CHEM-E8135 Microfluidics and BioMEMS

Microfluidics 2: Capillary flow

20.1.2021

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Lecture structure

Last week, we looked at pressure driven laminar flow.

This week, we continue by looking at **capillary pressure** driven laminar flow. All that we learned last week still applies.

<u>Both topics will be continued in the exercises.</u>

Next week we discuss diffusion and adsoprtion and their significance in microfluidics.

Intended learning outcomes for lecture 2:

This lecture mainly deals with the course ILO2:

ILO2: The student is familiar with liquid and solute interactions with surfaces. The student understands surface energy, wetting and capillary filling

Key concepts:

Laplace pressure / Capillary pressure

Capillary filling microfluidic channels

Advantages and disadvantages of capillary filling

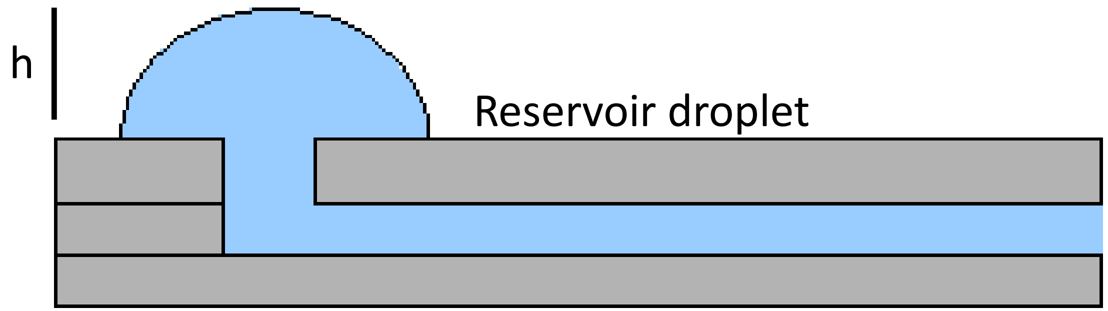
Some components/applications of capillary filling microfluidics

Scaling: surface forces vs body forces

- Surface area to volume ratio scales as d^{-1}
- Microsystems often dominated by surface effects
- An example:

3-phase cont

Advancing contact ang



Case 1: The flow channel is a microchannel: 100 µm x 100 µm x 100 mm, Volume 1 µl

Volume: Hydrostatic pressure from reservoir ≈ 10 Pa
Area: Capillary pressure from channel ≈ 3000 Pa

Surface dominated!

Case 2: The flow channel is a garden hose: 1 cm x 1 cm x 10 m , Volume 1 liter

Volume: Hydrostatic pressure from reservoir ≈ 1000 Pa
Area: Capillary pressure from channel ≈ 30 Pa

Volume dominated!

Bond number: Does gravity matter?

Dimensionless number that characterizes the ratio of surface forces to body forces

Capillary pressure= γ /L *Hydrostatic pressure*= ρaL

a = acceleration (for gravity, $9.81 m/s^2$) L = characteristic length scale

γ = surface tension ρ = density

$$Bo = \frac{\rho a L^2}{\gamma}$$

If Bo < 1 the system is dominated by surface forces (opposed to body forces)

For water, Bo = 1 at around **1 mm** range. Microfluidic channels are usually smaller than this so gravity typically does not matter in microfluidics.

Example from previous page, water in a channel with 100 µm and 1 cm dimensions:

Bo $\approx 1.4 * 10^{-3}$ (for 100 µm)

Bo ≈ 14 (for 1 cm)

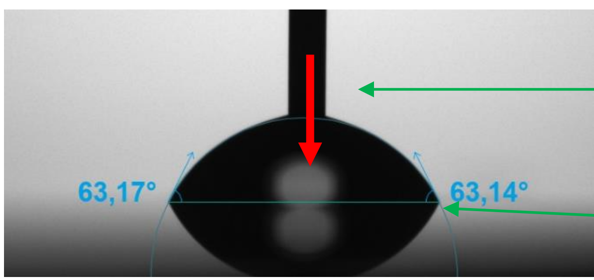
Summary of contact angles

Contact angle θ is the parameter needed for understanding how much water adheres on a surface.

