2009 and 2010
- The best known descriptive statistic to characterize how two variables' values are aligned is **correlation**
 - here, the correlation is 0.92
 - next: what does that mean?



Source: FLEED teaching data
`scatter earn_t1, mcolor(navy%25) msize(vsmall)`

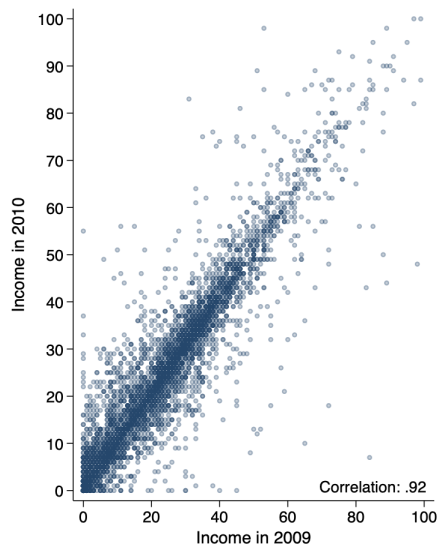
- To get to correlation, we need to first define the **covariance** of Y and X

$$\text{Cov}(X, Y) = \mathbb{E}[X - \mathbb{E}(X)]\mathbb{E}[Y - \mathbb{E}(Y)]$$

- ... and its empirical counterpart

$$\widehat{\text{Cov}}(X, Y) = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

- Here, the covariance is 256.6
 - a hard number to interpret



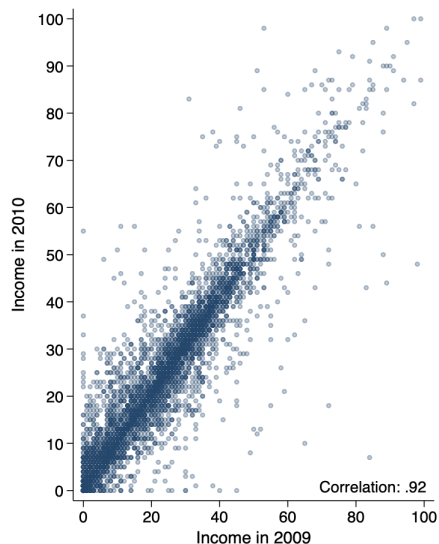
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- Pearson correlation coefficient is a scaled covariance

$$\text{Cor}(X, Y) = \frac{\text{Cov}(X, Y)}{SD(X)SD(Y)}$$

that varies between $-1 \leq \text{Cor}(X, Y) \leq 1$

- just makes the number easier to interpret



Source: FLEED teaching data

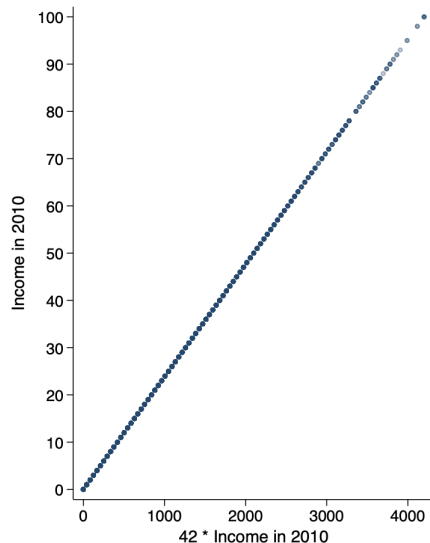
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- More examples
 - correlation 1

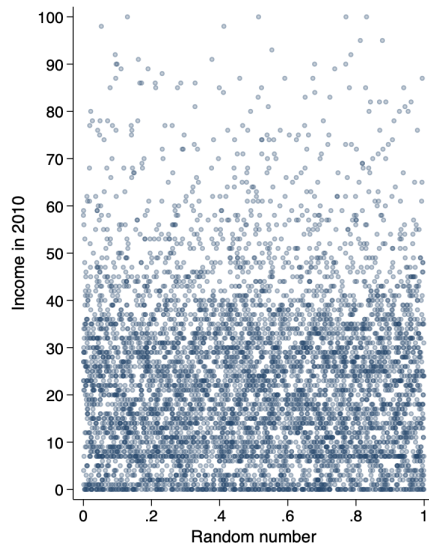


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- More examples
 - correlation 1
 - correlation 0.009

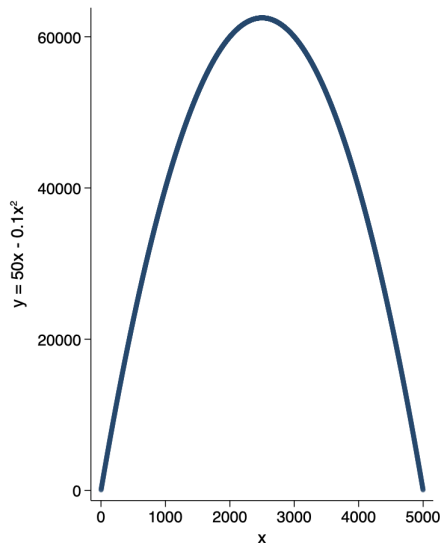


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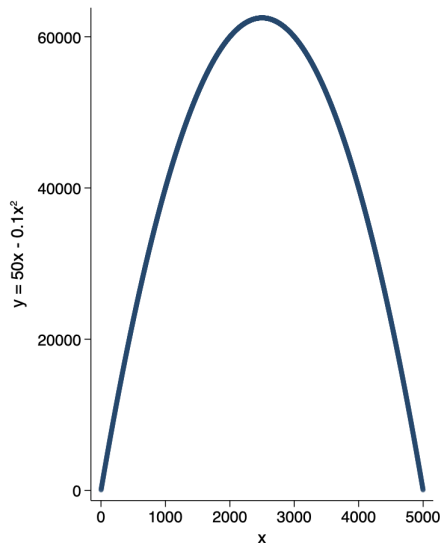


