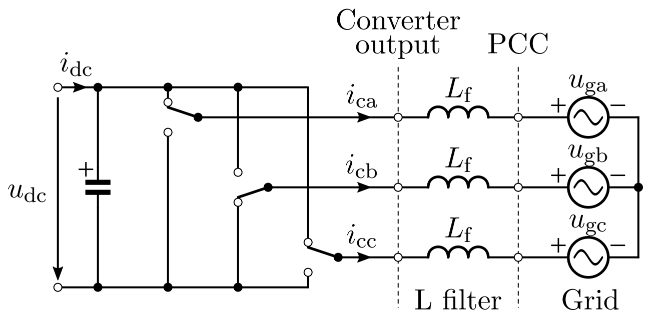
Problem 1: Grid support, current and voltage limits

The figure shows a grid-connected converter equipped with an L filter ($L_\mathrm{f} = 2$ mH). A solar generator feeds the constant power of 80 kW to the DC bus. The maximum AC-side current of the converter is $i_\mathrm{max} = \sqrt{2} \cdot 200$ A (peak). The grid frequency is 50 Hz and the nominal grid voltage is $u_\mathrm{N} = \sqrt{2/3} \cdot 400$ V (peak, phase-to-neutral). Assume the converter to be lossless and to operate in the steady state.

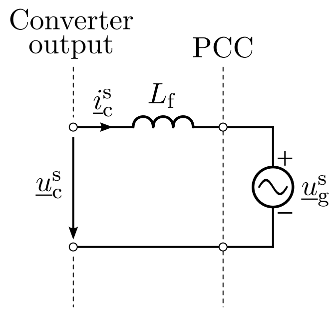
(a) The displacement power factor at the PCC is kept at unity. Calculate the power p_g fed to the grid, the current magnitude $|\underline{i}_\mathrm{c}|$, and the converter voltage magnitude $|\underline{u}_\mathrm{c}|$.

(b) The grid voltage decreases to $|\underline{u}_\mathrm{g}| = 0.8u_\mathrm{N}$ due to a symmetrical grid fault. Calculate $|\underline{i}_\mathrm{c}|$ and $|\underline{u}_\mathrm{c}|$. What would happen if the grid voltage decreased to $|\underline{u}_\mathrm{g}| = 0.5u_\mathrm{N}$?

(c) For supporting the grid at $|\underline{u}_\mathrm{g}| = 0.8u_\mathrm{N}$, the converter feeds the maximum reactive power q_g to the PCC, taking the current limit i_max into account. Calculate q_g and $|\underline{u}_\mathrm{c}|$. What is the minimum value for the DC-bus voltage u_dc in this operating condition?



Grid-connected converter



Space-vector equivalent circuit

Solution

(a) The power fed to the grid is $p_\mathrm{g} = 80$ kW since the converter is assumed to be lossless. In grid-voltage coordinates,

$$\underline{u}_\mathrm{g} = u_\mathrm{gd} + \mathrm{j}u_\mathrm{gq} = u_\mathrm{g} + \mathrm{j}0 = \sqrt{2/3} \cdot 400 \text{ V} = 326.6 \text{ V}$$

holds. The converter output current is $\underline{i}_\mathrm{c} = i_\mathrm{cd} + \mathrm{j}i_\mathrm{cq}$. Hence, the real power and reactive power at the PCC are

$$p_\mathrm{g} = \frac{3}{2}\,\mathrm{Re}\left\{\underline{u}_\mathrm{g}\underline{i}_\mathrm{c}^*\right\} = \frac{3}{2}u_\mathrm{g}i_\mathrm{cd} \qquad q_\mathrm{g} = \frac{3}{2}\,\mathrm{Im}\left\{\underline{u}_\mathrm{g}\underline{i}_\mathrm{c}^*\right\} = -\frac{3}{2}u_\mathrm{g}i_\mathrm{cq} \tag{1}$$

respectively. The displacement power factor is unity, corresponding to the reactive power $q_\mathrm{g} = 0$. The d and q current components are

$$i_\mathrm{cd} = \frac{2p_\mathrm{g}}{3u_\mathrm{g}} = \frac{2 \cdot 80 \text{ kW}}{3 \cdot 326.6 \text{ V}} = 163.3 \text{ A} \qquad i_\mathrm{cq} = -\frac{2q_\mathrm{g}}{3u_\mathrm{g}} = 0 \tag{2}$$

