General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. The (attempted) solutions should be submitted via email to the assistant at the start of the exercise session on February 3. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.

Energy confinement times and ignition

A central figure of merit within the field of fusion technology is the fusion triple product. The fusion triple product relates the fuel density, n_{DT}, fuel temperature, T, and energy confinement time, τ_E, to successful, self-sustained fusion burn. The triple product for Deuterium-Tritium fusion is

$$n_{DT} T \tau_E \geq 10^{22} \text{ keV s m}^{-3}.$$

Calculate the required confinement times for ignition in the following confinement concepts:

a) **Magnetic confinement:** a tokamak with fuel density $n \sim 10^{20}$ m^{-3} and temperature $T \sim 10$ keV. Compare the value to the one foreseen for ITER.

b) **Inertial confinement:** a laser fusion device with $n \sim 10^{32}$ m^{-3} and fuel temperature of $T \sim 10$ keV. Compare the obtained value to the ratio of the fuel sound speed ($c_s \sim 10^5$ m/s) to the fuel capsule radius ($R \sim 100$ μm). What confines the plasma in this concept?

c) **Gravitational confinement:** a star with fuel density $n \sim 10^{23}$ m^{-3} and fuel temperature $T \sim 1$ keV. Assume the stars are fuelled by D-T fusion for comparative purposes. How does τ_E again compare to the ones calculated before?

Exercise 2.

Power densities and wall power loads in different fusion reactors

Estimate the power densities and wall power loads in the following fusion reactors:

a) **Tokamak:** A tokamak operating with a 50-50 D-T fuel mix with major radius $R =$ 5.5 m and minor radius $a = 1.8$ m. Assume flat temperature and density profiles (= constant temperature and density) with $n_D = n_T = 10^{20}$ m^{-3} and $T_e = T_i = 10$ keV. Use the low temperature approximation from the "Fusion principles" lecture notes to determine the mean reaction rate.

b) **Z-pinch:** A Z-pinch with $R = 10$ cm operating with a 50-50 DT fuel mix. The fusion reactions occur in a narrow rope with a diameter of 40 μm. The DT density in the rope is $n_{DT} = 4.5 \times 10^{28}$ m^{-3}. The burn-up of the fuel is 30% and the pulse rate, f, of the device is 1 Hz. See figure 1 for illustration of the assumed geometry.

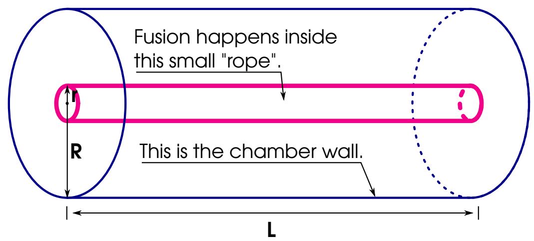

Figure 1: An illustration of the assumed z-pinch geometry in exercise 2b).

c) **Fusion in stars:** A full deuterium star of the size of the Sun. The radius of the star is $R = 7 \times 10^8$ m. Assume that the fusion reactions are produced inside the effective radius of $R_{\text{eff}} = 0.6R$, flat density and temperature profiles in this central ($R < R_{\text{eff}}$) region with $n_D = 1 \times 10^{21}$ m^{-3} and $T = 1$ keV. You do not need to consider wall power loads in this case.

d) Compare and comment on the power densities and wall heat loads achieved in the example fusion reactors and the example star. For comparison, the power density in the core of the Sun is estimated to be about 280 Wm^{-3}.

Exercise 3.

