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WINDING DESIGN

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Contents

Literature

I have used the following books in the construction of the text in this lecture material:

Richter R. Elektrische Maschinen I.: Allgemeinen Berechnungselemente und Gleichstrommaschinen. Zweite Auflage, Verlag Birkhäuser, Basel, Germany, 1951, 630 p.

Richter R. Elektrische Maschinen II: Synchronmaschinen und Einankerumformer. Zweite Auflage, Verlag Birkhäuser, Basel / Stuttgart, Germany, 1953, 707 p.

Richter R. Elektrische Maschinen IV. Inductionmaschinen. Verlag von Julius Springer, Berlin, Germany, 1936, 440 p.

Richter R. Ankerwicklungen für Gleich- und Wechselstrommaschinen. Verlag von Julius Springer, Berlin, Germany, 1920, 423 p.

Vogt K. Elektrische Maschinen: Berechnung rotierender elektrischer Maschinen. Dritte Auflage, VEBVerlag Technik, Berlin, Germany, 1983, 500 p.

Details and figures of winding constructions in Finnish and the collection of terminology in winding technique is found in Finnish, Swedish, English, German, and Estonian from

Jokiniemi M., Jokinen T. Terminology of winding in electrical machines (Sähkökoneiden käämityssanasto, in Finnish). Helsinki University of Technology, Laboratory of Electromechanics, Espoo, Finland, Report 49, 1996, 45p.

1 Reluctance model of magnetic circuit

1.1 Reluctance model

The flux distribution in an electrical machine will be defined conventionally by representing the whole machine cross-section by a reluctance network. The straightforward network, the reluctance chain of only the main flux path, is sufficient for the daily machine design. In both the solutions, the network is constructed basing on the material properties and on the reluctance models of sections in the cross-section; tooth on the slotting area, yoke sector of a pole pitch, and the most meaningful section, the air gap sector of a pole pitch.

The reluctance of a section will be defined by

$$R_{\mathrm{m}} = \frac{l_{\Phi}}{\mu A_{\Phi}} \tag{1.1}$$

where

- μ is the permeability of the section of magnetic circuit,
- A_{Φ} the cross-section of the section perpendicular to the flux flow, and
- l_{Φ} the length of the flux path inside the section.

The reluctance network is constructed by assembling the reluctances of the sections, either only in one direction, as for the model of the main flux path, or in radial and circumferential directions, as for the model of the machine cross-section. For 3-D models, also the axial direction has to be added to the network. For traditional machine constructing, the 2-D model is enough. For very short machine, as the axial-flux machines, the 3-D model is needed.

1.2 Basic reluctance model

A reluctance model of a region of flux path is presented in Fig. 1.1 a. The modelling is two-dimensional. The reluctances in both the directions are set on both sides of the node and having half of the reluctance value of the whole region.

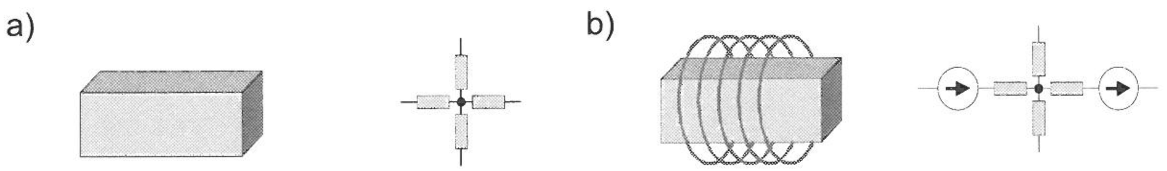


Fig. 1.1 Reluctance model
a) Reluctance model of a region, only the radial and circumferential axis are available. The flux has only two-dimensional variation of route on the flux path. The reluctance value of an axis is divided on both sides of the node.
b) Active reluctance model for a region with MMF source. The MMF source in the reluctance model has the same excitation direction as the real MMF source. The MMF source of an axis is divided on both sides of the node.

The source of the reluctance model is the MMF (magneto-motive force) of the magnetic circuit. A source, a winding on the section, is presented in Fig. 1.1 b. In the reluctance model, the source is described by two MMF sources placed on the axis of the excitation of the winding. The MMF sources are placed on both sides of the node and having half of the value of the winding MMF.

The machine cross-section may be described using four basic flux-path regions presented in Fig. 1.2. The reluctances of the basic regions, the radial reluctance R_{mr} and the circumferential reluctance $R_{\mathrm{m}\varphi}$, are

- for a sector region in Fig. 1.2 a

$$\begin{cases} R_{\mathrm{mr}} = \dfrac{1}{\mu l}\dfrac{1}{\alpha}\ln\left(\dfrac{r_{\mathrm{out}}}{r_{\mathrm{in}}}\right) \\ R_{\mathrm{m}\varphi} = \dfrac{1}{\mu l}\dfrac{\alpha}{\ln\left(\dfrac{r_{\mathrm{out}}}{r_{\mathrm{in}}}\right)} \end{cases} \tag{1.2}$$

where the quantities are

- μ permeability,
- l axial length,
- α angular width,
- r_{out} outer radius, and
- r_{in} inner radius.

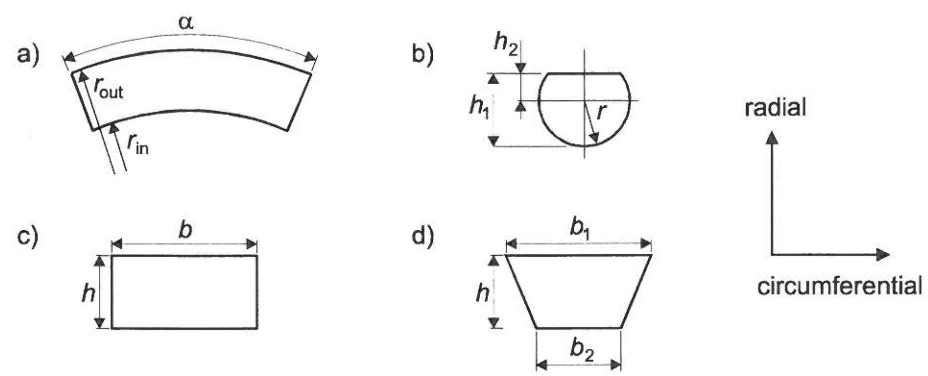


Fig. 1.2 The basic regions of the machine cross-section
a) sector, b) segment of a circle, c) rectangle, and d) symmetric trapezium.
The plane on the right defines the flux directions solved in the equations of permeance.

- for a segment of circle in Fig. 1.2 b

$$\begin{cases} R_{\mathrm{m}r} = \dfrac{1}{2\mu l}\left[\arcsin\left(\dfrac{h_2}{r}\right) + \arcsin\left(\dfrac{h_1 - h_2}{r}\right)\right] \\ R_{\mathrm{m}\varphi} = \dfrac{1}{\mu l}\dfrac{2}{\arcsin\left(\dfrac{h_2}{r}\right) + \arcsin\left(\dfrac{h_1 - h_2}{r}\right)} \end{cases} \tag{1.3}$$

where the quantities are

h_1 total radial height,
h_2 displacement from the centre line, and
r radius.

- for a rectangle in Fig. 1.2 c

$$\begin{cases} R_{\mathrm{m}r} = \dfrac{1}{\mu l}\dfrac{h}{b} \\ R_{\mathrm{m}\varphi} = \dfrac{1}{\mu l}\dfrac{b}{h} \end{cases} \tag{1.4}$$

where the quantities are

h radial height, and
b circumferential width.

- for a symmetric trapezium in Fig. 1.2 d

$$\begin{cases} R_{\mathrm{m}r} = \dfrac{1}{\mu l}\dfrac{h}{b_1 - b_2}\ln\left(\dfrac{b_1}{b_2}\right) \\ R_{\mathrm{m}\varphi} = \dfrac{1}{\mu l}\dfrac{b_1 - b_2}{h}\dfrac{1}{\ln\left(\dfrac{b_1}{b_2}\right)} \end{cases} \tag{1.5}$$

1.3 Reluctance network of a pole pitch

In a synchronous machine, the excitation current in the pole winding is the main MMF source for the flux component, the main flux, from the rotor to the air gap. The flux distribution on a region of one pole pitch is presented in Fig. 1.3 a.

The flux goes from the rotor pole, through the air gap, to the stator. The stator has no slots in Fig. 1.3 a. The flux distribution in the air gap depends only on the reluctance distribution of the air-gap region and on the reluctivity distribution in the stator iron.

