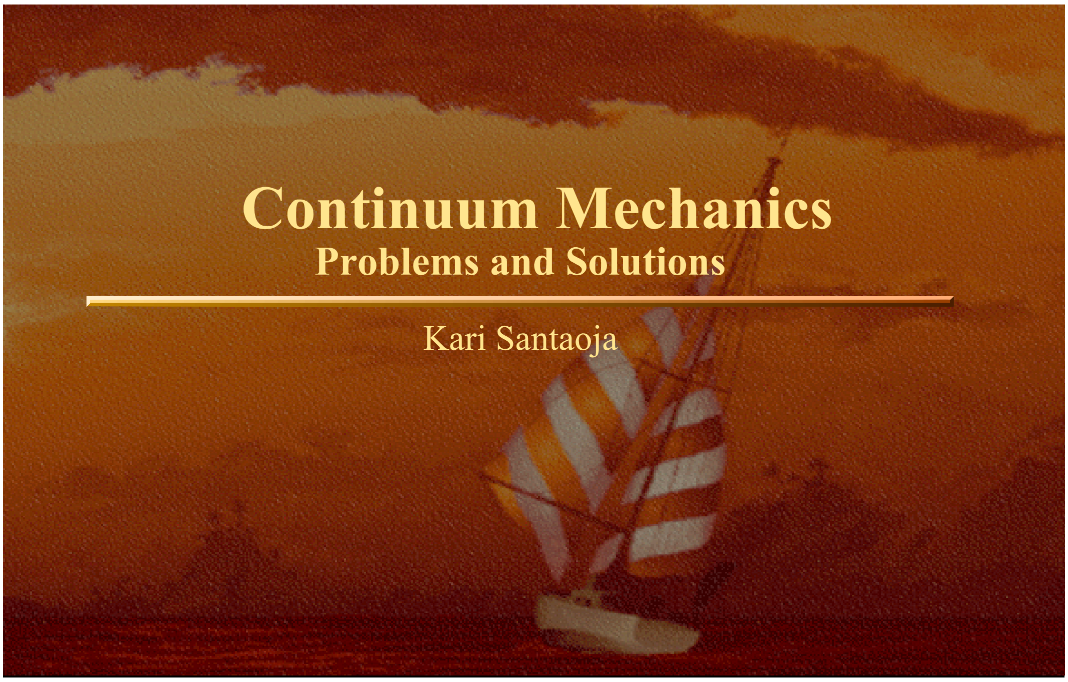

Continuum Mechanics and Material Modelling
MEC-E8002

Formulary 2021

Three important axioms are:

AX 2 $$U \;:=\; U(Y_\gamma, S)\,. \tag{1}$$

and

AX 3 $$\mathrm{d}U \;=\; \text{đ}W + \text{đ}Q^* \tag{2}$$

and finally

AX 4 $$\text{đ}Q^* \;=\; T\,\mathrm{d}S\,. \tag{3}$$

The following equations hold:

$$\boldsymbol{\sigma} \;=\; \rho_0\, \frac{\partial u\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, s, h(\vec{X})\big)}{\partial(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}})} \tag{4}$$

and

$$T \;=\; \frac{\partial u\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, s, h(\vec{X})\big)}{\partial s} \qquad \text{and finally} \qquad \boldsymbol{\beta} \;:=\; -\rho_0\, \frac{\partial u\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, s, h(\vec{X})\big)}{\partial \boldsymbol{\alpha}}\,. \tag{5}$$

The following equations hold:

$$s \;=\; -\,\frac{\partial \psi\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial T} \tag{6}$$

and

$$\boldsymbol{\sigma} \;=\; \rho_0\, \frac{\partial \psi\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}})} \qquad \text{and} \qquad \boldsymbol{\beta} \;=\; -\rho_0\, \frac{\partial \psi\big(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial \boldsymbol{\alpha}}\,. \tag{7}$$

and further

$$s \;=\; \frac{\partial g\big(\boldsymbol{\sigma}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial T} \tag{8}$$

and finally

$$\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}} \;=\; \rho_0\, \frac{\partial g\big(\boldsymbol{\sigma}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial \boldsymbol{\sigma}} \qquad \text{and} \qquad \boldsymbol{\beta} \;=\; \rho_0\, \frac{\partial g\big(\boldsymbol{\sigma}, \boldsymbol{\alpha}, T, h(\vec{X})\big)}{\partial \boldsymbol{\alpha}}\,. \tag{9}$$

