Problem set 2, 20.01.2021:

(11.1) One mole of ideal monatomic gas is confined in a cylinder by a piston and is maintained at a constant temperature T_0 by thermal contact with a heat reservoir. The gas slowly expands from V_1 to V_2 while being held at the same temperature T_0. Why does the internal energy of the gas not change? Calculate the work done by the gas and the heat flow into the gas.

(11.2) Show that, for an ideal gas,

$$\frac{R}{C_V} = \gamma - 1$$

and

$$\frac{R}{C_p} = \frac{\gamma - 1}{\gamma},$$

where C_V and C_p are the heat capacities per mole.

(12.2) Assume that gases behave according to a law given by $pV = f(T)$, where $f(T)$ is a function of temperature. Show that this implies

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{V}\frac{df}{dT}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{p}\frac{df}{dT}$$

Show also that

$$\left(\frac{\partial Q}{\partial V}\right)_p = C_p\left(\frac{\partial T}{\partial V}\right)_p$$

$$\left(\frac{\partial Q}{\partial p}\right)_V = C_V\left(\frac{\partial T}{\partial p}\right)_V.$$

In an adiabatic change, we have that

$$dQ = \left(\frac{\partial Q}{\partial p}\right)_V dp + \left(\frac{\partial Q}{\partial V}\right)_p dV = 0$$

Hence show that pV^γ is a constant.

(12.3) Explain why we can write

$$đQ = C_p dT + A dp$$

and

$$đQ = C_V dT + B dV$$

where A and B are constants. Subtract these equations and show that

$$(C_p - C_V)dT = BdV - Adp$$

and that at constant temperature

$$\left(\frac{\partial p}{\partial V}\right)_T = \frac{B}{A}$$

In an adiabatic change, show that

$$dp = -\left(\frac{C_p}{A}\right)dT$$

$$dV = -\left(\frac{C_V}{B}\right) dT$$

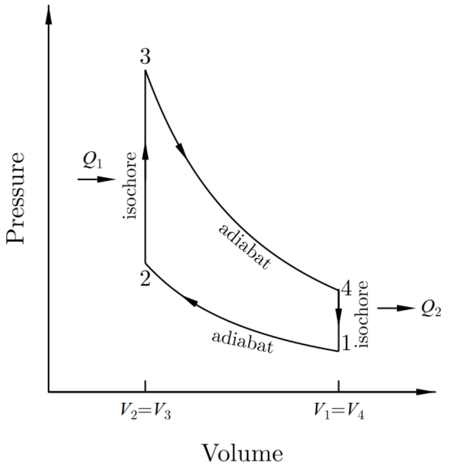
Hence show that in an adiabatic change, we have that

$$\left(\frac{\partial p}{\partial V}\right)_{\text{adiabatic}} = \gamma \left(\frac{\partial p}{\partial V}\right)_T$$

$$\left(\frac{\partial V}{\partial T}\right)_{\text{adiabatic}} = \frac{1}{1-\gamma}\left(\frac{\partial V}{\partial T}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_{\text{adiabatic}} = \frac{\gamma}{1-\gamma}\left(\frac{\partial p}{\partial T}\right)_V$$

(13.5) Show that the efficiency of the standard Otto cycle (see below) is $1 - r^{1-\gamma}$, where $r = {}^{V_1}/_{V_2}$ is the compression ratio. The **Otto cycle** is the four-stroke cycle in internal combustion engines in cars, lorries and electrical generators.



The number of the problem refers to the textbook.

(Problem A) The equation of state of a gas can be written in the form

$$p = nkT(1 + B_2 n)$$

where p is the mean pressure of the gas, T its absolute temperature, $n \equiv {}^{N}/_{V}$ the number of molecules per unit volume, and $B_2 = B_2(T)$ is the second virial coefficient. B_2 is an increasing function of the temperature.

Find how the mean internal energy E of this gas depends on its volume V, i.e., find an expression for $\left(\frac{\partial E}{\partial V}\right)_T$. Is it positive or negative?

Deadline for Problem set 2: 29th January at 10:00 a.m.
Send the solutions to bayan.karimi@aalto.fi