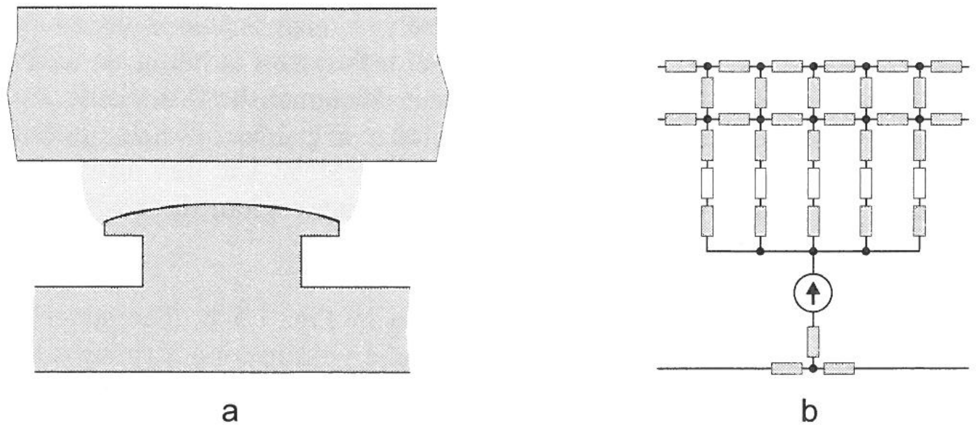


Fig. 1.3 Radial flux distribution in the air gap on one pole pitch.
a) Unslotted stator, excitation by rotor winding on the pole.
b) Simplified reluctance network to define the flux distribution.

The reluctance network of one pole pitch is presented in Fig. 1.3 b. The stator yoke has been divided into two rings. The rings are modelled by sectors having reluctances both on the radial and on the circumferential axis. The available flux routes are in the plane defined by the radial and circumferential axes.

The model of the air gap consists of five air-gap sectors connected to five stator sectors. The reluctance model of a sector in the air gap has only the radial reluctance of the sector, the white reluctance in Fig. 1.3 b. The available flux routes are limited to the radial direction. No circumferential flux from one flux tube to another one is allowed.

In the rotor, the pole shoe is modelled according to the air-gap model, by five sectors, connected to the equivalent air-gap sectors. The pole body is modelled only by a single radial reluctance connected to two circumferential reluctances of the rotor yoke. The circumferential reluctance of the yoke is defined for the pole pitch and divided to both sides of the node. The source of the modelled reluctance network is the MMF of the pole winding represented as a single MMF source equal to the whole MMF value of the winding. The MMF value affecting in the air gap is presented as a rectangle in Fig. 1.4. The positive bar means that the MMF source affects to the positive flux direction. The negative bar is for the opposite pole of the neighbouring pole pitch. The sinus line is the equivalent fundamental wave used in the design bases on the reluctance network of the main flux path. The amplitude equals to the height of the bar.

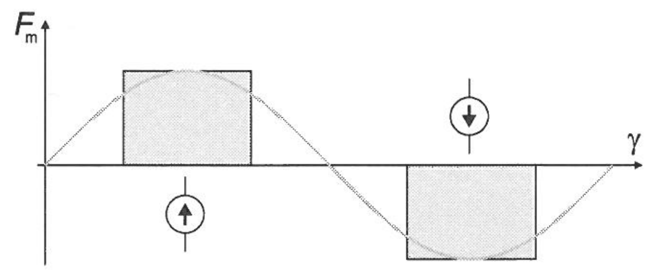


Fig. 1.4 MMF source distribution in air gap, caused by the pole winding in rotor

In a permanent-magnet synchronous machine, a permanent magnet pole is the source of the main flux in the machine, Fig. 1.5. The stator has slots for the stator winding, so the periphery of the stator in the air gap is not smooth. This means a change to the flux distribution in the machine. The flux distribution in the air gap is presented in Fig. 1.5 a. The flux goes from the air gap to the stator teeth, partly through the opening of the slots. The flux distribution in the air gap depends on the reluctance distribution in the air-gap region, the reluctances in the stator teeth, and the reluctance distribution in the stator yoke.

The reluctance network of one pole pitch is presented in Fig. 1.5 b. The stator has been divided to two sections modelled by slot-pitch sectors: the yoke section forming a reluctance ring with circumferential reluctances and the slotting section as a collection of radial reluctances of the teeth.

The model of the air gap consists of five air-gap sectors connected to five stator teeth. The reluctance model of a sector in the air gap has only the radial reluctance of the sector, the white reluctance in Fig. 1.5 b. The available flux routes are limited to the radial direction. No circumferential flux from one flux tube to another one is allowed. The flux routes through the slot opening to the tooth are taken into account in the air-gap reluctances by Charter factor, k_C.

Basically, the machine doesn't notice the type of the MMF source at all. In the sense of the magnetic circuit, the reluctance network of the rotor in a permanent-magnet machine, see Fig. 1.5 b, is equivalent to the reluctance network of the rotor with the pole winding, see Fig. 1.3 b. The only difference is that the radial reluctance of the wound pole body, Fig.1.3 b, is divided to two reluctances in Fig. 1.5 b, the reluctance of the permanent magnet (the black component) and the radial reluctance of the connection of the magnet to the rotor yoke. The reason is the different material properties of these regions.

There are two sources in the reluctance model: the MMF of the magnet and the MMF of the stator winding. The MMF of the magnet is presented as a single MMF source equal to the whole MMF value of the magnet. The MMF of the stator winding is presented as a collection of MMF sources placed on the branches of the teeth.

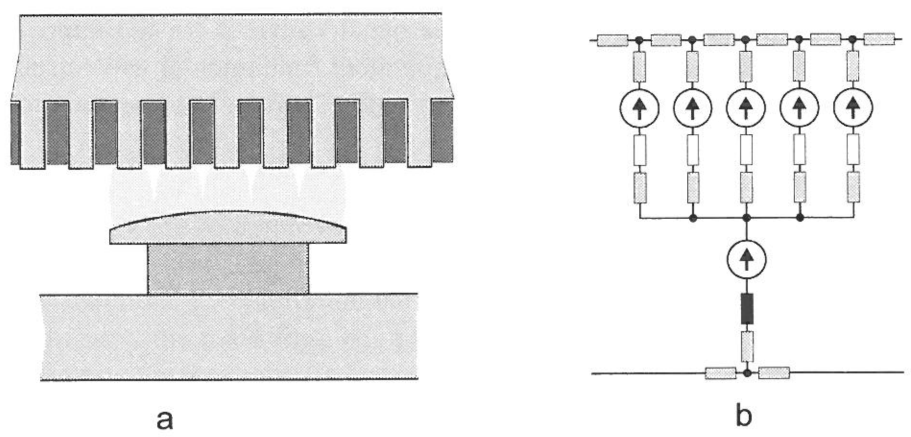


Fig. 1.5 Radial flux distribution in the air gap on one pole pitch.
a) Slotted stator, excitation by the permanent magnet of the pole body.
b) Simplified reluctance network to define the flux distribution.

The MMF value from the magnet affecting in the air gap equals to the MMF presented in Fig. 1.4. The positive bar means that the MMF source affects to the positive flux direction. The sinus line is the equivalent fundamental wave used in the design based on the reluctance network of the main flux path. The amplitude equals to the height of the bar

$$F_{\mathrm{m}} = h_{\mathrm{M}} H_{\mathrm{c}} \tag{1.6}$$

where

h_{M} is the radial height of the magnet, the height parallel to the flux flow, and
H_{c} coercitive field strength of the magnet.

The MMF distribution in the air gap due to the stator winding, a symmetric three phase winding, is presented in Fig. 1.6. The circumferential periphery in the figure is two pole pitches. The sinus line is the fundamental wave of the MMF distribution. The positive value means that the winding affects to the positive flux direction in the reluctance model. If only the fundamental MMF wave is taken into account, the MMF sources used in the reluctance network are defined by the value of the MMF wave at the stator slot pitch γ_i

$$F_{\mathrm{m}} = \hat{f}_{\mathrm{m}} \sin \gamma_i = \frac{2\sqrt{3}}{\pi} N_{\mathrm{eff}} \hat{i}_{\mathrm{s}} \sin \gamma_i \tag{1.7}$$

where

N_{eff} is the effective number of turns in the winding and
i_{s} the phase current.

To take into account all the MMF harmonics the MMF distribution of the winding has to be defined. The current $i_{s,i}$ in a slot i causes to the MMF distribution an increment of

$$F_{\mathrm{m}i} = N_{\mathrm{slot}} i_{\mathrm{s},i} \tag{1.8}$$

where N_{slot} is the effective number of turns in the slot.

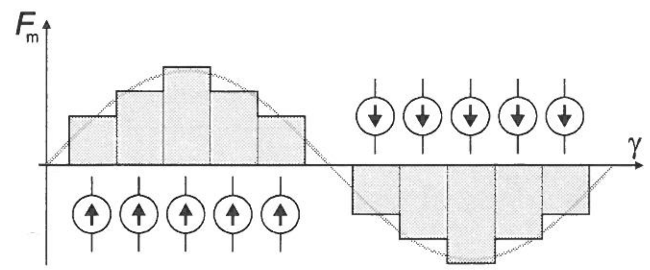


Fig. 1.6 MMF source distribution in air gap, caused by the stator winding in slots.
The height of the bars mean the value of the MMF source in the teeth.
The sinus curve means the fundamental wave of the MMF distribution.

The superposition of all the slots, after balancing of the positive and negative sides, gives the total MMF distribution of the winding. The form of the MMF distribution is still a collection of bars. Every bar means a MMF source in the tooth at the position of the bar, the slot pitch in Fig. 1.6. The height of the bar is the value of the MMF source.

Another way to define the place of sources in the reluctance network is to use MMF sources on the circumferential branches at the position of the slots. This is done in the reluctance network of permanent magnet machine in Fig. 1.7 a. The MMF source of the slot pitch is placed on the circumferential branch of yoke model. The values of the MMF sources are given as bars in Fig. 1.7 b. The height of the bar, from Eq. (1.8), is the value of the MMF source.