Two important equations read

$$\rho_0\, \dot{u} \;=\; \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \rho_0\, r - \vec{\nabla} \cdot \vec{q} \tag{10}$$

and

$$\Phi \;:=\; \rho_0\, T \dot{s} + \vec{\nabla} \cdot \vec{q} - \frac{\vec{\nabla} T}{T} \cdot \vec{q} - \rho_0\, r \qquad (\;\geq\; 0\;)\,. \tag{11}$$

The following expressions hold:

$$\boldsymbol{\sigma} \;=\; \mu\, \rho_0\, \frac{\partial \varphi_{\mathrm{mech}}(\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}, \dot{\boldsymbol{\alpha}}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h)}{\partial \dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}} \tag{12}$$

and

$$\boldsymbol{\beta} \;=\; \mu\, \rho_0\, \frac{\partial \varphi_{\mathrm{mech}}(\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}, \dot{\boldsymbol{\alpha}}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h)}{\partial \dot{\boldsymbol{\alpha}}} \tag{13}$$

and further

$$\varphi_{\mathrm{mech}}(\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}, \dot{\boldsymbol{\alpha}}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h) = \mu \left(\frac{\partial \varphi_{\mathrm{mech}}}{\partial \dot{\boldsymbol{\varepsilon}}^{\mathrm{i}}} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{i}} + \frac{\partial \varphi_{\mathrm{mech}}}{\partial \dot{\boldsymbol{\alpha}}} : \dot{\boldsymbol{\alpha}} \right) . \qquad \textbf{(14)}$$

and even further

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{i}} = \rho_0 \, \frac{\partial \varphi^{\mathrm{c}}_{\mathrm{mech}}(\boldsymbol{\sigma}, \boldsymbol{\beta}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h)}{\partial \boldsymbol{\sigma}} \qquad \textbf{(15)}$$

and

$$\dot{\boldsymbol{\alpha}} = \rho_0 \, \frac{\partial \varphi^{\mathrm{c}}_{\mathrm{mech}}(\boldsymbol{\sigma}, \boldsymbol{\beta}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h)}{\partial \boldsymbol{\beta}} . \qquad \textbf{(16)}$$

and

$$\varphi^{\mathrm{c}}_{\mathrm{mech}}(\boldsymbol{\sigma}, \boldsymbol{\beta}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h) = (1 - \mu) \left[\frac{\partial \varphi^{\mathrm{c}}_{\mathrm{mech}}}{\partial \boldsymbol{\sigma}} : \boldsymbol{\sigma} + \frac{\partial \varphi^{\mathrm{c}}_{\mathrm{mech}}}{\partial \boldsymbol{\beta}} : \boldsymbol{\beta} \right] . \qquad \textbf{(17)}$$

and

$$- \frac{\vec{\nabla} T}{T} = \mu_{\mathrm{ther}} \, \rho_0 \, \frac{\partial \varphi_{\mathrm{ther}}(\vec{q}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h)}{\partial \vec{q}} \qquad \textbf{(18)}$$

and finally

$$\varphi_{\mathrm{ther}}(\vec{q}; \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{i}}, \boldsymbol{\alpha}, T, h) = \mu_{\mathrm{ther}} \frac{\partial \varphi_{\mathrm{ther}}}{\partial \vec{q}} \cdot \vec{q} . \qquad \textbf{(19)}$$

This is sometimes useful

$$f(x_0 + \Delta) = f(x_0) + \frac{\partial f(x_0)}{\partial x} \Delta + \frac{1}{2!} \frac{\partial^2 f(x_0)}{\partial x^2} \Delta^2 + \frac{1}{3!} \frac{\partial^3 f(x_0)}{\partial x^3} \Delta^3 + \ldots \qquad \textbf{(20)}$$

Do you know this one?

$$\chi^2(\bar{a}) := \sum_{i=1}^{N} \left(\frac{\varepsilon^{\mathrm{d}}_i - \varepsilon(t_i; \bar{a})}{\gamma_i} \right)^2 . \qquad \textbf{(21)}$$

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