Contact angle is measured by optical or force methods.

Contact angle is a property of a liquid-solid-fluid 3-phase system.

In theory (and on ideally smooth and homogenous surfaces), contact angle is given by Young's equation. $\gamma_{lv} \cos(\theta) = \gamma_{sv} - \gamma_{sl}$



Advancing contact angle measurement: volume of droplet is increasing

3-phase contact line. We are measuring from this point

In practice, due to contact angle hysteresis, there are many metastable contact angles so that $\theta_{rec} < \theta < \theta_{adv}$

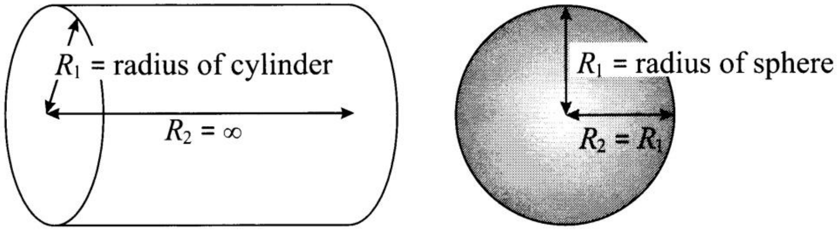
Contact angle is needed for capillary filling. Typically advancing contact angle θ_{adv} is the correct parameter to use.

Laplace pressure

- There is a pressure difference across a curved **liquid** surface.

$$\Delta P = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Young-Laplace equation



Calculating radius of curvature

For a spherical droplet or bubble: $$\Delta P = \frac{2\gamma}{R}$$

1 µl spherical water droplet in air, $\Delta P \approx 140$ Pa

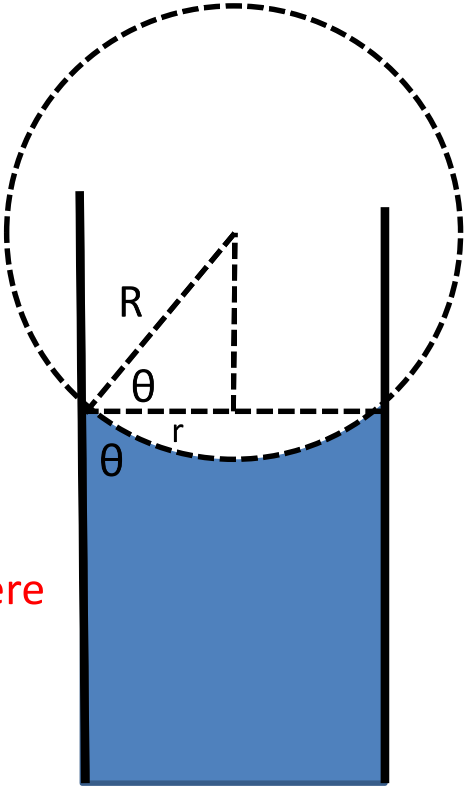
1 µl spherical air bubble in water, $\Delta P \approx -140$ Pa

Pressure is higher inside a spherical bubble or a droplet!

Capillary pressure and contact angle

Liquid makes a contact angle of θ with a capillary with radius r.