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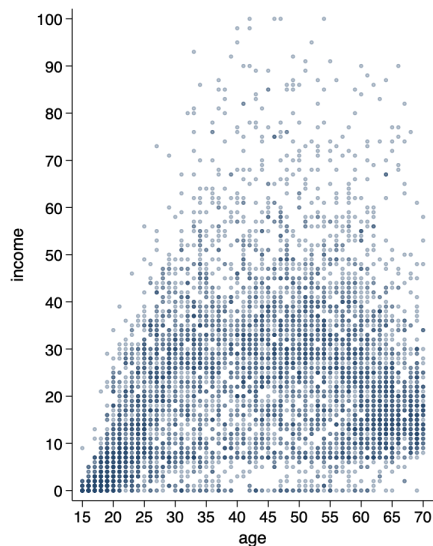
that varies between $-1 \leq \text{Cor}(X, Y) \leq 1$

- just makes the number easier to interpret
- More examples
 - correlation 1
 - correlation 0.009
 - correlation 0
- Correlation measures a linear dependence
 - the point: possible to have perfect dependence and zero correlation



Association between age and income

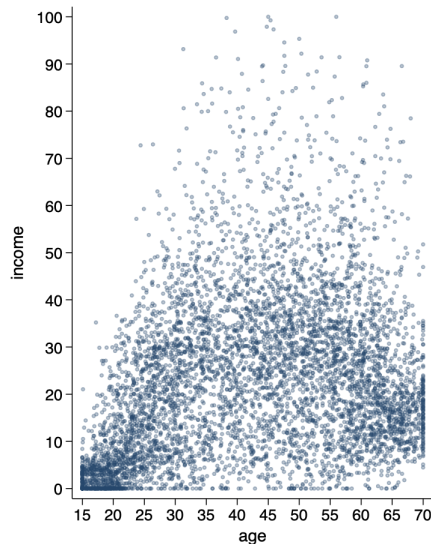
- Another example: How does income vary with age?
 - scatter plot of the full data (correlation = .28)



Source: FLEED teaching data
scatter earn age, 'opt'

Association between age and income

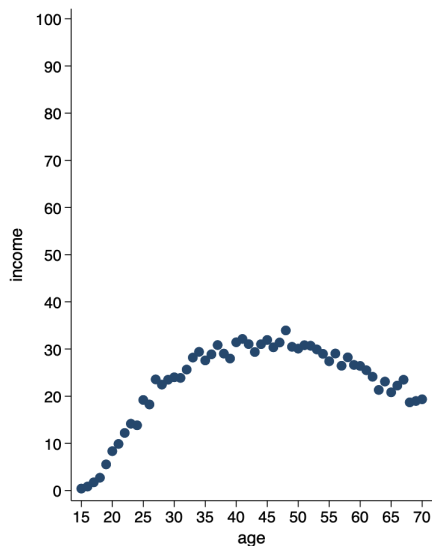
- Another example: How does income vary with age?
 - scatter plot of the full data (correlation = .28)
 - adding a little bit of noise sometimes makes the pattern more visible



Source: FLEED teaching data
scatter earn age, 'opt' jitter(10)

Association between age and income

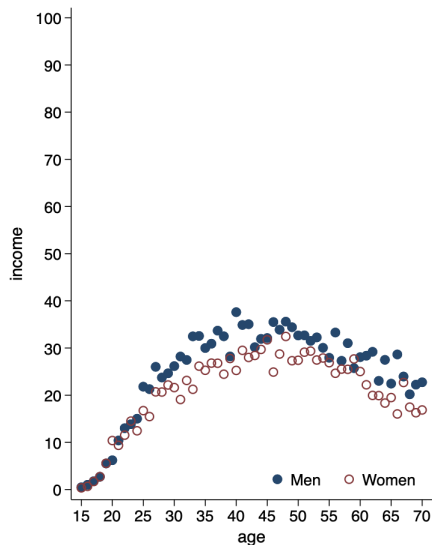
- Another example: How does income vary with age?
 - scatter plot of the full data (correlation = .28)
 - adding a little bit of noise sometimes makes the pattern more visible
- Getting back to conditional expectation
 - sample averages by age



```
collapse (mean) earn, by(age)
scatter earn age, 'opt'
```