The stepped MMF curve, in Fig. 1.7 b, equals to the shape of the collection of MMF bars in Fig. 1.6. This means the same MMF distribution in the air gap. The fundamental wave of the MMF distribution, the sinus curve in Fig. 1.7 b, equals the fundamental wave of the MMF distribution, the sinus curve in Fig. 1.6.

The direction of the MMF sources in the reluctance network, Fig. 1.7 a, is defined along the flux flow. The direction of the MMF sources in the MMF distribution curve, Fig. 1.7 b, are defined by the value of the MMF source, a negative value turns the direction of source. The sources of reluctance network are the sources of the positive half-wave below the MMF distribution curve, Fig. 1.7 b.

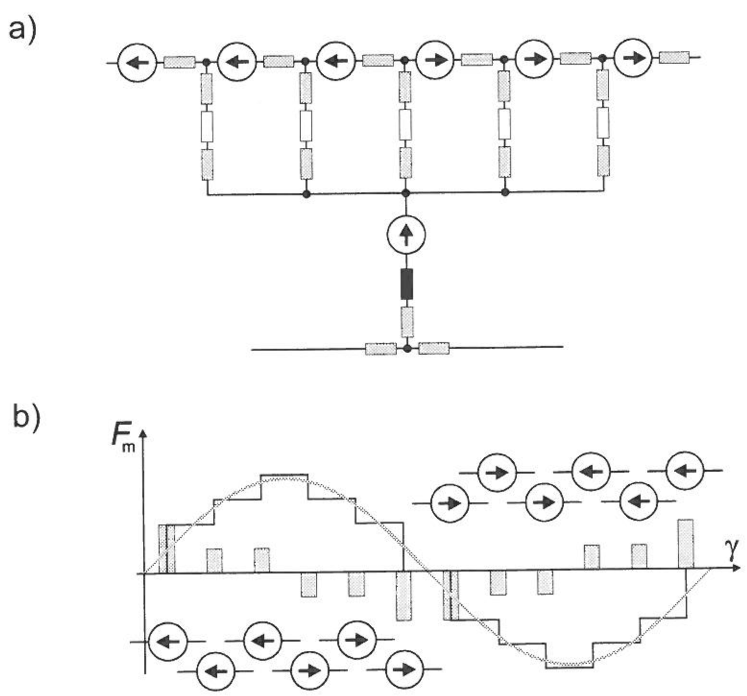


Fig. 1.7 MMF sources of stator winding in slots placed on the circumferential branch of the yoke model.
a) Simplified reluctance network to define the flux distribution of permanent magnet machine.
b) MMF distribution in air gap caused by the stator winding in slots.
The height of the bars mean the value of the MMF source in the yoke.
The stepped curve shows the MMF distribution in air gap caused by the winding.
The sinus curve is the fundamental wave of the MMF distribution.

2 Slot winding

2.1 Basis

To define the winding construction, the air-gap periphery of the machine will be divided into equal parts, pole pitches τ_p

$$\tau_p = \frac{\pi D_i}{2p} \tag{2.1}$$

where

D_i is the air-gap diameter and
p number of pole pairs.

The design of windings bases on the pole pitches. One pole pitch is 180 electrical degrees, 180^e. The pole pitch consists of zones of phases. The width of a zone τ_z is the pole pitch τ_p divided by m, the number of phases of the winding

$$\tau_z = \frac{\tau_p}{m} = \frac{\pi D_i}{2pm} \tag{2.2}$$

The zones are named sequentially and symmetrically for the phases. Every phase has both a positive and a negative zone. The angle between the positive zones of the phases is the angle between the excitation axes of the phase windings

$$\alpha_z = \frac{360^e}{m} \tag{2.3}$$

The number of slots in a zone is q, the number of slots per pole and per phase

$$q = \frac{Q}{2pm} \tag{2.4}$$

where Q is the number of slots in the winding.

If the number q is an integer number, the winding is an integral slot winding. Every zone of the winding includes the same number of slots, equal to q.

If the number q is a fractional number, the winding is a fractional slot winding. The number of slots per pole and per phase, q, gives the average number of slots in a zone. The winding includes two kinds of zones:

- zones having the number of slots equal to the integer number nearest higher than q, and
- zones having the number of slots equal to the integer number nearest smaller than q.

The number q for any fractional slot winding may be written as

$$q = \frac{Q}{2pm} = \frac{\mathrm{z}}{\mathrm{n}} \tag{2.5}$$

where both z and n are integers and they don't have any common factor.

The rules to place the coils of a symmetric slot winding into the slots:

- Every phase winding has to create into the air gap a flux density distribution having a sinusoidal fundamental wave with a wave length equal to the length of two pole pitches.
- All the phase windings have to generate in the air gap flux-density distributions being identical in shape.
- The flux-density distributions generated by the phase windings must deviate by the same phase-shift, symmetrically to each other along the air-gap periphery.

2.2 Phasor representation

In the basic study of windings, the radial flux-density spatial-distribution in the air gap is developed to harmonic series by Fourier analysis. The fundamental spatial-harmonic b_p of the radial flux-density distribution is the harmonic of order p having the wave length equal to the length of two pole pitches, peripheral air-gap length of one pole pair. The fundamental spatial-harmonic is not always the lowest order spatial-harmonic of the Fourier analysis.

The fundamental harmonic b_p rotates along the air-gap periphery by angular velocity ω_p caused by the rotation of rotor (or rotation of the excitation field). The rotating fundamental wave b_p is a time and space function of the form

$$b_p(t,\gamma) = \hat{b}_p \sin(\gamma - \omega_p t) \tag{2.6}$$

where

$\hat{b}_p$	is the peak value of the rotating flux-density distribution,
t	the time,
γ	the spatial co-ordinate of pole pair ($\gamma = 180^{\mathrm{e}}$ means one pole pitch), and
ω_p	the angular velocity of the rotating harmonic.

The rotating fundamental harmonic b_p induces an electromotive force (EMF) to a conductor placed in a slot. For the following study, the conductor may be replaced by a coil wound through the slot and around the yoke, a slot coil. The flux linkage ψ_{b} of this coil doesn't depend on the form of the coil outside the slot. The EMF induced in the coil, e_{b}, is defined by the spatial co-ordinate of the slot γ_{b} and the rotating fundamental flux-density harmonic b_p

$$e_{\mathrm{b}}(t) = -\frac{\mathrm{d}\psi_{\mathrm{b}}}{\mathrm{d}t} = \frac{\omega_p}{2} N_{\mathrm{coil}}\, \tau_{\mathrm{p}} l_{\mathrm{i}}\, \hat{b}_p \cos(\gamma_{\mathrm{b}} - \omega_p t) \tag{2.7a}$$

where

N_{coil}	is the number of turns in the coil and
l_{i}	the axial length of the air gap.

Divisor "2" in Eq. (2.7a) bases on the fact that only half of the flux, equivalent to the flux-density harmonic, flows through the slot coil.

In the same way, a rotating flux-density spatial-distribution harmonic b_ν of the whole air gap periphery induces an EMF $e_{\mathrm{b}\nu}$ in the slot coil. The spatial co-ordinate of the slot is γ_{b}, in electrical degrees. The induced EMF $e_{\mathrm{b}\nu}$ is

$$e_{\mathrm{b}\nu}(t) = -\frac{\mathrm{d}\psi_{\mathrm{b}\nu}}{\mathrm{d}t} = \frac{\omega_\nu}{2} N_{\mathrm{coil}}\, \tau_{\mathrm{p}} l_{\mathrm{i}}\, \hat{b}_\nu \cos\left(\frac{\nu\gamma_{\mathrm{b}}}{p} - \omega_\nu t\right) \tag{2.7b}$$

The fundamental EMF e_{b} can be presented as a phasor in the complex plane

$$\vec{e}_{\mathrm{b}} = \frac{\omega_p}{2} N_{\mathrm{coil}}\, \tau_{\mathrm{p}} l_{\mathrm{i}}\, \hat{b}_p e^{j(\gamma_{\mathrm{b}} - \omega_p t)} = \hat{e}_{\mathrm{b}} e^{j(\gamma_{\mathrm{b}} - \omega_p t)} \tag{2.8}$$

A real coil consists of coil-sides in two slots, e.g. slots b_1 and b_2, and of a connection of the coil-sides (end-winding) outside the slots. The spatial co-ordinates of the slots are $\gamma_{\mathrm{b}1}$ and $\gamma_{\mathrm{b}2}$. The width of the coil, the coil pitch, is $\gamma_{\mathrm{b}1} - \gamma_{\mathrm{b}2}$. The EMF of the real coil, e_{coil}, is the sum EMF of the two coil-sides where the EMF of the right coil-side has to be added in the opposite direction

$$\vec{e}_{\mathrm{coil}} = \vec{e}_{\mathrm{b}1} - \vec{e}_{\mathrm{b}2} = \hat{e}_{\mathrm{b}1} e^{j(\gamma_{\mathrm{b}1} - \omega_p t)} - \hat{e}_{\mathrm{b}2} e^{j(\gamma_{\mathrm{b}2} - \omega_p t)} \tag{2.9}$$