1. What is the curvature of the meniscus?
2. What is the Capillary pressure?



1. From the triangle in the figure we get:

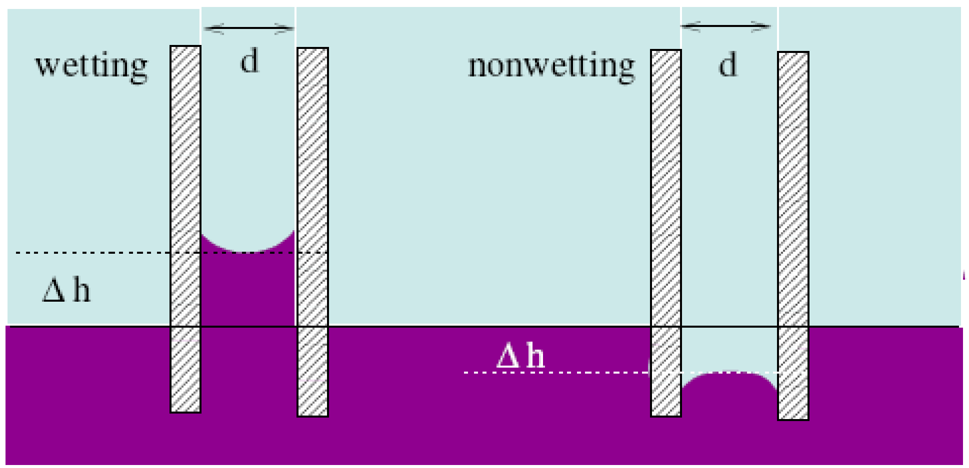
$r/R = \cos(\theta)$ so $R = r/\cos(\theta)$

Curvature of the sphere is defined as $-2/R$ (bubble) so
Curvature = $-2 \cos(\theta) / r$

2. Capillary pressure is the corresponding Laplace pressure:
$P_{cap} = \gamma$ * Curvature $= -2\gamma \cos(\theta) / r$

Capillary pressure is the Laplace pressure in a ”capillary”, where the radii of curvature depend on the dimension and the contact angle.

Capillary rise and depression



Hydrostatic pressure $\rho g \Delta h = -4 * \gamma * \cos(\theta)/d$ Capillary pressure

hydrophilic or wetting materials liquid is sucked in, $\Delta h > 0$

hydrophobic or non-wetting materials liquid is pushed out, $\Delta h < 0$

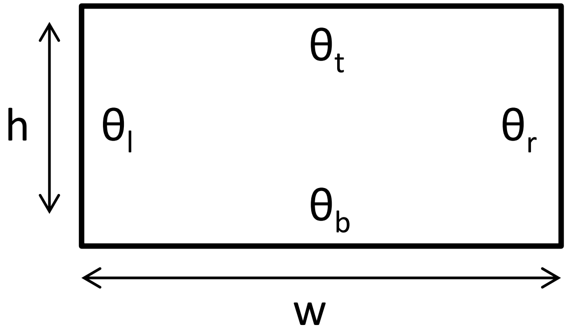
Note that θ is the only material parameter of the capillary that is needed, not e.g., the solid surface energy γ_{sv} or the solid-liquid interfacial energy γ_{sl}

Capillary filling of microfluidic channels

The main differences to classical capillary rise:

Horizontal vs vertical

- Capillary filling continues *until the channel network is full*



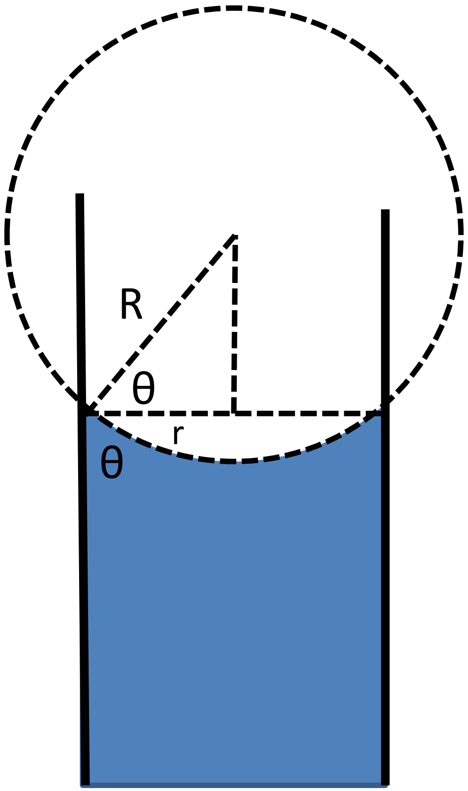
Geometry usually non circular and nonuniform materials

- Hydrophilic walls contribute to filling, hydrophobic oppose it

- The capillary pressure is calculated **at the filling front** (and possibly the de-wetting front), already filled areas contribute only to flow resistance.