The power balance in a deuterium star

Continue the analysis of the example star introduced in exercise 2c). Assume the surface temperature of the star to be a constant 6420 K, with unknown central temperature. All the other parameters are the same as introduced in the exercise 2c).

a) What is the central temperature of the star, assuming the star is a black body? Assume the only radiation losses are due to black body radiation

$$P_{bb} = \sigma_{SB} T_S^4 A,$$

where $\sigma_{SB} = 5.67 \times 10^{-8}$ Wm^{-2}K^{-4} and find the equilibrium temperature. You may use the low temperature ($T < 25$ keV) approximation of the mean reaction rate for D-D fusion

$$\langle \sigma v \rangle_{D-D} = 2.33 \times 10^{-14} T^{-2/3} \exp -18.76 T^{-1/3} \quad \text{cm}^3\text{s}^{-1}.$$

b) Next, calculate the central temperature of the star, assuming it to be transparent, i.e. only considering Bremsstrahlung radiation:

$$P_{Br} = c_B n_e^2 T_e^{1/2},$$

where $c_B = 1.71 \times 10^{-38}\ \ \mathrm{Wm^3eV^{-1/2}}$.

Exercise 4.

Inertial confinement

In a laser fusion device, a spherical symmetric fusion target is compressed to 1000 times the initial density: $\rho = 1000\rho_0$, or $R = 0.1R_0$. The initial density of the fuel pellet, consisting of cooled, solid D-T, is $\rho_0 = 0.23\ \mathrm{g/cm^3}$ and the radius is $R_0 = 0.2\ \ \mathrm{cm}$.

a) What is the minimum energy of the laser pulse $E_{\min}$ required, if a temperature of 5 keV is required for the ignition? Assume that the fuel compresses adiabatically, i.e. $pV^\gamma = \mathrm{const}$ in the process, with $\gamma = 5/2$.

b) What is the fusion gain factor (the ratio of fusion energy released to the energy needed)? The burn-up B, the fraction of the fuel that fuses, is needed, and can be calculated according to:

$$B = \frac{\rho R}{\rho R + 6\frac{\mathrm{g}}{\mathrm{cm^2}}},$$

where ρ is the mass density of the fuel.

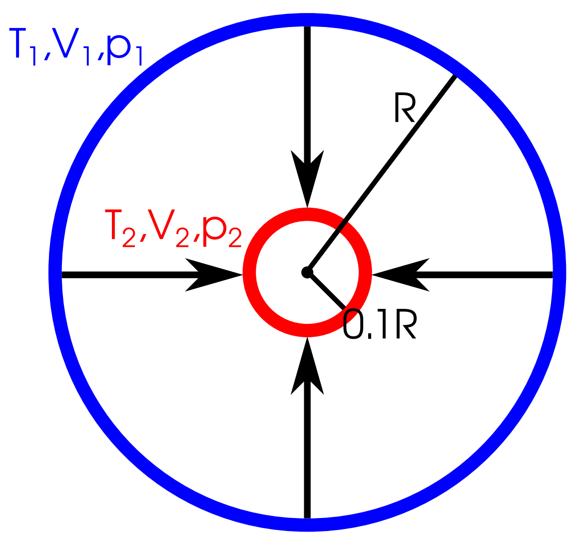

Figure 2: Illustration of the compression in laser fusion.

Constants:

1 eV $= 1.602 \times 10^{-19}$ J
$m_p = 1.673 \times 10^{-27}$ kg
$m_n = 1.675 \times 10^{-27}$ kg
$N_A = 6.022 \times 10^{23}$ mol^{-1}
$k_B = 1.381 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}

Power equations assuming pure hydrogenic (Z=1) plasma:

Fusion power density: $P_f = \alpha n_i n_j \langle \sigma v \rangle E_f$,

where E_f represents the produced energy per a fusion reaction, n_i and n_j are the fuel isotope densities, n_e the electron density, T_S the surface temperature of the black body, and A the surface area of the black body. The α parameter in the fusion power density equation is 1 for D-T fusion, and 1/2 for D-D fusion. This parameter arises due to the fact that when calculating the fusion cross-section integral ($\langle \sigma v \rangle$) for like particle collisions (D-D), every collision is counted twice. This should not be confused with the 1/4-factor that arises in the D-T fusion cross-section with 50-50 % fuel mixture due to $n_D = n_T = n_e/2 \rightarrow n_D \times n_T = (1/4) \times n_e$. More information can be found in e.g. [1].

References:

[1] J. P. Freidberg, Plasma Physics and Fusion Energy, Cambridge University Press, 2007, p.44