Two-level system (TLS)

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I. SINGLE-ELECTRON BOX (SEB) AS A TWO-LEVEL SYSTEM

Consider SEB as schematically shown in Fig. 1(a). There is an integer number n of electrons on the island distributed on the capacitors as

$$-ne = C_g(U - V_g) + CU, \tag{1}$$

where $U = (-ne + C_g V_g)/C_\Sigma$ is the potential of the island and V_g is the gate voltage. Here $C_\Sigma = C_g + C$. The energy of the capacitors in SEB is given by

$$E_{\rm cap} = \frac{1}{2}C_g(U - V_g)^2 + \frac{1}{2}CU^2 = \frac{(ne)^2}{2C_\Sigma} + \text{const.}, \tag{2}$$

where const. refers to the terms which are not dependent on n. The free energy of SEB is then

$$E = E_{\rm cap} - Q_g V_g, \tag{3}$$

where $Q_g V_g$ is the work done by the voltage source. Since $Q_g = C_g(V_g - U)$ then by considering only the terms dependent on n we have

$$E = \frac{(ne)^2}{2C_\Sigma} - \frac{C_g V_g}{C_\Sigma} ne + \text{const.}, \tag{4}$$

or

$$E = E_C(n - n_g)^2, \tag{5}$$

where $E_C = \frac{e^2}{2C_\Sigma}$ and $n_g = \frac{C_g V_g}{e}$. In the gate range $0 < n_g < 1$, we may consider SEB as a TLS with

$$\begin{aligned}\Delta E &= E(n=1) - E(n=0)\\ &= E_C(1 - 2n_g).\end{aligned} \tag{6}$$

II. HEAT, WORK AND INTERNAL ENERGY OF A TWO-LEVEL SYSTEM (TLS)

Heat is associated to transitions in the TLS, $đQ = dP_1 \Delta E$, where $P_1 = \frac{1}{1+e^{\beta \Delta E}}$ is the population in the excited state. Work corresponds to lifting the level of the excited particles (electrons by the voltage source in a SEB),

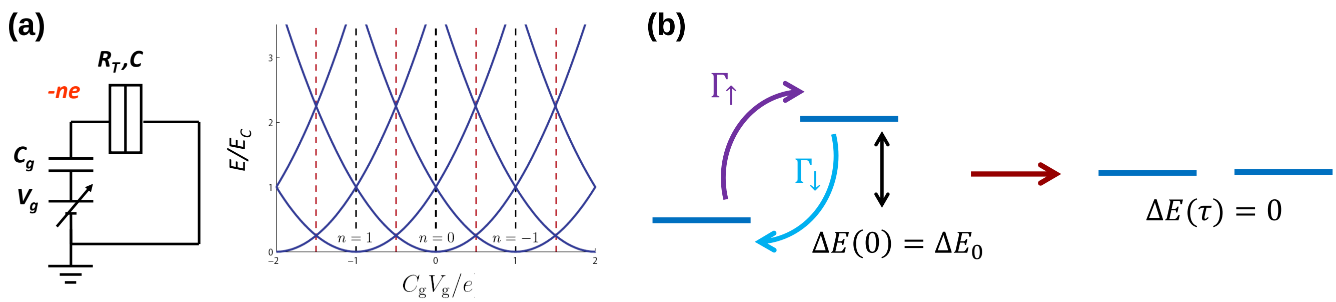

FIG. 1. (a,left) Single-electron box (SEB). (a,right) Energies of different charge states n. (b) Two-level system moved from finite energy difference initially to degeneracy at time τ. In the text the adiabatic ramp is discussed.

$đW = P_1 d(\Delta E)$.

$$\begin{aligned} dU &= đQ + đW \\ &= dP_1 \Delta E + P_1 d(\Delta E) \\ &= d(P_1 \Delta E). \end{aligned} \tag{7}$$

Thus in going from initial state i to final state one f, we have

$$\Delta U = U^{(f)} - U^{(i)} = P_1^{(f)} \Delta E^{(f)} - P_1^{(i)} \Delta E^{(i)}. \tag{8}$$

III. HEAT AND ENTROPY IN REVERSIBLE DRIVE OF A TLS

We may write $\dot{Q} = dQ/dt$ as

$$\dot{Q} = \Delta E \dot{P}_1. \tag{9}$$

Then we obtain the total heat from the bath in the process of changing the level spacing from a finite value ΔE_0 to 0 over time τ as

$$\Delta Q = \int_0^\tau dt \Delta E(t) \dot{P}_1(t) = -\int_0^\tau dt \frac{d\Delta E(t)}{dt} P_1(t), \tag{10}$$

where the second step is obtained by partial integration and by observing that the boundary term vanishes when $\beta \Delta E_0 \gg 1$.

Assuming linear change $\frac{d\Delta E(t)}{dt} = -\Delta E_0/\tau$ and quasistatic (reversible) ramp where $P_1(t) = 1/(1 + e^{\beta \Delta E(t)})$, we obtain the heat from the bath in this process as

$$\Delta Q^{(0)} = \frac{\Delta E_0}{\tau} \int_0^\tau dt \frac{1}{1 + e^{\beta \Delta E(t)}}. \tag{11}$$

Letting $\beta \Delta E_0 \to \infty$, we have (with $\beta = 1/(k_B T)$)

$$\Delta Q^{(0)} = k_B T \ln 2. \tag{12}$$

This corresponds to entropy production $\Delta S = \Delta Q^{(0)}/T$ of

$$\Delta S = k_B \ln 2 \tag{13}$$

in the TLS.