Capillary pressure is calculated here

Capillary Pressure, rectangular



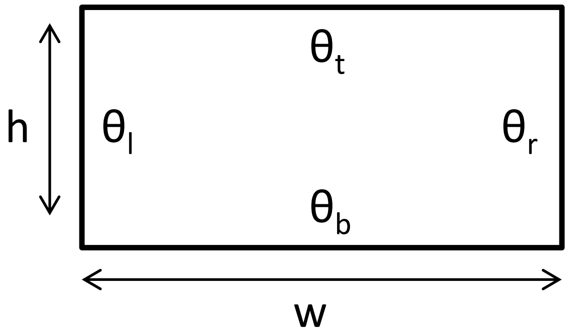
Capillary pressure is the Laplace pressure of a microfluidic channel.

Laplace pressure: $$\Delta P = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Thought experiment: if θ in the figure was smaller, would the dashed circle be bigger or smaller?

In a rectangular channel, with height h and width w, and all walls having different contact angle, the capillary pressure becomes:

$$\Delta P = -\gamma \left(\frac{\cos\theta_t + \cos\theta_b}{h} + \frac{\cos\theta_l + \cos\theta_r}{w} \right)$$

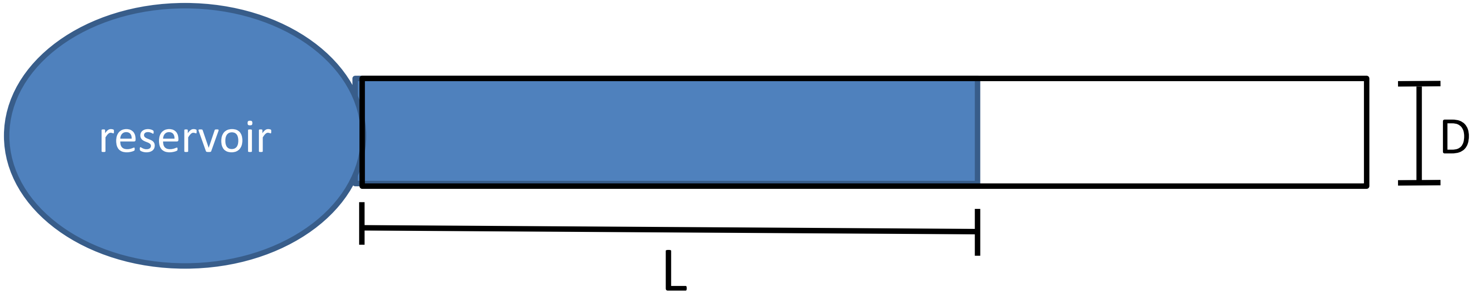


Washburn equation

Washburn relation gives the **filling rate** of a channel with constant capillary pressure. A circular cross section is assumed in the formula below.

$$L^2 = \frac{\gamma D cos\theta}{4\mu} t$$

,where L is the length of channel that is filled at time t, D is the diameter of a (circular) channel , μ is viscosity and γ surface tension and θ the contact angle.



The Washburn $L \sim t^{1/2}$ relation holds for laminar flow systems with constant cross-sectional geometry in the process of filling under constant pressure.

Washburn relation also holds for random networks of small capillaries typically found in porous materials, such as paper.

Bonus slide: Washburn equation derivation

A simple derivation for circular tube and starting from Hagen Poiseuille's equation:

$$Q = -\frac{\pi r^4}{8\mu L}\Delta P$$

$$v_{ave} * A = v_{ave} * \pi r^2 = -\frac{\pi r^4}{8\mu L}\Delta P$$

Separate the volume and velocity contributions to volumetric flow

$$v_{ave} = -\frac{r^2}{8\mu L}\Delta P = -\frac{r^2}{8\mu L} * \frac{2\gamma cos\theta}{r} = \frac{\gamma r cos\theta}{4\mu L}$$

Insert capillary pressure 2γcosθ/r

$$v_{ave} = dL/dt = \frac{\gamma r cos\theta}{4\mu L}$$

For a channel in the process of filling!