Association between age and income

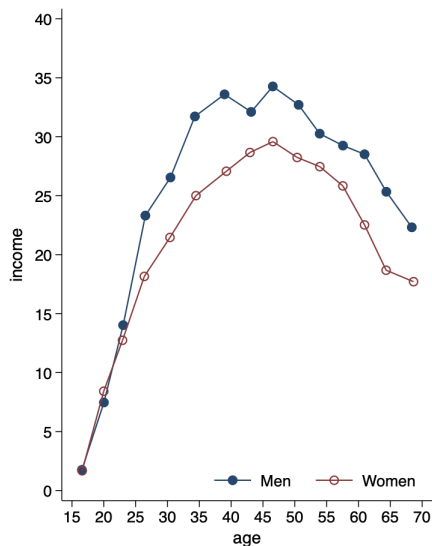
- Another example: How does income vary with age?
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 - sample averages by age
 - age and gender



see course website for the full code

Association between age and income

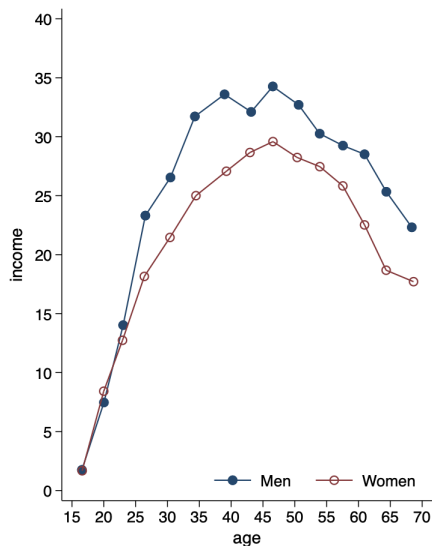
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- Getting back to conditional expectation
 - sample averages by age
 - age and gender
 - aggregating a bit to reduce noise



see course website for the full code

Association between age and income

- Another example: How does income vary with age?
 - scatter plot of the full data (correlation = .28)
 - adding a little bit of noise sometimes makes the pattern more visible
- Getting back to conditional expectation
 - sample averages by age
 - age and gender
 - aggregating a bit to reduce noise
- General lesson: looking at the data in several ways almost always a good idea



see course website for the full code

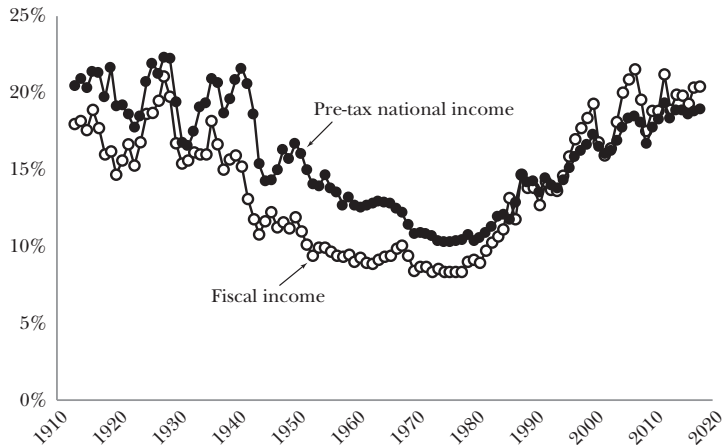
Examples:
Recent work on the widening U.S. income distribution

- We now have tools to understand the basic results of the income distribution literature
 - top percent shares
 - changes over the entire distribution
 - group averages
 - social mobility
- Much of this research is based on tax data
 - available over long time periods and many countries, but earlier periods limited to the top (historically, only the rich paid taxes)
 - tax records never capture all income → ongoing work to deal with the missing parts



Source: [The Economist](#), 28 Nov 2019

Share of Income Earned by the Top 1 Percent

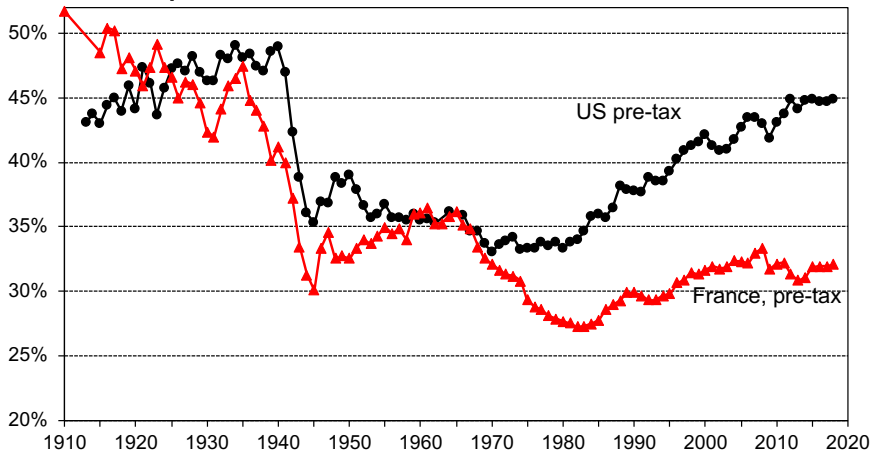


- US top 1% share based on tax data only and [Distributional National Accounts](#) by PSZ

Note: This figure compares the share of fiscal income earned by the top 1 percent tax units (from Piketty and Saez 2003, updated series including capital gains in income to compute shares but not to define ranks, to smooth the lumpiness of realized capital gains) to the share of pre-tax national income earned by the top 1 percent equal-split adults (from Piketty, Saez, and Zucman 2018, updated September 2020, available on WID.world).