If the slots b1 and b2 deviate from each other by one pole pitch, a full-pitch coil, the EMF of the coil is

$$\vec{e}_{\mathrm{coil}} = \vec{e}_{\mathrm{b}1} - \vec{e}_{\mathrm{b}2} = 2\hat{e}_{\mathrm{b}} e^{j(\gamma_{\mathrm{b}1} - \omega t)} \tag{2.10}$$

where

$$\hat{e}_{\mathrm{b}} = \frac{1}{2} \omega_p \mathrm{N}_{\mathrm{coil}}\, \tau_{\mathrm{p}} l_{\mathrm{i}}\, \hat{b}_p \tag{2.11}$$

2.3 Basic winding

A rotating flux-density spatial-distribution harmonic, sinusoidal in space, induces in the coil-sides an alternating voltage, sinusoidal in time. The EMF phasors of the coil-sides form a phasor star. The angle α_{n} between the EMF phasors of neighbouring coil-sides is, in electrical degrees,

$$\alpha_n = 2\pi \frac{p}{Q} \tag{2.12}$$

If the number of slots Q is divisible by the number of pole pairs, p, the phasor star of every pole pair is identical. This means that the winding has a basic winding of one pole pair. The phasors of this kind of winding are presented in Fig. 2.1. The phasors are collected as groups of the pole pair. The groups are presented as rings of phasor stars inside each other, Fig. 2.1 b. The number of phasors in one group is

$$Q^* = \frac{Q}{p} \tag{2.13}$$

The angle between the neighbouring phasors α_z in the phasor star and the angle α_n between the neighbouring coil-sides are identical.

If the number of slots Q^* is divisible by 2, the basic winding is an integral slot winding. Otherwise, the basic winding is a fractional slot winding.

The basic winding is the smallest independent and symmetric part of a winding. The complete winding is a collection of basic windings where the basic windings are connected either all in parallel or all in series or partly in parallel and partly in series. The properties of the basic winding are identical to the properties of the whole winding.

If the number of slots Q is not divisible by the number of pole pairs, p, the phasor stars of the neighbouring pole pairs are not identical. The full phasor star of the basic winding needs more phasors than only the phasors of one pole pair. The full phasor star needs phasors of p^* pole pairs before the phasor star starts to repeat itself, Fig. 2.2. The phasors of one full phasor star forms a group of phasors of p^* pole pairs. The number of phasors in this group is

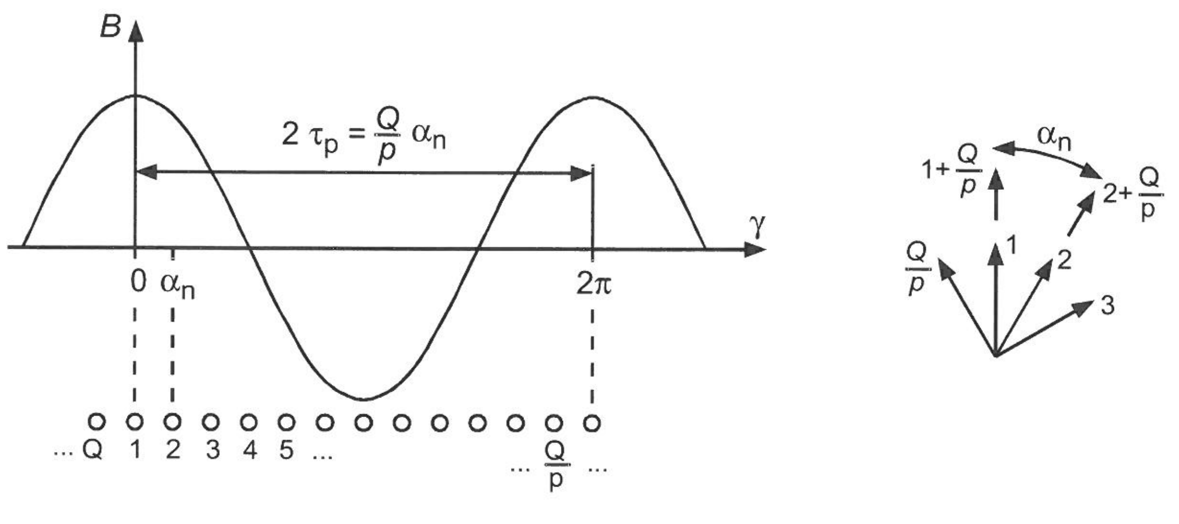


Fig. 2.1 The number of slots Q is divisible by the number of pole pairs p.
a) The slots and the air-gap flux-density distribution along the air-gap periphery, and
b) the phasor star equivalent to the slotting and the repetition of the phasor star.

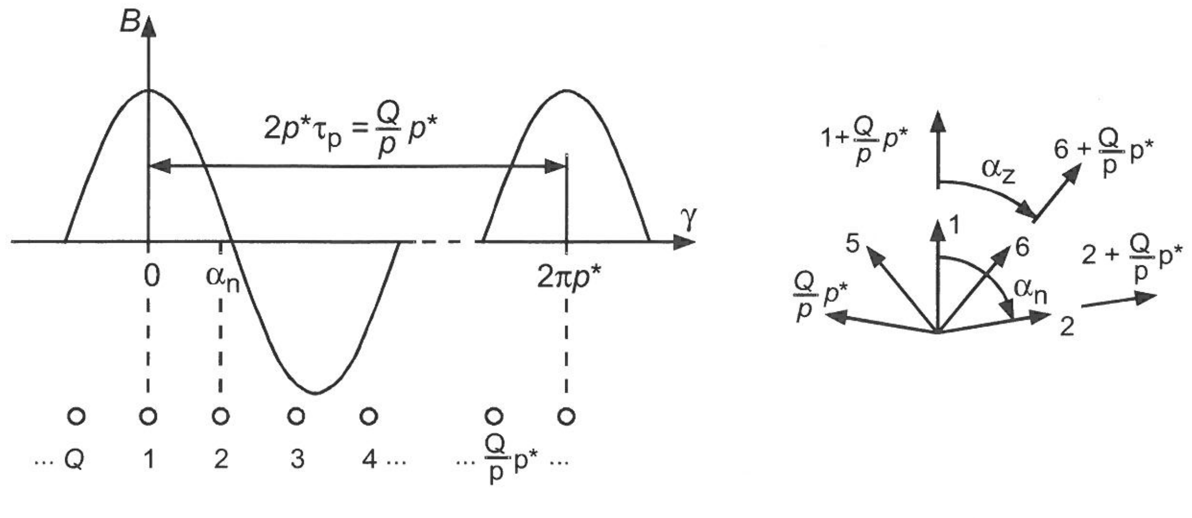


Fig. 2.2 The number of slots Q is not divisible by the number of pole pairs p.
a) The slots and the air-gap flux-density distribution along the air-gap periphery, and
b) the phasor star equivalent to the slotting and the repetition of the phasor star.

$$Q^* = p^* \frac{Q}{p} \tag{2.14}$$

This phasor group defines a basic winding having Q^* slots and p^* pole pairs. The basic winding is a fractional slot winding. The number of basic windings in the whole winding is

$$t = \frac{p}{p^*} \tag{2.15}$$

The angle between the neighbouring phasors α_z in the phasor star is a portion of the angle α_n between the neighbouring coil-sides

$$\begin{cases} \alpha_n = \dfrac{p}{t}\alpha_z = p^*\alpha_z \\ \alpha_z = 2\pi \dfrac{t}{Q} \end{cases} \tag{2.16}$$

For the harmonic ν of the flux-density distribution the phasor star is constructed like for the fundamental harmonic. The only difference is the angle between the neighbouring coil-sides $\alpha_{n\nu}$ and the angle $\alpha_{z\nu}$ between the phasors

$$\begin{cases} \alpha_{n\nu} = \dfrac{\nu}{p}\alpha_n \\ \alpha_{z\nu} = \dfrac{\nu}{p}\alpha_z \end{cases} \tag{2.17}$$

2.4 Rules for symmetry

A multiphase winding is symmetrical if

- the winding, by a multiphase excitation, creates in the air gap a rotating fundamental harmonic of the flux-density distribution, or
- the rotating fundamental harmonic of the flux-density distribution in the air gap induces a symmetric multiphase voltage into the winding.

First rule of symmetry:

The number of coils in every phase is equal and in overall the number is an integer number.

- For a one-layer winding

$$\frac{Q}{2m} = pq = \text{integer} \tag{2.18a}$$

- For a two-layer winding

$$\frac{Q}{m} = 2pq = \text{integer} \tag{2.18b}$$

Second rule of symmetry:

The magnetic axes of the phase windings must deviate by the same angle to each other.

This means that the angle between the phases is divisible by the angle α_z between the neighbouring phasors in the phasor star

$$\frac{\alpha_v}{\alpha_z} = \frac{Q}{m\,\mathrm{t}} = \text{integer} \tag{2.19}$$

The integer slot winding is always symmetrical. The first rule of symmetry is fulfilled because both the number of pole pairs p and the number of slots per pole and per phase q are integer numbers. The second rule of symmetry is fulfilled because the number of pole pairs of the basic winding p^* is one ($p^* = 1$ equals to $\mathrm{t} = p$).