$$LdL = \frac{\gamma r cos\theta}{4\mu}dt$$

$$\frac{L^2}{2} = \frac{\gamma r cos\theta}{4\mu}\mathrm{t} + \mathrm{C}$$

C=0 since L=0 at t=0

$$L^2 = \frac{\gamma D cos\theta}{4\mu}\mathrm{t}$$

Which is the classical Washburn equation if one inserts cosθ=1

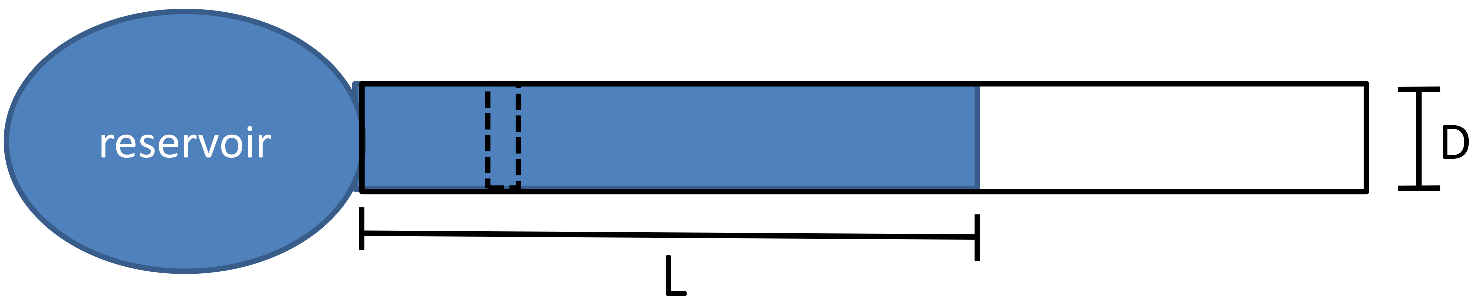
Washburn equation, two ways to think about it

Why is L ~ $t^{½}$?
Two ways to think about it.

$$L^2 = \frac{\gamma D}{4\mu} t \qquad Q = -\frac{\pi r^4}{8\mu L}\Delta P$$

#1: Consider the entire filled part as a whole. The pressure difference stays the same (P_{cap}) but the flow resistance keeps increasing as a function of L. Hence dL/dt ~ 1/L

#2: Consider the flow through a small section of the filled line (dashed line). The resistance of that part stays the same but the pressure gradient P_{cap} / L decreases as 1/L.



Qualitatively, the reason is that the same pressure needs to drag an increasingly large liquid column behind it.

Capillary driven microfluidics

Capillary pressure driven microfluidics follows all the same laws as any pressure driven microfluidics, Hagen-Poiseuille's law etc.

1. Calculate capillary pressure
2. Otherwise utilize the same protocol as for pump driven microfluidics.

The only complications are that for a simple capillary filling system, the **hydraulic resistance changes as the channel fills** and if there are any **changes** in the **dimensions** or **materials** of the channel, then the **capillary pressure changes** as well.