The current magnitude is $|\underline{i}_\mathrm{c}| = 163.3$ A. The converter output voltage in synchronous coordinates is

$$\underline{u}_\mathrm{c} = L_\mathrm{f}\frac{\mathrm{d}\underline{i}_\mathrm{c}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{g}L_\mathrm{f}\underline{i}_\mathrm{c} + \underline{u}_\mathrm{g}$$

In grid-voltage coordinates, the steady-state voltage is

$$\underline{u}_\mathrm{c} = \mathrm{j}\omega_\mathrm{g}L_\mathrm{f}\underline{i}_\mathrm{c} + u_\mathrm{g} = u_\mathrm{g} - \omega_\mathrm{g}L_\mathrm{f}i_\mathrm{cq} + \mathrm{j}\omega_\mathrm{g}L_\mathrm{f}i_\mathrm{cd} \tag{3}$$

Hence, the components are

$$\begin{aligned} u_\mathrm{cd} &= u_\mathrm{g} - \omega_\mathrm{g}L_\mathrm{f}i_\mathrm{cq} = 326.6 \text{ V} \\ u_\mathrm{cq} &= \omega_\mathrm{g}L_\mathrm{f}i_\mathrm{cd} = 2\pi \cdot 50 \text{ rad/s} \cdot 2 \text{ mH} \cdot 163.3 \text{ A} = 102.6 \text{ V} \end{aligned}$$

The magnitude of the converter output voltage is

$$u_\mathrm{c} = \sqrt{u_\mathrm{cd}^2 + u_\mathrm{cq}^2} = \sqrt{(326.6\text{ V})^2 + (102.6\text{ V})^2} = 342.3 \text{ V}$$

(b) From (2), the d and q current components are

$$i_\mathrm{cd} = \frac{2p_\mathrm{g}}{3u_\mathrm{g}} = \frac{2\cdot 80\text{ kW}}{3\cdot 0.8\cdot 326.6\text{ V}} = 204.1\text{ A} \qquad i_\mathrm{cq} = -\frac{2q_\mathrm{g}}{3u_\mathrm{g}} = 0$$

Hence, the current magnitude is $|\underline{i}_\mathrm{c}| = 204.1$ A, which is well below the maximum current $i_\mathrm{max} = \sqrt{2}\cdot 200$ A $= 282.8$ A. Using (3), the converter voltage is $\underline{u}_\mathrm{c} = (261.3 + \mathrm{j}128.3)$ V and the magnitude is $|\underline{u}_\mathrm{c}| = 291.0$ V.

If the grid voltage decreased down to 50%, the required converter current 326.6 A would be above the maximum current. Hence, the converter would trip in this condition (unless the power supplied to the DC bus is reduced).

(c) The d and q current components are

$$i_\mathrm{cd} = 204.1\text{ A} \qquad i_\mathrm{cq} = -\sqrt{i_\mathrm{max}^2 - i_\mathrm{cd}^2} = -\sqrt{(282.8\text{ A})^2 - (204.1\text{ A})^2} = -195.8\text{ A}$$

Accoding to (1), the reactive power fed to the PCC is

$$q_\mathrm{g} = \frac{3}{2}\,\mathrm{Im}\left\{\underline{u}_\mathrm{g}\underline{i}_\mathrm{c}^*\right\} = -\frac{3}{2}u_\mathrm{g}i_\mathrm{cd} = -\frac{3}{2}\cdot 0.8\cdot 326.6\text{ V}\cdot(-195.8\text{ A}) = 76.7\text{ kVAr}$$

Using (3), the converter voltage is $\underline{u}_\mathrm{c} = (384.3 + \mathrm{j}128.3)$ V and the magnitude is $|\underline{u}_\mathrm{c}| = 405.1$ V.