2.5 Basic fractional-slot winding

The number of slots per pole and per phase, q, for a fractional-slot basic-winding is

$$q = \frac{Q^*}{2p^*m} = \frac{\mathrm{z}}{\mathrm{n}} \tag{2.20}$$

where Q^* is the number of slots and p^* the pole-pair number of the basic winding.

The first rule of symmetry for a one-layer winding gets the form

$$\frac{Q^*}{2m} = p^*\frac{\mathrm{z}}{\mathrm{n}} = \text{integer} \quad\Rightarrow\quad \frac{p^*}{\mathrm{n}} = \text{integer} \tag{2.21a}$$

This means that a fractional-slot one-layer winding is not possible with $p = 1$.

The first rule of symmetry for a two-layer winding gets the form

$$\frac{Q^*}{m} = 2p^*\frac{\mathrm{z}}{\mathrm{n}} = \text{integer} \quad\Rightarrow\quad \frac{2p^*}{\mathrm{n}} = \text{integer} \tag{2.21b}$$

This means that the only fractional-slot two-layer winding for $p = 1$ is with $\mathrm{n} = 2$.

The second rule of symmetry for a fractional slot winding gets the form

$$\frac{\alpha_v}{\alpha_z} = \frac{Q}{m\,\mathrm{t}} = \frac{2pqm}{m\,\mathrm{t}} = \frac{2\mathrm{z}\frac{p}{\mathrm{n}}}{\mathrm{t}} = \text{integer} \tag{2.22}$$

where t is the largest common factor of the slot number Q and the pole-pair number p.

The rule is fulfilled only by

$$\mathrm{t} = \mathrm{k}\frac{p}{\mathrm{n}} = \text{integer}, \quad \text{where k} = 1 \text{ or } 2 \tag{2.23}$$

There are two types of fractional-slot basic-windings according to the number of slots Q^*:

Ratio Q^* / m is even:

The winding with Q^*-slots may be constructed either as a one-layer or as a two-layer winding. The winding is a first order basic-winding.

The definitions for the first order winding are

$$\begin{cases} \dfrac{Q}{m\mathrm{t}} = \text{even} \\ \mathrm{n} = \text{odd} \\ \mathrm{t} = \dfrac{p}{\mathrm{n}} \\ \alpha_{\mathrm{n}} = \mathrm{n}\alpha_z \end{cases} \qquad \begin{cases} Q^* = \dfrac{Q}{\mathrm{t}} \\ p^* = \dfrac{p}{\mathrm{t}} = \mathrm{n} \\ \mathrm{t}^* = 1 \end{cases} \tag{2.24}$$

Ratio Q^* / m = odd:

The winding of Q^*-slots may be constructed only as a two-layer winding. The one-layer winding needs an even number of slots. The one-layer winding has to have two basic windings, together $2Q^*$ slots. Both the one-layer $2Q^*$-slot winding and the two-layer Q^*-slot winding are second order basic-windings.

The definitions for the second order winding are

$$\begin{cases} \dfrac{Q}{mt} = \text{odd} \\ \text{n} = \text{even} \\ \text{t} = 2\dfrac{p}{\text{n}} \\ \alpha_\text{n} = \dfrac{\text{n}}{2}\alpha_\text{z} \end{cases} \qquad \text{one-layer winding} \begin{cases} Q^* = 2\dfrac{Q}{\text{t}} \\ p^* = 2\dfrac{p}{\text{t}} = \text{n} \\ \text{t}^* = 2 \end{cases} \qquad \text{two-layer winding} \begin{cases} Q^* = \dfrac{Q}{\text{t}} \\ p^* = \dfrac{p}{\text{t}} = \dfrac{\text{n}}{2} \\ \text{t}^* = 1 \end{cases} \tag{2.25}$$

2.6 One-layer winding

A one-layer winding has one coil side in each slot. The number of coils in the winding is half of the number of slots Q. The following instructions base on a three phase winding, but the instructions may also be solved for any phase number m.

An example of a zone distribution for a two pole, three phase winding is presented in Fig. 2.3. The zones for the phases are named according to the following instructions:

1 Choose the starting point - one slot. This is the first slot belonging to a positive zone "**+A**" for the phase A. The width of the zone is 60^e (= $180^\text{e} / m$).

2 Move 120^e (= $360^\text{e} / m$) along the air-gap periphery, e.g. to the rotational direction of the flux density distribution, clockwise in Fig. 2.3. The zone starting from this position is

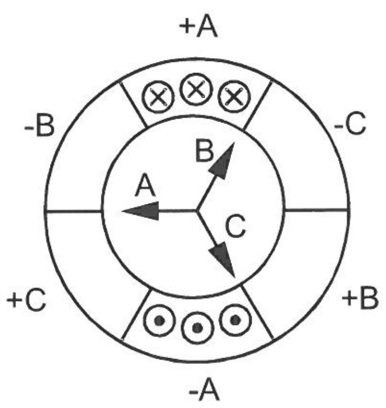


Fig. 2.3 The air-gap periphery is divided into zones of the one-layer winding. The magnetic axes of the phases are marked by arrows. The rotation of phases is clockwise.

the positive zone "**+B**" for the phase B. The width of the zone is 60^e. Between these two named zones there is an unnamed zone.

3 Continue to the same direction by moving 120^e. This is the positive zone "**+C**" for the phase C. The width of the zone is 60^e.

These three named zones are the positive zones of the first pole pair. If there are more pole pairs the naming of "**+A**", "**+B**" and "**+C**" zones has to be continued. The moving by 120^e has to be continued until the next target zone is the first named zone.

4 When all the positive zones are named, the unnamed zones left shall be named as negative zones. The naming starts from the first positive "**+A**" zone. Moving 180^e along the air-gap periphery gives the first negative zone. It is the negative zone "**-A**" for the phase A. The width of the zone is 60^e.

5 Continue to the same direction by moving 120^e. This is the negative zone "**-B**" for the phase B. The width of the zone is 60^e.

6 Continue to the same direction by moving 120^e. This is the negative zone "**-C**" for the phase C. The width of the zone is 60^e.

These three named zones are the negative zones of the first pole pair. If there are more pole pairs the naming of zones "**-A**", "**-B**" and "**-C**" has to be continued. The moving by 120^e has to be continued until all the unnamed zones are named.

Two alternative one-layer windings are presented in Fig. 2.4 for a winding with 18 slots and two poles. Only one phase of the windings is drawn. The number of slots per pole and per phase in the windings is an integer number, q = 3. The pole pitch of the winding is $\tau_\text{p} = mq = 9$. The alternative windings present the two ways to build coils for a one-layer integral-slot winding:

a) All the coils have the same coil pitch, Fig. 2.4 b.

The winding design is started from the leftmost slot of the positive zone "+A", phase A. The first slot is "0" in Fig 2.4 b. The left side of the first coil will be set into the slot "0". The right side of the first coil should be set into a slot of a distance of one pole pitch τ_p from the slot "0" to the rotational direction of the flux density distribution, into the slot "9". Slot "9" is the leftmost slot of the negative zone "–A" of phase A.

The left side of the following coil for the phase A should be set into the slot "1" in the positive zone "+A". Because the coil pitch of all the coils is the same, the right coil-side comes to the slot "10", a distance of one pole pitch τ_p from the slot "1". And so on.

The quantity of the coils in phase A is q. All the left coil-sides of these coils will be set into the slots of the positive zone "+A". All the right coil-sides will be set into the slots of the negative zone "–A".

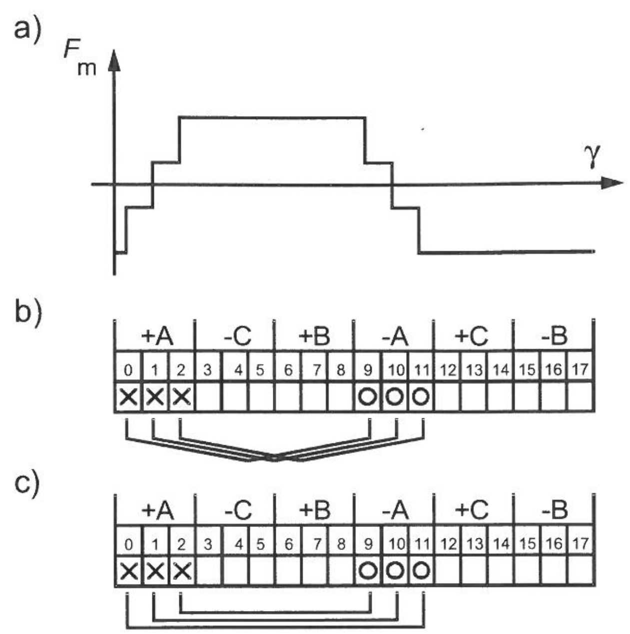


Fig. 2.4 The layout of one-layer winding with $q = 3$.
a) Distribution of MMF F_m of phase A,
b) End-winding made as diamond winding, and
c) End-winding made as concentric winding.

The coils are crossing each other in the end-windings, both sides of the core. The end-winding forms a diamond image, so the winding is called a diamond winding.

Previously in this example, the connection of the coils in a coil group has not been specified. There are two basic ways to connect a coil to another. All the coils in a coil group shall be connected in series, generally.