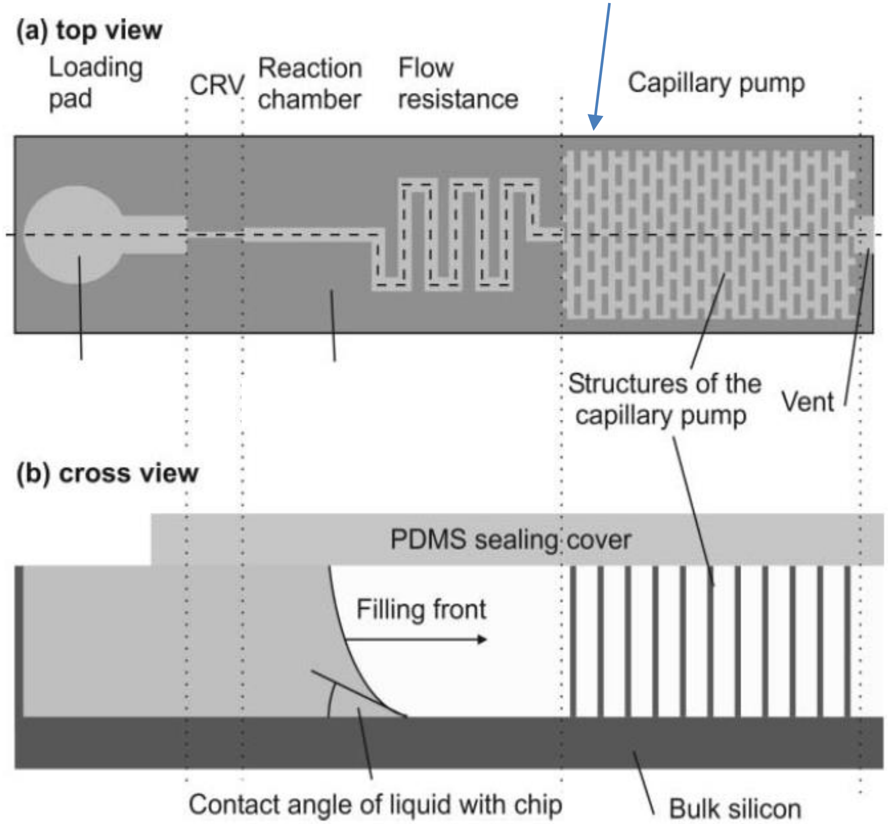
Capillary pump

Parallel channels reduce hydraulic resistance in the capillary pump

How to get *constant flow rate over long time*?

Answer:

1. Design a capillary pump: a channel or network channel with constant capillary pressure and high volume.
2. Add a flow resistor which has much higher hydraulic resistance than the capillary pump. This way the flow does not slow down noticeably as the pump fills.



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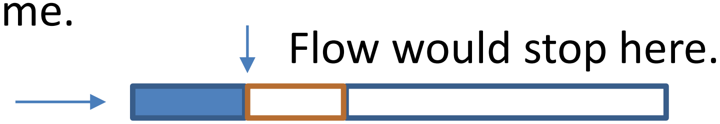
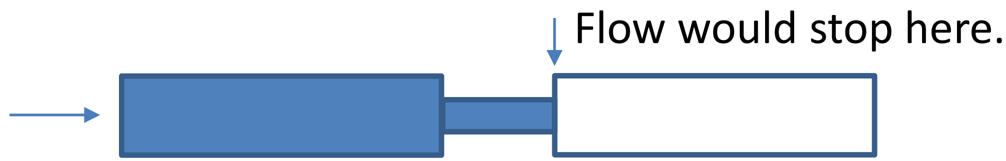
Some basic components

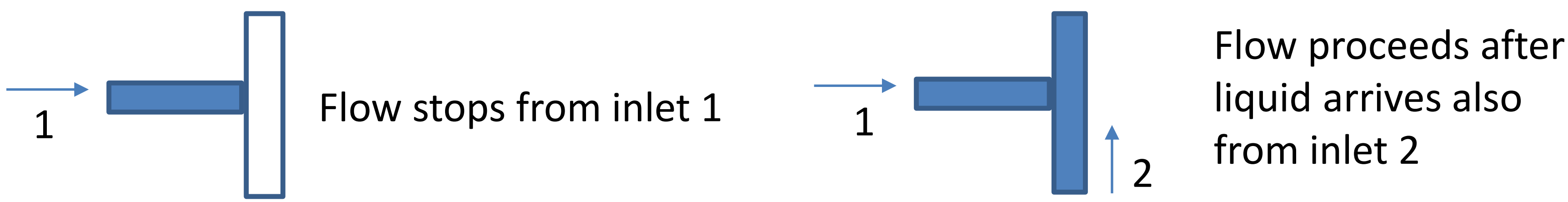
Hydrophobic stop valve: a hydrophobic part of the channel, needs pressure to overcome.



Geometric stop valve: an **expanding** cross section of the channel, needs pressure to overcome.



Delay valve: A T intersection that is a geometric valve in one direction. Used for e.g. timed delays or to synchronize flow from 2 directions.



