General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. The (attempted) solutions should be submitted via email to the assistant at the start of the exercise session on March 10th. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Chapter 10 in Fusion Physics [IAEA Publishing, 2012, ISBN 978-92-0-130410-0] might prove itself a useful resource for this exercise set. A PDF version of the book can be downloaded from the IAEA website by searching for the title. The course website should also have a link to the PDF under "Additional resources". Alternatively, the paper "*The physics issues that determine inertial confinement fusion target gain and driver requirements: A tutorial*" by M.D. Rosen (Physics of Plasmas, 1999) is an alternative source.

Exercise 1.
Inertial confinement fusion concepts

a) Explain the difference between direct and in-direct drive in inertial confinement fusion.

b) Explain the difference between fast and central ignition?

c) What limits the performance of inertial confinement fusion?

d) What are the main benefits of compressing the fuel?

e) How does "rocket equation" relate to ICF?

f) What are the main subsystems of an inertial confinement fusion power plant?

Exercise 2.
The Lawson criterion in ICF

a) In the context of ICF, the Lawson criterion is often expressed in terms of the fuel mass density, ρ, and the fuel pellet radius, R. Derive the Lawson criterion, $n\tau_E > 10^{20}$ m^{-3}s, in terms of ρR. Assume a fuel temperature of 10 keV. The fuel mass density can be expressed using the particle number density and particle mass, $\rho = nm$, while the confinement time can be estimated using the radius of a sphere and the ion sound speed: $\tau \sim R/c_s$.

b) The density of solid hydrogen is approximately 0.2 g cm^{-3}. Calculate the required radius of a spherical fuel pellet to satisfy the Lawson criterion, assuming that the fuel is not compressed.

c) What is the mass of the resulting D-T fuel pellet.

d) Calculate a simple estimate for the energy required to heat this fuel pellet uniformly from 0 to 10 keV, i.e. calculate the thermal energy change for the fuel pellet. Neglect any inefficiencies in the the laser system, and any other caveats. The lasers of the National Ignition Facility (NIF) have energies of about 1.8 MJ, how does this compare to the value you calculated?

e) How do the results of b), c), and d) change, if the fuel pellet is compressed to a density of about 400 g cm^{-3}? What kind of conclusion can you draw from this comparison?

Exercise 3.
The gain factor in ICF

a) The gain factor has been defined as Q = *energy produced / energy used* in previous exercises. Calculate the maximum *theoretical* gain factor in inertial fusion when the fuel pellet is heated uniformly to ideal ignition (α-heating overcomes Bremsstrahlung losses). Consider both D-T (ideal ignition requires $T = 4$ keV) and with D-^{3}He fuel (ideal ignition requires $T = 30$ keV).

b) How does a burn-up ratio of 30% affect the gain factor?

c) How does a combined laser absorption and hydrodynamic efficiency of 10 % affect the gain factor, considering the burn-up ratio as well?

d) How does considering further losses due to electric conversion and laser efficiency affect the gain factor, is the resulting gain factor viable for energy production? For a realistic power plant, you should have about $Q = 100$.

e) If the answer is no, how can the gain factor be increased?

Exercise 4.
The burn-up ratio Derive the equation for the fraction of fuel burned in inertial confinement fusion:

$$\phi = \frac{\rho R}{\rho R + H_B},$$

where R is the fuel pellet radius, ρ is the mass density, and $H_B \approx 7$ g cm^{-2} is the burn parameter, as presented in the lecture slides (Inertial Confinement Fusion, slide 10).

Assume 50-50 D-T fuel and write the differential equation for the fuel density:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \text{fusion reaction rate}$$

then integrating from zero to confinement time.