In the first way, the neighbouring coils are connected in series. The first coil, from slot "0" to slot "9", is connected to the next one, from slot "1" to slot "10", by connecting the right side of the first coil in slot "9" to the left side of the neighbouring coil in slot "1". The connection between the coils goes to opposite direction, to the left, then the winding direction. The coils are forming closed circles or laps partly inside each other, so the winding is called a lap winding.

In the second way, the coils of all the zone pairs are connected in series. The first coil, from slot "0" to slot "9", is connected to the next one, from slot "1" to slot "10", by connecting the right side of the first coil in slot "9" to the left side of the neighbouring coil in slot "1". The difference is now that the direction of this coil connection is along the winding direction, to the right. The coils follow each other by rounding the whole slotting periphery. The end-windings of both ends form continuous waves round the winding, so the winding is called a wave winding.

b) The coils have different coil pitch, Fig. 2.4 c.

The winding design is started with the coil of shortest coil pitch from the rightmost slot of the positive zone "+A", phase A. This is the slot "2" in Fig 4c. The left side of the

first coil will be set into the slot "2". The right side of the first coil should be set, to the rotational direction of the flux density distribution, into the leftmost slot of the negative zone "–A" of phase A, into the slot "9".

The left side of the following coil for the phase A should be set into the slot "1" in the positive zone "+A". The coil pitch of this coil is two slot-pitches wider than the width of the first coil. The right coil-side comes to the next free slot of the negative zone "-A", into the slot "10".

Correspondingly, all the q coils in the phase A will be set every time into the slots outside the previous coil. All the left coil-sides will be set into the slots of the positive zone "+A" and the right coil-sides into the slots of the negative zone "–A".

Because the coils are around each other, the winding is called a concentric winding. There is also another way to name the winding: the end-winding forms a level image. When the coils of the other phases B and C should be set into the slots of their positive and negative zones, there comes a problem. The end-winding levels of the phases are the same and the coils push each other. The solution is to force the levels of the end-winding of phases A, B or C to deviate from the horizontal level. In this example, the only solution for the end-winding region is that the levels of both the B and C phases will be changed. The end-winding should have three levels, so the winding is called a 3-level concentric winding. Other solutions could be that the end-winding could have one level, a 1-level concentric winding, or two levels, a 2-level concentric winding.

The coil of largest coil pitch in Fig. 2.4 c has its left side in slot 0 and the right side in slot 11. The coil pitch is 11 slot pitches, over one pole pitch. The coil pitch of the two other coils is 9 and 7 slot pitches. The average value of the coil pitch for the coil group is 9 slot pitches equal to the pole pitch.

For the phases B and C, the placing of the coils bases on the symmetry of the winding. For an integral slot winding, the placing order is exactly the same as for the phase A starting from any positive zone of the target phase. For a fractional slot winding, the placing is more restricted and the starting zone depends on the symmetry of the whole winding. Sometimes the best choice for the starting zone of phases B and C is the zone "+B" in the place of 120° (mechanical degrees) and the zone "+C" in the place of 240° to the rotational direction of the flux density distribution from the starting zone of phase A.

The quantity of the coil sides in a zone of a phase is q. Generally, the winding is designed for equal number of coil sides for the positive and for the negative zone. Then the winding will be formed by full coils. The coils of a phase and of a pole form a coil group. The coils in a coil group are generally connected in series, but the coil groups of a phase winding may be connected either in parallel or in series. In an integral slot winding, the coil group equals to the phase winding of basic winding: the coil groups of a phase could be connected either all in parallel or all in series or partly in parallel and partly in series. The rule for the parallel connection is that the EMFs of the coils for parallel connection have equal amplitude and equal phase-shift.

In all the coils of a slot winding, the number of effective turns is equal. The parallel and series connections between the coils shall not affect to the flux-density distribution in the air gap.

All the connections are made outside the slots, in the end-winding region. This affects to the place demands of the coil ends, the amount of coil material, manufacturing costs of winding, and to the electrical properties, the resistance and leakage reactance of the winding.

The MMF of the windings will be defined by a fictive current flowing in a phase. A time instant will be chosen when the phase A has its highest value, the peak value "1". The other phases B and C have the value "-0.5" to fulfil the symmetry of the phase currents. When the fictive current "1" flows in a slot of the positive zone "+A" in Figs. 2.4 b and 2.4 c, the effect of the current to the MMF distribution is an increment of "1". When the same current flows in a slot of the negative zone "-A", the effect of the current to the MMF distribution is a decrement of "1". In the positive zone of phase A, all the slots have a positive increment of MMF, a positive fictive current. In the negative zone of phase A, all the slots have a negative increment of MMF, a negative fictive current. Both of the windings, Figs. 2.4 b and 2.4 c, give the same MMF distribution, Fig. 2.4 a.

2.7 Two-layer winding

A two-layer winding has two coil sides in each slot in two layers. The coil side on the bottom of the slot belongs to the inner (lower) layer and the coil side closer to the air gap to the outer (upper) layer. The number of coils in the winding is the number of slots Q of the winding. Half of the coils are on the positive zones and half on the negative zones. This gives the name "divided coil groups of pole pair" for the coil groups of the two-layer winding.

A two-layer winding is constructed like a one-layer winding by dividing the air-gap periphery to zones of phases, in Fig. 2.5. The zones for the two-layer winding will be formed as for the one-layer winding. The only difference is that the ring of zones should be divided into two parts, the outer ring of zones and the inner ring of zones. The MMF distribution of a one-layer integral-slot winding and a two-layer integral-slot winding with the same value of q is the same. If the number of effective turns in the coil of the one-layer winding is N_{slot}, in the two-layer winding the number of effective turns in a coil is $N_{slot}/2$.

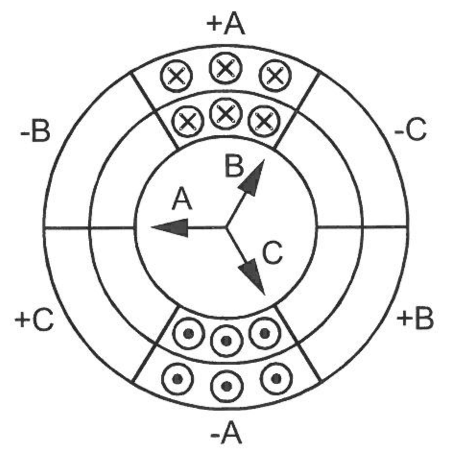


Fig. 2.5 The air-gap periphery is divided into zone rings of the two-layer winding. The magnetic axes of the phases are marked by arrows. The rotation of phases is clockwise.

The winding design starts from the placement of the coils into the slots like in the one-layer winding. The first coil has its right side on the lower layer of the negative zone "-A", phase A. The left side of the first coil should be set, against the rotational direction of the flux density distribution, into a slot of the positive zone "+A" of phase A. There are two alternatives of placing the coil-side: either in the upper layer or in the lower layer. Both the alternatives give the same MMF distribution for the coil in the air gap. The placement of the coils defines the manufacturing of the coils: the coil pitch and the form of coils.

Three alternative coil placements for the phase A are presented for a winding with 18 slots and two poles in Fig. 2.6. The number of slots per pole and per phase in the windings is an integer number, q = 3. The pole pitch of the winding is $\tau_p = mq = 9$. The chosen coil pitch is $W = 9$.

The first alternative is the basic two-layer winding where the right coil-side is in the lower layer and the left coil-side in the upper layer, Fig. 2.6 b. In the second and third alternative coil placements, the left and right coil-sides of a coil are on the same layer, Figs. 6 c and 6 d. These alternatives may be used for round wire winding wound by automatic winding tools. All the three alternatives give the same MMF distribution in the air gap.

a) Basic two-layer winding, Fig. 2.6 b.

The leftmost slot of the positive zone "+A" of upper layer, phase A, is the first slot "0". The first coil has its left side in the upper layer in slot "0" and its right side in the lower

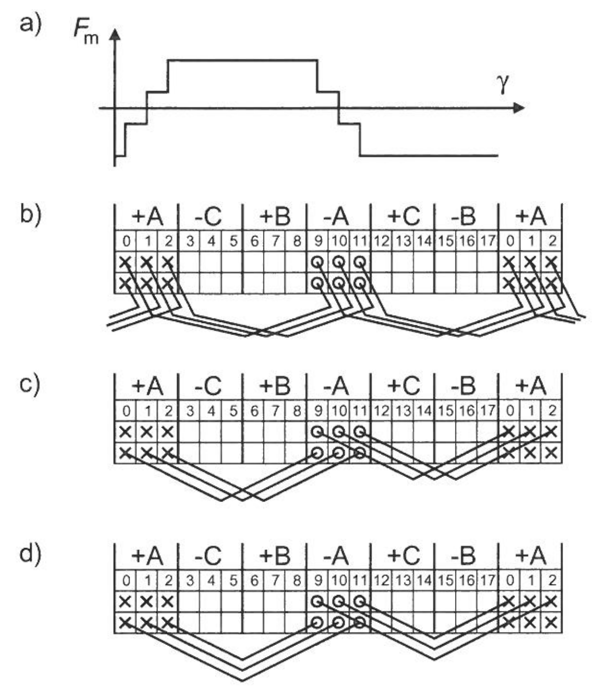


Fig. 2.6 Two-layer windings having the same distribution of MMF F_m.
a) Distribution of MMF of phase A
b) The basic two-layer winding, diamond winding
c) Coil sides on the same layer, coils having the same coil pitch, diamond winding
d) Coil sides on the same layer, coils having different coil pitch, concentric winding

layer of slot "9", the leftmost slot of the negative zone "-A" of lower layer. The coil pitch is $W = 9$.