Hydrophilic channel, full

Hydrophobic channel, full

Hydrophilic channel, empty

Hydrophobic channel, empty

Programmed capillary flow

Capillary pump

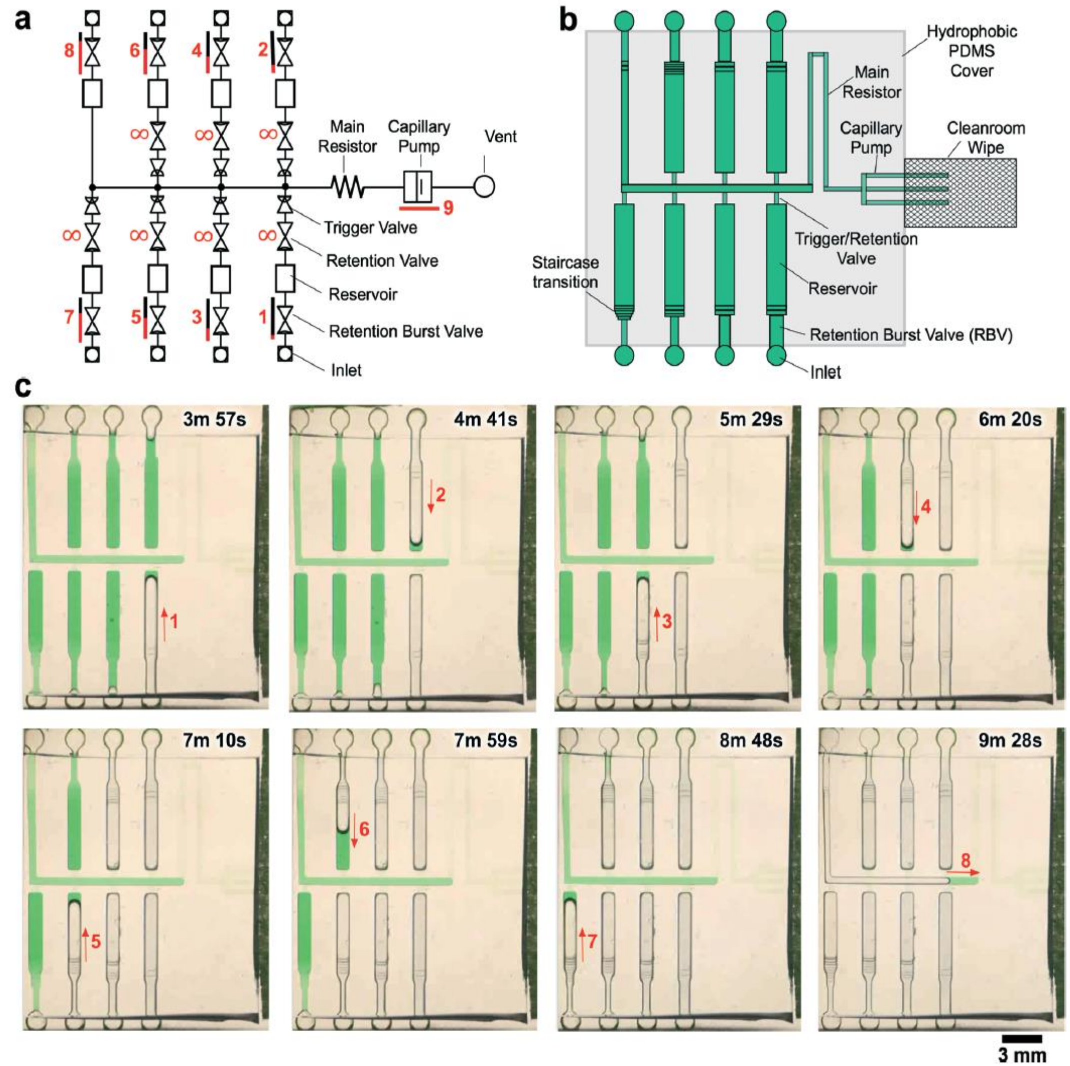
Flow resistor

Geometric trigger valves

"Retention burst valves" with tailored dimensions (=tailored capillary pressures) so that they drain in order.