Source: [Saez and Zucman \(2020\)](#), *Journal of Economic Perspectives*.

Top 10% Income Shares in the US and France, 1910-2018

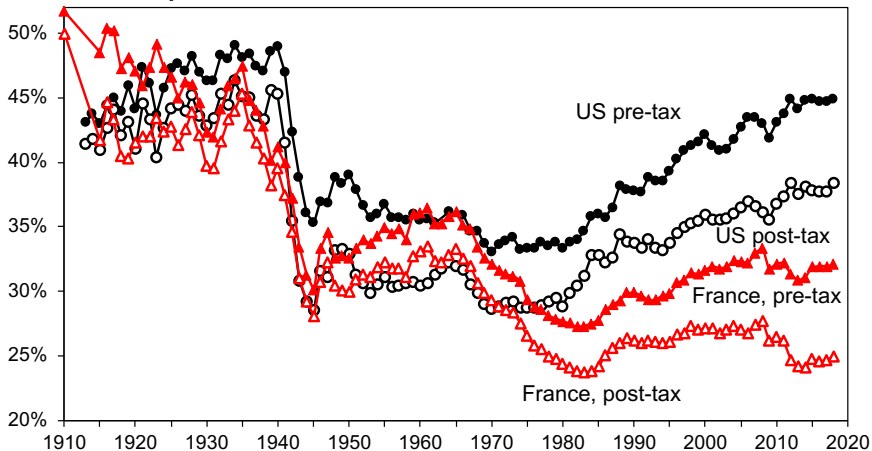


Top income shares of pretax national income among adults (income within married couples equally split).
Source is Piketty, Saez, Zucman (2018) for US and Piketty et al. (2020) for France.

- Comparable measures constructed for many countries and made available through the [WID database](#)

Source: [Saez \(2021\)](#), AEA Distinguished Lecture.

Top 10% Income Shares in the US and France, 1910-2018

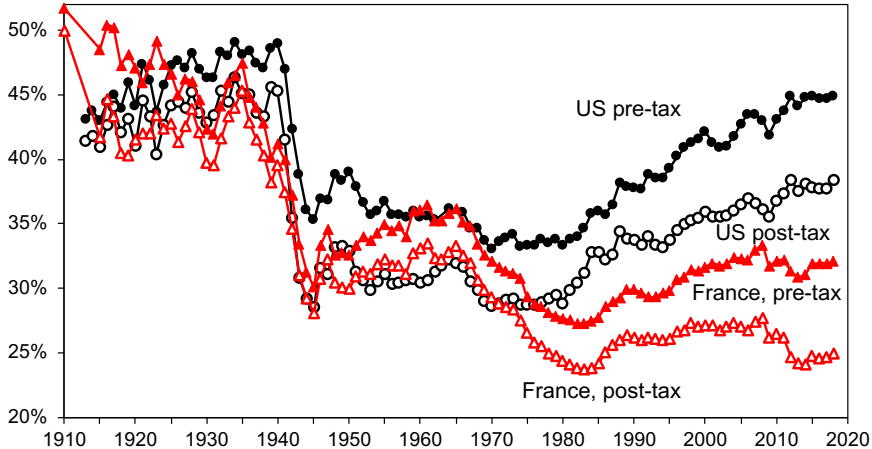


Top income shares of pretax and posttax national income among adults (income within married couples equally split). Source is Piketty, Saez, Zucman (2018) for US and Piketty et al. (2020) for France.

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- Taking into account taxes and transfers matters

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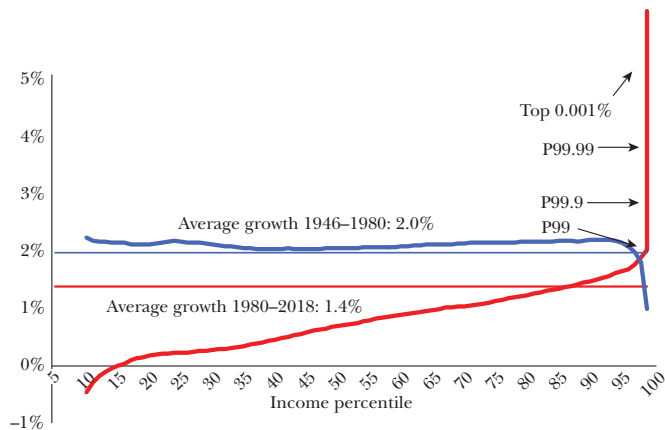


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- Comparable measures constructed for many countries and made available through the [WID database](#)
- Taking into account taxes and transfers matters
- How about the rest of the population?