The following coil comes into the slots to the right side of the first coil. The quantity of the coils in the positive zone "+A" in phase A is q. All the left coil-sides of these coils will be set into the slots of the positive zone "+A" and the right coil-sides into the slots of the negative zone "–A".

In addition, the other q coils of phase A are still unused. These should be set starting from the first slot "9" of the negative zone "-A". The left side of the first "negative" coil will be set in the upper layer in slot "9" and its right side in the lower layer of slot "18". Because in this example there are only 18 slots, the right side comes in the lower layer of slot "0". The coil direction goes to the right, to the rotational direction of the flux density distribution.

In practice, this alternative of winding construction needs to be done continuously, in contrary to the previously presented one-layer windings. In one-layer winding, it is possible that, at first, the coils of phase A are set into the slots, then the coils of phase B or C, and, at last, the coils of the last phase. In the basic two-layer winding, this is not possible. The coils have to have the correct order inside the slot, the correct layer-order.

The winding construction starts from the leftmost slot "0" of the positive zone "+A", phase A. The left side of the first coil (of phase A) shall come there, on the top of the slot, but now it will be left free, outside the target slot. The right side of the coil will be set on the bottom of the slot "9". The next stage is to leave the left side of the second coil outside the target slot "1". The right side of the coil will be set on the bottom of the slot "10". The same happens for all the q coils of the phase A.

The winding construction is come to the negative zone of phase C, "-C". The winding construction continues from the leftmost slot "3" of the negative zone "-C". The left side of the first coil (of phase C) will be left free outside the slot. The right side of the coil will be set on the bottom of the slot "12". By this way whole the periphery of the slotting is filled by one coil-side in a slot.

The next stage is to start the upper layer construction. It starts from the leftmost slot "0" of the positive zone "+A" on the upper layer, phase A. The left side of the first coil (of phase A) comes there, on the top of the slot. The left side of the second coil (of phase A) will be set into the slot "10", on the top of the slot. And so on. By this way the whole periphery of the slotting is filled by two coil-sides in a slot.

The coils are crossing each other in the end-windings, both sides of the core. The end-winding forms a diamond image, so the winding is called a diamond winding.

b) Coil sides on the same layer. All the coils have the same coil pitch, Fig. 2.6 c.

This method is suitable mainly for an integer slot winding. All the coils for lower layer are set in the bottom of the slot. Then the coils for the upper layer are set on the top of the slot.

The winding design is started from the leftmost slot of the positive zone "+A" of lower layer, phase A. The first slot is "0". The left side of the first coil will be set on the bottom of the slot "0". The right side should be set now on the bottom of the slot "9", one pole pitch τ_p from the slot "0" to the rotational direction of the flux density distribution. Slot "9" is the leftmost slot of the negative zone "–A" of lower layer, phase A. The following coils for the phase A should be set into the slots "1" - "10" and "2" - "11".

The construction of phase B starts from the leftmost slot of the zone "+B", the slot "6". The coils of phase B are set on the bottom of the slots "6" - "15", "7" - "16", and "8" - "17". The construction of phase C starts from the leftmost slot of the zone "+C", the slot "12". The coils of phase C are set in the slots "12" - "3", "13" - "4", and "14" - "5". The lower layer is ready.

The next stage is to start the upper layer construction. The design is started from the leftmost slot "W" of the negative zone "-A" on the upper layer, phase A. The chosen coil pitch is $W = 9$, so the left side of the first coil will be set on the top of the slot "9". The right side should be set on the top of the slot "0", one pole pitch τ_p from the slot "9" to the rotational direction of the flux density distribution. The following coils for the phase A should be set to the slots "10" - "1" and "11" - "2".

The construction of phase B starts from the slot "15", the leftmost slot of the zone "-B". The coils of phase B come to the slots "15" - "6", "16" - "7", and "17" - "8". The construction of phase C starts from the leftmost slot of the zone "-C", the slot "3". The coils of phase C come to the slots "3" - "12", "4" - "13", and "5" - "14". The upper layer is ready.

The winding is called a diamond winding.

c) Coil sides on the same layer. The coils have different coil pitch, Fig. 2.6 d

This method is suitable mainly for an integer slot winding. All the coils for lower layer are set in the bottom of the slots. Then the coils for the upper layer are set on the top of the slots.

The winding design is started from zone "+A" of the lower layer, phase A, with a coil having the smallest coil pitch. The left side of the first coil will be set on the bottom of the slot "2", the rightmost slot of the zone. The right side should be set on the bottom of the slot "9", the leftmost slot of the zone "–A" of the lower layer, phase A. The following coils for the phase A should be set into the slots "1" - "10" and "0" - "11", on the bottom of the slots.

The construction of phase B starts from the slot "8", the rightmost slot of the zone "+B". The coils of phase B come to the slots "8" - "15", "7" - "16", and "6" - "17". The construction of phase C starts from the slot "14", the rightmost slot of the zone "+C". The coils of phase C come to the slots "14" - "3", "13" - "4", and "12" - "5". The lower layer is ready.

The next stage is the construction of the upper layer. The design starts from the rightmost slot of the zone "-A" of phase A. The starting slot depends on the phase-shift between the negative zone "-A" and the positive zone "+A" of the layer. The phase-shift is the chosen coil pitch W. The chosen coil pitch is $W = 9$, so the leftmost slot of the zone "-A" is slot "9" and the rightmost slot "11". The coils of phase A, in slots "11" - "0", "10" - "1", and "9" - "2", are set on the top of the slots.

The construction of phase B starts from the rightmost slot of the zone "-B", the slot "17". The coils of phase B come into the slots "17" - "6", "16" - "7", and "15" - "8" on the top of the slots. The construction of phase C starts from the rightmost slot of the zone "-C", the slot "5". The coils of phase C come to the slots "5" - "12", "4" - "13", and "3" - "14" on the top of the slots. The coils of phase "C" fulfil the upper layer.

The winding is called a concentric winding.

The quantity of the coil-sides in a zone of a phase is q. The coils of a phase and of a pole form a coil group. For every pole pair there is two coil groups for a phase. The coils in a coil group are generally connected in series, but the coil groups of a phase winding may be connected either in parallel or in series. In an integral slot winding the coil group equals to the phase winding of the basic winding: the coil groups of a phase could be connected all in parallel, all in series, or partly in parallel and partly in series. The rule for the parallel connection is that the EMFs of the coils for parallel connection have equal amplitude and equal phase-shift.

2.8 Chorded coil

In two-layer winding, the average coil pitch is adjustable. Generally, in the windings the coil pitch should be equal to the pole pitch or shorter than the pole pitch. The use of two zone-layers gives a possibility to rotate the zones of the outer layer, in Fig. 2.5 to counter-clockwise direction. The same time the coil pitch, W, comes smaller than the pole pitch. The shift between the inner and outer layers, the chord of coil, will be defined as slot pitches

$$\tau_n = \frac{\pi D_i}{Q} \tag{2.26}$$

The effective value of the flux through a chorded coil is smaller than through a pole-pitch coil. That means a smaller value of the induced voltage. The reason for the chording is to regulate the amplitude or the harmonic contents of the induced voltage of the winding. Especially, the chorded coils are used in synchronous generators where the chording is used to improve the quality of the voltage wave form. For the delta-connected windings, the coils are chorded to remove the induced third harmonic.

The effect of chorded coils is presented in Fig. 2.7 where two chorded coils don't react to a certain harmonic in the air-gap flux-density distribution. The full-pitch coil reacts to all the harmonics of flux density, Fig. 2.7 a, and the 3rd and the 5th harmonic of the flux density induce a corresponding harmonic voltage into the coil. The 5th harmonic of flux density doesn't induce any 5th harmonic voltage into the chorded coil in Fig. 2.7 b. The 3rd harmonic of flux density doesn't induce any 3rd harmonic voltage into the chorded coil in Fig. 2.7 c.