$P_1 > P_2 > \ldots > P_{pump}$

(due to sign convention, this means the pump has highest capillary pressure, followed by 8, 7, 6…)



3812 | Lab Chip, 2016, 16, 3804–3814

Advantages and disadvantages

Advantage: No need for separate pumps

- Ease of operation
- Low cost
- Many components have been designed to achieve more complex pre-programmed fluidic flow profiles.

Disadvantages:

-Flow is not flexible and cannot be changed on fly. Very complex and exact flow profiles are not possible.

-Places requirements for the materials/coatings. One might want to use coating X for other purposes but the coating could alter capillary flow.

Overall, capillary filling fluidics are most suitable for applications where simple fluidics is sufficient and there is a clear advantage in not using pumps.

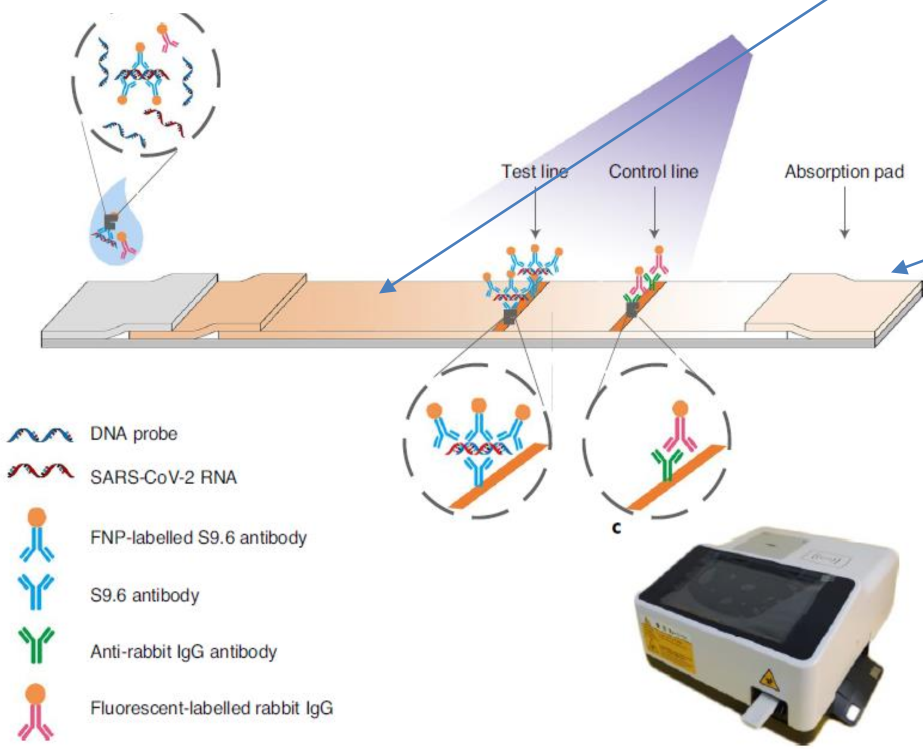
For example: home diagnostics, field diagnostics (e.g. developing countries).

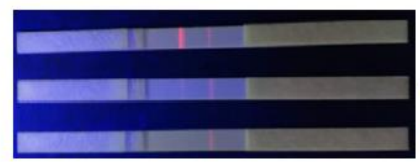
Lateral flow devices

Typical examples of capillary filling ("lateral flow") devices already in use:
-Lateral flow immunoassays
-Lateral flow home pregnancy tests.

Instead of channels, porous hydrophilic nitrocellulose membrane

Absorption pad ≈ Capillary pump





Covid19 lateral flow immunoassay.

Review

Laplace pressure and Capillary pressure

Capillary filling of microfluidic channels: basic components and design strategies, advantages and disadvantages.

Reading material

Chapter 5, surface tension, from a book “Physics of Continuous matter” by B. Lautrup. Pages 69-83 (mostly about surface tension basics, not about capillary filling)

Available from link: http://www.cns.gatech.edu/~predrag/courses/PHYS-4421-13/Lautrup/surface.pdf