Average Annual Income Growth Rates

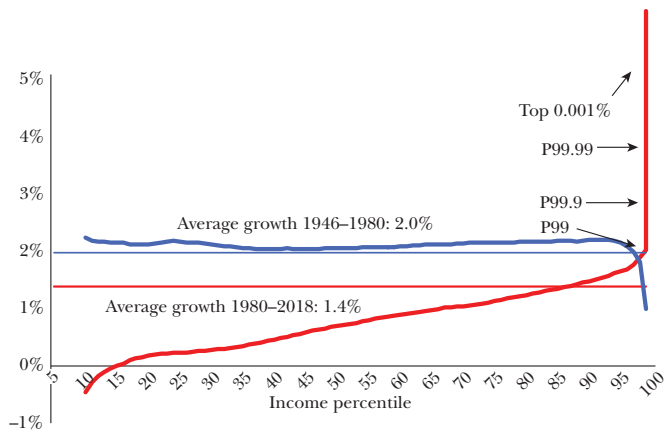


Source: Saez and Zucman (2019b).

Note: This figure depicts the annual real pre-tax income growth per adult for each percentile in the 1946-1980 period (in blue) and 1980-2018 period (in red). From 1946 to 1980, growth was evenly distributed with all income groups growing at the average 2 percent annual rate (except the top 1 percent which grew slower). From 1980 to 2018, growth has been unevenly distributed with low growth for bottom income groups, mediocre growth for the middle class, and explosive growth at the top.

Source: Saez and Zucman (2020), Journal of Economic Perspectives.

Average Annual Income Growth Rates



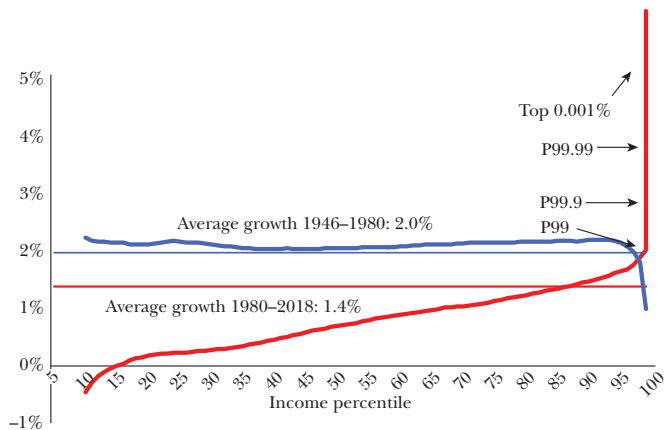
- **1946–1980:** roughly 2% annual income growth across the distribution among "the 99%"

Source: Saez and Zucman (2019b).

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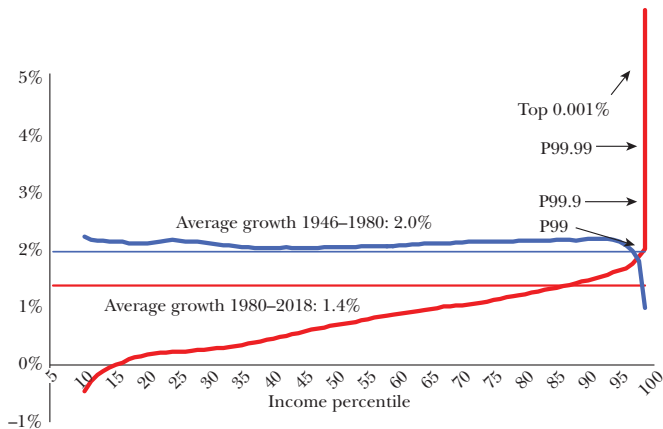
- **1946–1980:** roughly 2% annual income growth across the distribution among "the 99%"
- **1980–2018:** income growth faster among the more wealthy even among "the 99%"; the very top very different than the rest

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Average Annual Income Growth Rates



- **1946–1980:** roughly 2% annual income growth across the distribution among "the 99%"
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- Next: How is this figure constructed?

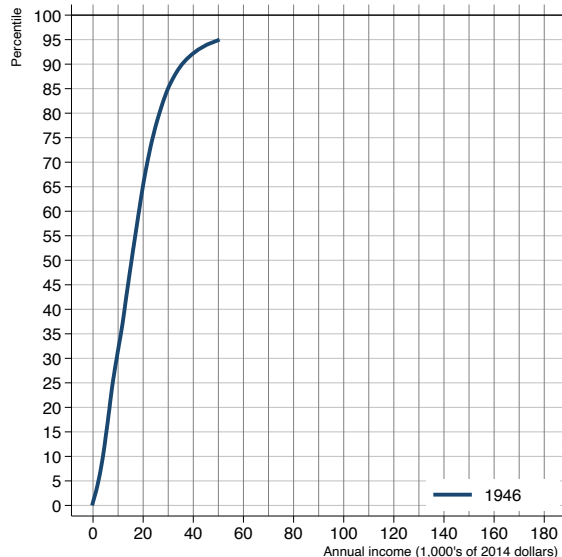
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The U.S. income distribution, 1962–2014, bottom 95 percentiles