The maximum achievable voltage magnitude is $u_\mathrm{max} = u_\mathrm{dc}/\sqrt{3}$ in the linear modulation region. Hence, the DC-bus voltage should $u_\mathrm{dc} > \sqrt{3}\cdot 405.1$ V $= 702$ V.

Problem 2: Refence calculation under unbalanced voltage conditions

The grid voltage is unbalanced due to an unbalanced fault. The PLL of the grid converter is able to decompose the measured grid voltage into the positive-sequence and negative-sequence components. Furthermore, the current controller is capable of feeding the desired positive- and negative-sequence current components as well as any desired harmonics. Calculate the grid current references in the following cases:

(a) Both the active power and reactive power are kept constant.

(b) The grid currents are controlled to be sinusoidal and balanced.

(c) The active power is kept constant and the grid currents are controlled to be sinusoidal.

The inputs to the current reference calculation are the active and reactive power references.

Solution

To simplify notation, no separate subsripts for reference quantities will be used.

(a) The instantaneous complex power is

$$\underline{s}_\mathrm{g} = \frac{3}{2}\underline{u}_\mathrm{g}\underline{i}_\mathrm{g}^*$$

Therefore, constant (or arbitrary) active and reactive powers are obtained if the grid current is

$$\underline{i}_\mathrm{g} = \frac{2}{3}\frac{\underline{s}_\mathrm{g}^*}{\underline{u}_\mathrm{g}^*} = \frac{2}{3}\frac{p_\mathrm{g} - \mathrm{j}q_\mathrm{g}}{\underline{u}_\mathrm{g}^*}$$

It is worth noticing that in this case the currents become distorted if the grid voltage is unbalanced.

(b) The instantaneous unbalanced grid voltage and current are expressed using the positive- and negative-sequence components:

$$\underline{u}_\mathrm{g} = \underline{u}_\mathrm{g+} + \underline{u}_\mathrm{g-} = u_\mathrm{g+}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{u+})} + u_\mathrm{g-}\mathrm{e}^{-\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{u-})} \tag{4a}$$

$$\underline{i}_\mathrm{g} = \underline{i}_\mathrm{g+} + \underline{i}_\mathrm{g-} = i_\mathrm{g+}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{i+})} + i_\mathrm{g-}\mathrm{e}^{-\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{i-})} \tag{4b}$$

The complex power is

$$\begin{aligned}\underline{s}_\mathrm{g} &= \frac{3}{2}\underline{u}_\mathrm{g}\underline{i}_\mathrm{g}^* = \frac{3}{2}(\underline{u}_\mathrm{g+} + \underline{u}_\mathrm{g-})(\underline{i}_\mathrm{g+} + \underline{i}_\mathrm{g-})^* \\ &= \underbrace{\frac{3}{2}\left(\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g+}^* + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g-}^*\right)}_{\underline{S}_\mathrm{g}=\,\text{constant}} + \underbrace{\frac{3}{2}\left(\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g-}^* + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g+}^*\right)}_{\tilde{\underline{s}}_\mathrm{g}=\,\text{oscillating}}\end{aligned} \tag{5}$$

where the constant term in the complex power is denoted by $\underline{S}_\mathrm{g}$ and the oscillating term by $\tilde{\underline{s}}_\mathrm{g}$. The sinusoidal balanced currents are obtained if $\underline{i}_\mathrm{g-} = 0$ holds. Under this condition, the desired grid current can be solved from the constant part,

$$\underline{S}_\mathrm{g} = \frac{3}{2}\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g+}^*$$

leading to the desired grid current

$$\underline{i}_\mathrm{g} = \underline{i}_\mathrm{g+} = \frac{2}{3}\frac{\underline{S}_\mathrm{g}^*}{\underline{u}_\mathrm{g+}^*} = \frac{2}{3}\frac{P_\mathrm{g} - \mathrm{j}Q_\mathrm{g}}{\underline{u}_\mathrm{g+}^*}$$

For calculating this reference, the control system has to be able to segregate the positive-sequence component $\underline{u}_\mathrm{g+}$ from the measured grid voltage $\underline{u}_\mathrm{g}$. This functionality can be included in the PLL. It can also be seen from (5) that both the active and reactive powers are oscillating, if the voltage is unbalanced.

