Microeconomics 3: Game Theory - Exam

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The exam is 2 hours and has a total of 120 points. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering. Point totals reflect the difficulty of the problem and give a rough estimate for how long the question should take.

1. Consider the following extensive form.

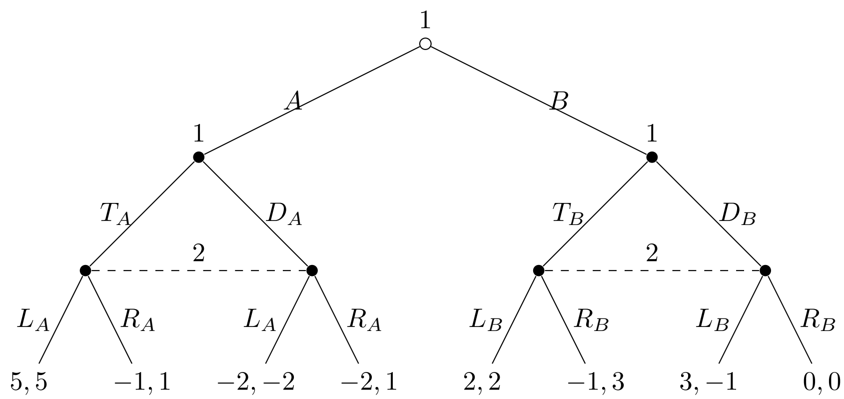

Figure 1: Extensive Form for Question 1

   (a) *(5 points)* Is this a game of perfect information? Why or why not?

   (b) *(5 points)* What are the subgames in this game?

   (c) *(10 points)* What is the Subgame Perfect Equilibrium of this game?

   (d) *(10 points)* Describe a system of beliefs that makes the strategy profile $((B, T_A, D_B), (R_A, R_B))$ sequentially rational.

   (e) *(10 points)* Define sequential equilibrium. Show that the strategy profile in (d) cannot be played in a sequential equilibrium.

2. This question concerns the following normal form game.

	L	R
T	$1, 10$	$8, 8$
B	$0, 0$	$10, 1$

Figure 2: Normal Form for Question 2

   (a) *(5 points)* What are the 3 Nash Equilibria of this game?

   Suppose the game is repeated once, and player's payoffs are the sum of the stage game payoffs (without discounting).

   (b) *(5 points)* Consider a subgame starting in the second period. What strategy profiles can be played in this subgame in a Subgame Perfect Equilibrium?

   (c) *(15 points)* Describe a Subgame Perfect Equilibrium where TR is played in the first period. Verify that it is a SPE.

   (d) *(15 points)* Consider the infinitely repeated game with common discount factor δ. For what values of δ does the following automaton describe a subgame perfect equilibrium.

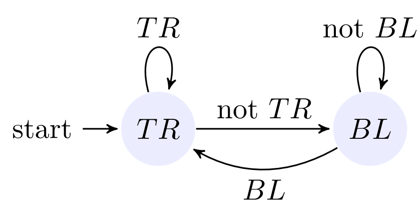

Figure 3: Automaton for 2d

3. Consider the following variant of a Cournot game. At the start of the game, each firm publicly chooses a level of advertising $a_i \in \mathbb{R}_+$ for cost $\frac{3}{2}a_i^2$. This choice of advertising increases demand for the product.

They then compete by choosing how much quantity $q_i \in \mathbb{R}_+$ to produce for 0 marginal cost, where inverse demand given by

$$P(q_1, q_2, a_1, a_2) = \max\{0, (a_1 + a_2 + 1) - \frac{2}{3}(q_1 + q_2)\},$$

so a firm's payoff is

$$q_i P(q_1, q_2, a_1, a_2) - \frac{3}{2}a_i^2.$$

(a) *(5 points)* Describe the firm's strategies in this game.

(b) *(10 points)* Fix levels of advertising a_1, a_2. What is the Nash Equilibrium in the corresponding subgame?

(c) *(10 points)* Describe the Subgame Perfect Equilibrium of this game.

(d) *(15 points)* Now suppose each firm chooses a_i and q_i simultaneously. What is the Nash Equilibrium of this game?