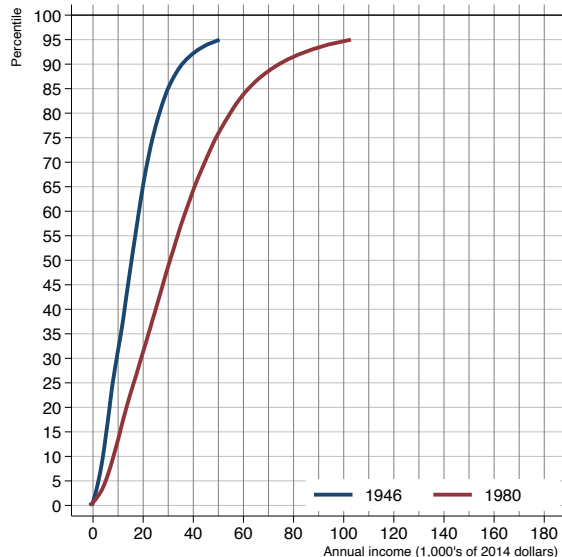
- Let's start with the CDF of income distribution in 1946
 - 90/10 percentile ratio: $\frac{35.5}{3.8} = 9.0$



Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, bottom 95 percentiles

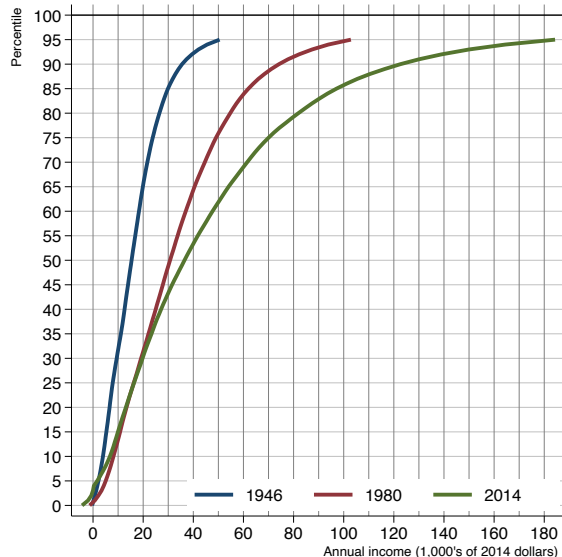
- Let's start with the CDF of income distribution in 1946
 - 90/10 percentile ratio: $\frac{35.5}{3.8} = 9.0$
- Adding the CDF for 1980 income
 - 90/10 percentile ratio: $\frac{74.2}{8.1} = 9.1$



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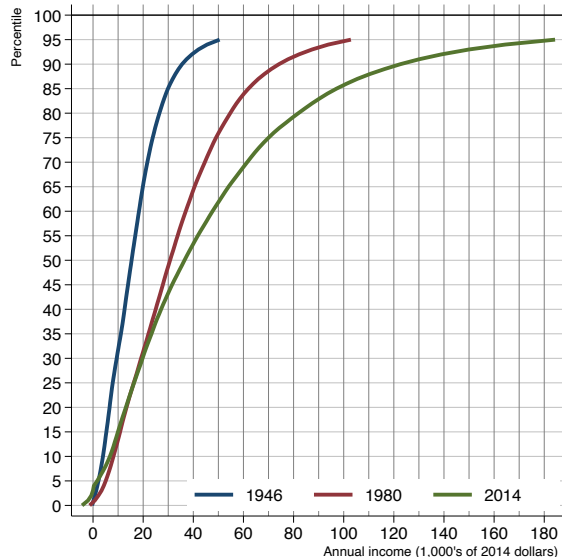
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Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, bottom 95 percentiles

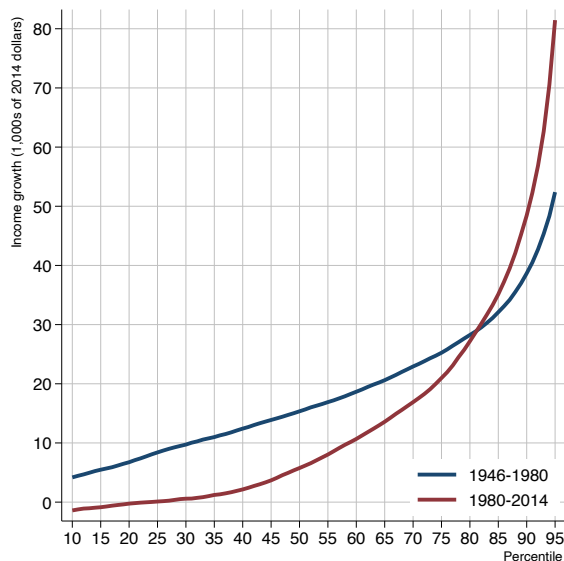
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- Adding the CDF for 2014 income
 - 90/10 percentile ratio: $\frac{122.6}{6.7} = 18.2$
- Horizontal distance btw the CDFs = dollar change for each percentile
 - these are not the same *people*; we are comparing percentiles
 - next: from dollar changes to annualized growth rates



Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, bottom 95 percentiles

- Let's first calculate dollar changes
 - i.e. horizontal distance btw CDFs

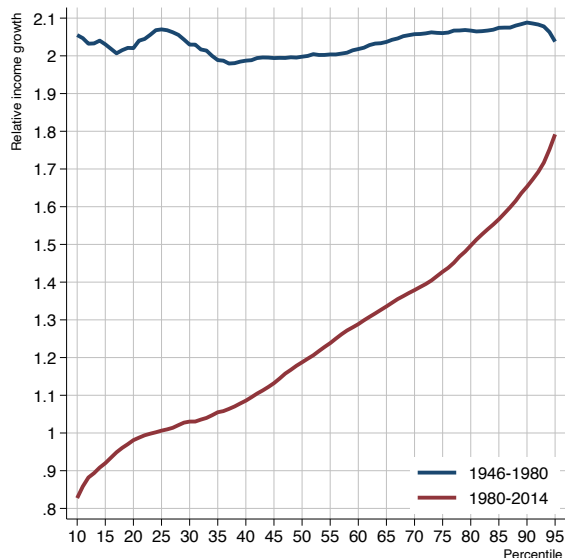


Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, bottom 95 percentiles

- Let's first calculate dollar changes
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- Then: relative change in income between years a and b for quantile τ

$$G = \frac{Q_b(\tau)}{Q_a(\tau)}$$



Source: Piketty, Saez, Zucman (2018) data appendix

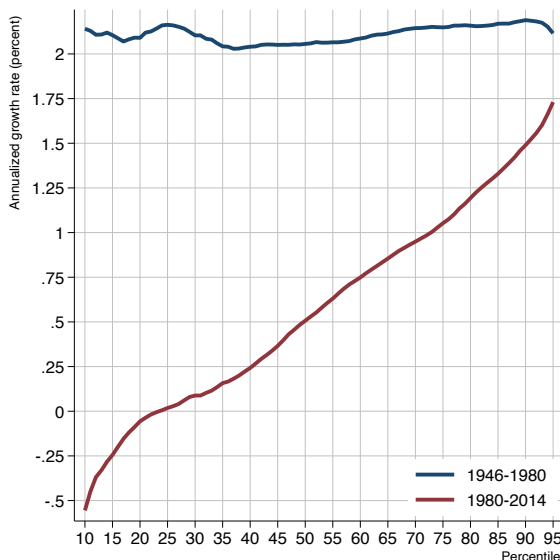
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- Finally: annualization, i.e. annual growth rate g that accumulates to G over 34 years

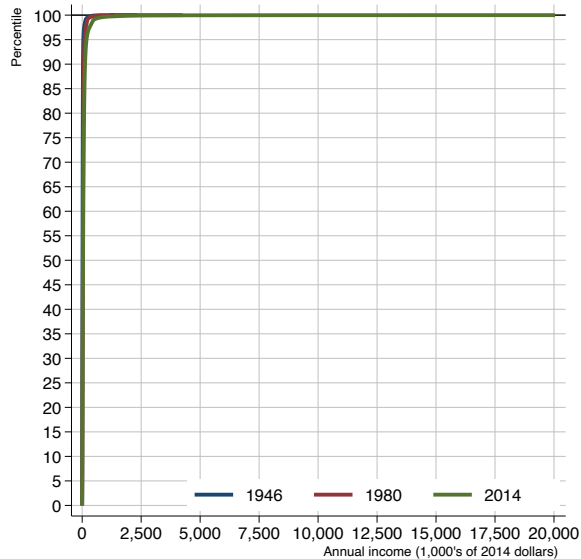
$$(1 + g)^{34} = G \Leftrightarrow g = G^{1/34} - 1$$



Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, full distribution

- CDFs for very skewed distributions are uninformative

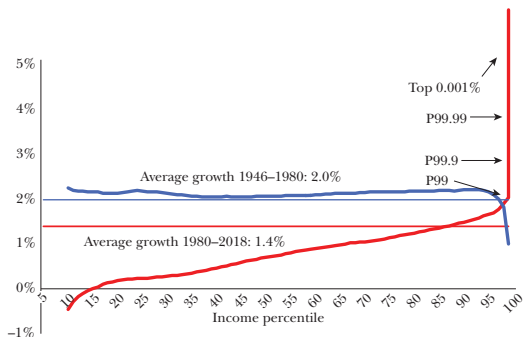


Source: Piketty, Saez, Zucman (2018) data appendix

The U.S. income distribution, 1962–2014, full distribution

- CDFs for very skewed distributions are uninformative ... but changes can nevertheless be made visible
- Next: other approaches to inequality
 - changes in expected wages across population groups over time
 - intergenerational mobility

Average Annual Income Growth Rates



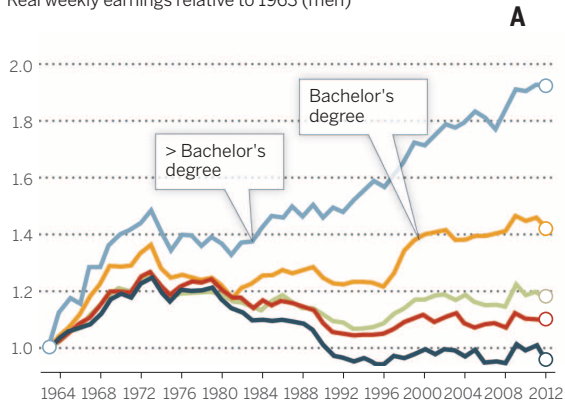
Source: Saez and Zucman (2019b).