(c) The positive- and negative-sequence components of (4) are repeated here:

$$\underline{u}_\mathrm{g+} = u_\mathrm{g+}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{u+})} \qquad \underline{u}_\mathrm{g-} = u_\mathrm{g-}\mathrm{e}^{-\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{u-})} \tag{6a}$$

$$\underline{i}_\mathrm{g+} = i_\mathrm{g+}\mathrm{e}^{\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{i+})} \qquad \underline{i}_\mathrm{g-} = i_\mathrm{g-}\mathrm{e}^{-\mathrm{j}(\omega_\mathrm{g}t+\phi_\mathrm{i-})} \tag{6b}$$

The oscillating part of the active power in (5) can be expressed as

$$\begin{aligned}
\tilde{p}_\mathrm{g} &= \frac{3}{2}\mathrm{Re}\{\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g-}^* + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g+}^*\} \\
&= \frac{3}{4}\left(\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g-}^* + \underline{u}_\mathrm{g+}^*\underline{i}_\mathrm{g-} + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g+}^* + \underline{u}_\mathrm{g-}^*\underline{i}_\mathrm{g+}\right) \\
&= \underbrace{\frac{3}{4}(\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g-}^* + \underline{u}_\mathrm{g-}^*\underline{i}_\mathrm{g+})}_{\text{rotates at } 2\omega_\mathrm{g}} + \underbrace{\frac{3}{4}(\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g-}^* + \underline{u}_\mathrm{g-}^*\underline{i}_\mathrm{g+})^*}_{\text{rotates at } -2\omega_\mathrm{g}}
\end{aligned}$$

where the second form follows from $\mathrm{Re}\{\underline{z}\} = (\underline{z}+\underline{z}^*)/2$ and the last form follows from (6). Therefore, the oscillating active power is $\tilde{p}_\mathrm{g} = 0$ if

$$\underline{i}_\mathrm{g-}^* = -\frac{\underline{u}_\mathrm{g-}^*\underline{i}_\mathrm{g+}}{\underline{u}_\mathrm{g+}} \tag{7}$$

Using (7), the constant active power becomes

$$\begin{aligned}
P_\mathrm{g} &= \frac{3}{2}\mathrm{Re}\left\{\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g+}^* + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g-}^*\right\} \\
&= \frac{3}{2}\mathrm{Re}\left\{\left(\underline{u}_\mathrm{g+}^* - \frac{\underline{u}_\mathrm{g-}\underline{u}_\mathrm{g-}^*}{\underline{u}_\mathrm{g+}}\right)\underline{i}_\mathrm{g+}\right\} = \frac{3}{2}\mathrm{Re}\left\{\left(\frac{\underline{u}_\mathrm{g+}\underline{u}_\mathrm{g+}^* - \underline{u}_\mathrm{g-}\underline{u}_\mathrm{g-}^*}{\underline{u}_\mathrm{g+}}\right)\underline{i}_\mathrm{g+}\right\} \\
&= \frac{3}{2}\frac{|\underline{u}_\mathrm{g+}|^2 - |\underline{u}_\mathrm{g-}|^2}{|\underline{u}_\mathrm{g+}|^2}\mathrm{Re}\left\{\underline{i}_\mathrm{g+}\underline{u}_\mathrm{g+}^*\right\}
\end{aligned} \tag{8}$$

The desired active power is obtained if

$$\underline{i}_\mathrm{g+} = \frac{2}{3}\frac{\underline{u}_\mathrm{g+}}{|\underline{u}_\mathrm{g+}|^2 - |\underline{u}_\mathrm{g-}|^2}P_\mathrm{g} \tag{9}$$

Similarly, the average of the reactive power is

$$\begin{aligned}
Q_\mathrm{g} &= \frac{3}{2}\mathrm{Im}\left\{\underline{u}_\mathrm{g+}\underline{i}_\mathrm{g+}^* + \underline{u}_\mathrm{g-}\underline{i}_\mathrm{g-}^*\right\} \\
&= \frac{3}{2}\mathrm{Im}\left\{\left(-\underline{u}_\mathrm{g+}^* - \frac{\underline{u}_\mathrm{g-}\underline{u}_\mathrm{g-}^*}{\underline{u}_\mathrm{g+}}\right)\underline{i}_\mathrm{g+}\right\} = -\frac{3}{2}\mathrm{Im}\left\{\left(\frac{\underline{u}_\mathrm{g+}\underline{u}_\mathrm{g+}^* + \underline{u}_\mathrm{g-}\underline{u}_\mathrm{g-}^*}{\underline{u}_\mathrm{g+}}\right)\underline{i}_\mathrm{g+}\right\} \\
&= -\frac{3}{2}\frac{|\underline{u}_\mathrm{g+}|^2 + |\underline{u}_\mathrm{g-}|^2}{|\underline{u}_\mathrm{g+}|^2}\mathrm{Im}\left\{\underline{i}_\mathrm{g+}\underline{u}_\mathrm{g+}^*\right\}
\end{aligned} \tag{10}$$

The desired average of the reactive power is obtained if

$$\underline{i}_{g+} = -j\frac{2}{3}\frac{\underline{u}_{g+}}{|\underline{u}_{g+}|^2 + |\underline{u}_{g-}|^2}Q_g \tag{11}$$

The expressions (7), (9), and (11) can be combined

$$\begin{aligned}\underline{i}_{g+} &= \frac{\underline{u}_{g+}}{|\underline{u}_{g+}|^2 - |\underline{u}_{g-}|^2}P_g - j\frac{\underline{u}_{g+}}{|\underline{u}_{g+}|^2 + |\underline{u}_{g-}|^2}Q_g \\ \underline{i}_{g-} &= -\frac{\underline{u}_{g-}\underline{i}^*_{g+}}{\underline{u}^*_{g+}}\end{aligned}$$

For calculating this reference, the control system has to be able to decompose the measured grid voltage into its positive- and negative sequence components, $\underline{u}_g = \underline{u}_{g+} + \underline{u}_{g-}$. In the balanced case $\underline{u}_{g-} = 0$ and the reference calculation reduces to the standard one.

The derivation for the constant reactive power $\tilde{q}_g = 0$ is analogous to the derivation shown here.

Remark: Example waveforms of these control strategies are shown in the figure below. The phase voltages are

$$\begin{aligned}u_{ga} &= 0.25u_g\cos(\omega_g t) \\ u_{gb} &= u_g\cos(\omega_g t - 2\pi/3) \\ u_{gc} &= u_g\cos(\omega_g t - 4\pi/3)\end{aligned}$$

The space vector transformation gives

$$\underline{u}_g = \underline{u}_{g+} + \underline{u}_{g-} = u_{g+}e^{j(\omega_g t + \phi_{u+})} + u_{g-}e^{-j(\omega_g t + \phi_{u-})}$$

where $u_{g+} = 0.75u_g$, $u_{g-} = 0.25u_g$, $\phi_{u+} = 0$, and $\phi_{u-} = \pi$. The strategies in Part (a), (b), and (c) are shown at $t = 0\ldots40$ ms, $t = 40\ldots80$ ms, and $t = 80\ldots120$ ms, respectively. The currents at $t = 120\ldots160$ ms correspond to the strategy $\tilde{q}_g = 0$. You can try to plot these waveforms using your computer.