Note: This figure depicts the annual real pre-tax income growth per adult for each percentile in the 1946–1980 period (in blue) and 1980–2018 period (in red). From 1946 to 1980, growth was evenly distributed with all income groups growing at the average 2 percent annual rate (except the top 1 percent which grew slower). From 1980 to 2018, growth has been unevenly distributed with low growth for bottom income groups, mediocre growth for the middle class, and explosive growth at the top.

Source: Saez and Zucman (2020), Journal of Economic Perspectives.

Changes in real wage levels of full-time U.S. workers by sex and education, 1963–2012

Real weekly earnings relative to 1963 (men)



Real weekly earnings relative to 1963 (women)

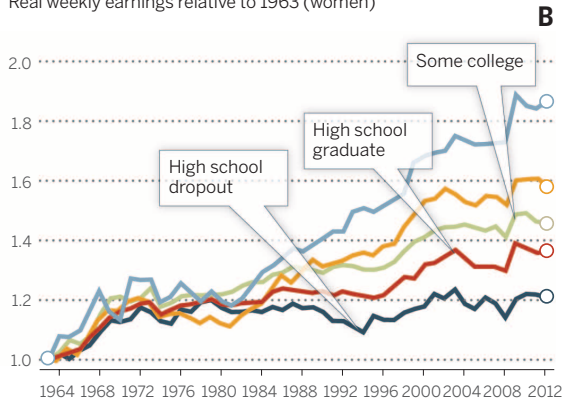


Fig. 6. Change in real wage levels of full-time workers by education, 1963–2012. (A) Male workers, (B) female workers. Data and sample construction are as in Fig. 3.

Source: Autor (2014), Science.

- Estimates over time for $\mathbb{E}[w|E = e, G = G]$, where w is weekly wage, E education level and G is gender. Wages are divided by 1963 group-specific average wages.

- A complementary way to think about inequality is based on the idea of equality of opportunities
 - the extent to which people compete on a “level playing field” vs. inherit their position

- A complementary way to think about inequality is based on the idea of equality of opportunities
 - the extent to which people compete on a “level playing field” vs. inherit their position
- An incomplete, but powerful measure

$$\mathbb{E}[p_c | P_p = p_p]$$

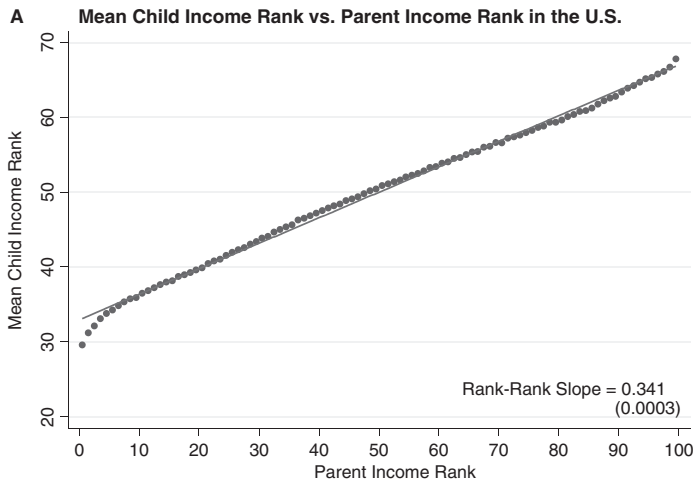
where p_c is the child's position in (lifetime) income distribution and p_p is her parent's position

Intergenerational mobility

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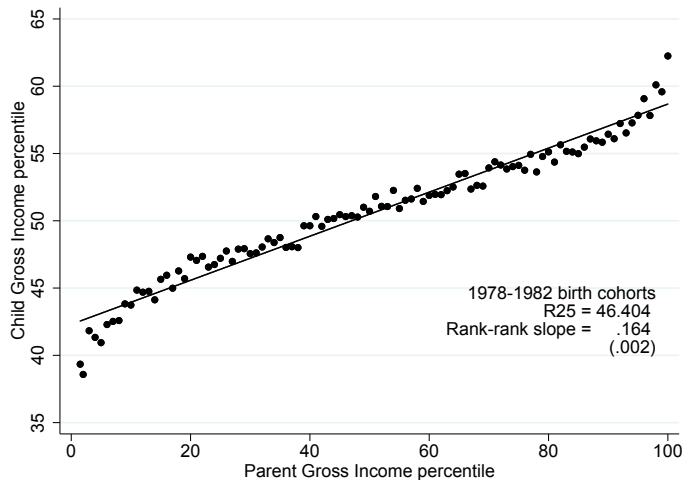
Children born in 1980–82. Their income is the mean of 2011–2012 family income (when the child is approximately 30 years old). Parent income is mean family income from 1996 to 2000. Children are ranked relative to other children in their birth cohort, and parents are ranked relative to all other parents. *Source: Chetty, Hendren, Kline and Saez (2014), Quarterly Journal of Economics.*

- A complementary way to think about inequality is based on the idea of equality of opportunities
 - the extent to which people compete on a “level playing field” vs. inherit their position
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... in Finland



Source: Unpublished, ongoing work.

- Conditional statistics and correlation
 - conditional expectation
 - joint, marginal and conditional distribution
 - correlation
 - cross tabulation, scatter plots
- Example: income inequality
 - top percent shares
 - changes over entire distribution
 - group averages
 - social (or intergenerational) mobility