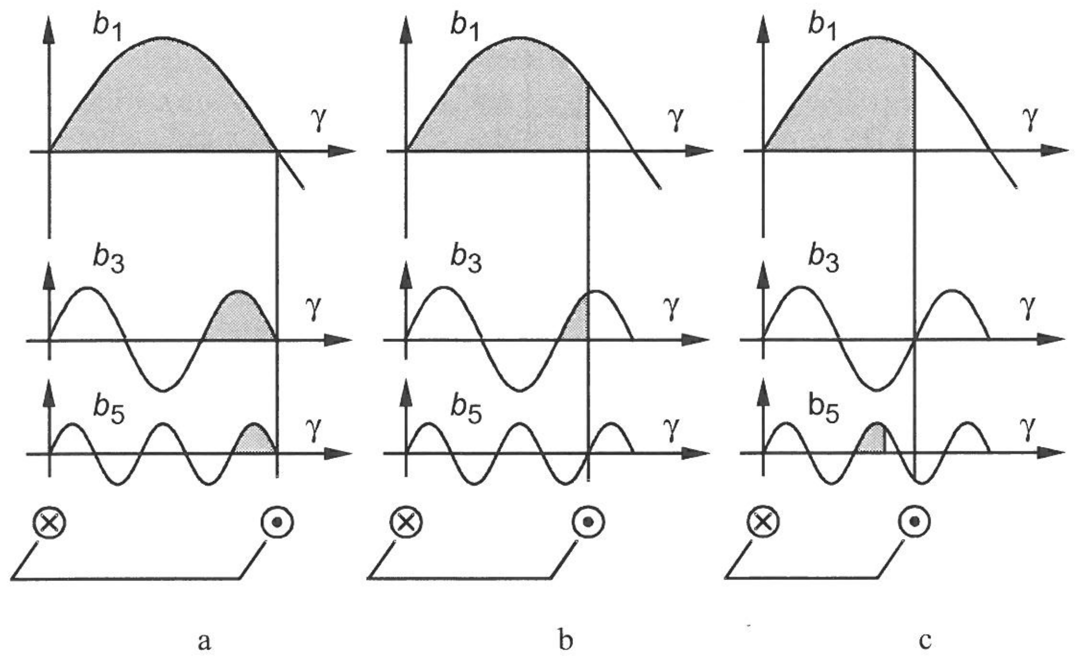


Fig. 2.7 Chorded coil to reduce the harmonics in induced voltage of a coil.
a) The basic full-pitch coil. The induced voltage includes 3rd and 5th harmonics.
b) No 5th harmonic in the induced voltage. The coil pitch is 4/5 of the pole pitch.
c) No 3rd harmonic in the induced voltage. The coil pitch is 2/3 of the pole pitch.

Some of the methods to perform the chording of the winding are presented for a 18 slot two-layer winding in Fig. 2.8:

- The starting position of the winding, Fig. 2.8 a. The winding is the basic two-layer winding with coil pitch equal to one pole pitch.

- The basic method of chording, the coil pitch is shortened, Fig 2.8 b. The width of all the coils is chorded by one slot pitch. In the winding schema, the change means a counter-clockwise rotation of the lower layer. The name and the width of the layer-zones stay in the rotation. In this example, the chording minimises the 7th harmonic in the voltage.

- A coil changes its placing zone, Fig. 2.8 c. A full-pitch coil of the basic two-layer winding is removed from its coil group and placed into another zone. The width of the coil doesn't change. The change of the placing zone has to be done for all the phases by equivalent coil displacements. In this example winding, the method prevents the 9th harmonic in the air-gap flux to induce any voltage component into the winding.

- A coil changes its placing layer, Fig. 2.8 d. This method gives the same "chording" as in the winding in Fig. 2.8 b. The method may be explained by "conductors". The lower layer conductors are shifted (rotated) to the left by one slot pitch. The leftmost conductor in the positive zone is removed from the lower layer and put into the upper layer in the same slot. The rightmost conductor in the negative zone is removed from the upper layer and put into the lower layer in the same slot. The placing slot is not changed, only

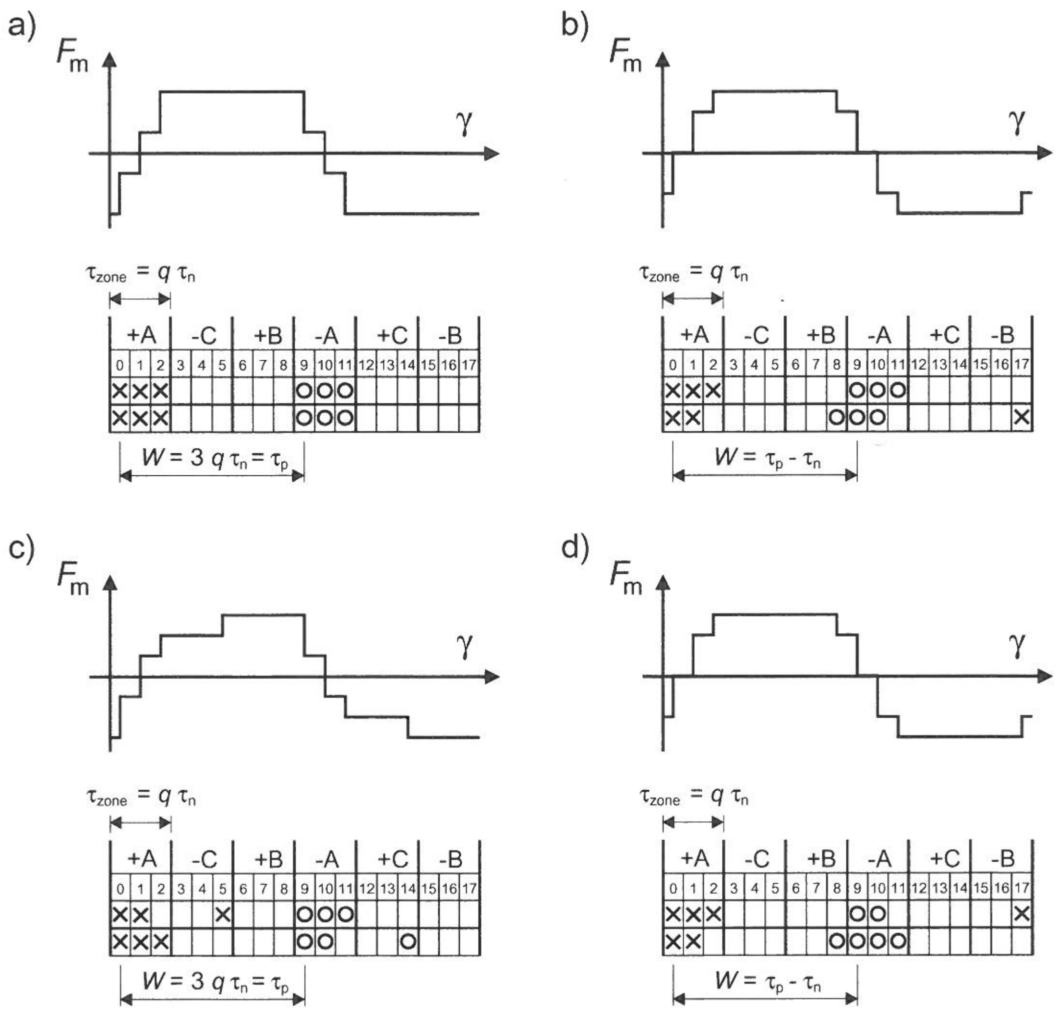


Fig. 2.8 Some examples to manage the chording of a two-layer winding.
a) The basic full-pitch two-layer winding.
b) Rotation of the lower layer. The coil pitch comes shorter than the pole pitch.
c) A coil changes its zone. The coil pitch is equal to the pole pitch.
d) A coil changes its layer. The coil pitch is equal to the pole pitch.

the placement of the conductors in the slot. The width of all the coils is still equal to the pole pitch.

2.9 Winding factor, integral slot winding

The EMF of a coil is the geometrical sum of the EMFs of the coil sides, see Eq. (2.9). For all the harmonics of the total induced EMF into the coil, the EMF may be written as a sum of all EMF harmonics of the positive and the negative coil side

$$\sum_{\nu=1}^{\infty} \vec{e}_{\mathrm{coil},\nu} = \sum_{\nu=1}^{\infty} \left(\vec{e}_{\mathrm{b1},\nu} - \vec{e}_{\mathrm{b2},\nu} \right) \tag{2.27}$$

In the design of winding, the calculation uses the arithmetic sum of the target EMF harmonic of a full-pitch coil, Eq. (2.10). The real width of the coil, the coil pitch, will be taken into account by a correction factor ξ_{coil}, the winding factor of the coil. For the ν^{th} EMF harmonic, the winding factor of the coil is the relation between the geometrical and arithmetical sums

$$\xi_{\mathrm{coil},\nu} = \sin\left(\nu \frac{\pi}{2}\right) \frac{\left|\vec{e}_{\mathrm{b1},\nu} - \vec{e}_{\mathrm{b2},\nu}\right|}{\left|\vec{e}_{\mathrm{b1},\nu}\right| + \left|\vec{e}_{\mathrm{b2},\nu}\right|} \tag{2.28}$$

The winding factor of the one-layer integral-slot winding may be defined by using only the EMF phasors in the basic winding: the EMF phasors of one phase and its positive zones. The number of coils in a zone is q. In the following, the coil placed in the slots b1 and b2 is the first coil of the positive and the negative zone, respectively, and the coil pitch is the pole pitch τ_{p}. The winding factor of the phase is

$$\xi_{\nu} = \sin\left(\nu \frac{\pi}{2}\right) \frac{\left|\sum_{i=0}^{q-1} \left(\vec{e}_{\mathrm{b1}+i,\nu} - \vec{e}_{\mathrm{b2}+i,\nu} \right)\right|}{\sum_{i=0}^{q-1} \left(\left|\vec{e}_{\mathrm{b1}+i,\nu}\right| + \left|\vec{e}_{\mathrm{b2}+i,\nu}\right| \right)} = \sin\left(\nu \frac{\pi}{2}\right) \frac{\left|\sum_{i=0}^{q-1} \vec{e}_{\mathrm{b1}+i,\nu}\right|}{\sum_{i=0}^{q-1} \left|\vec{e}_{\mathrm{b1}+i,\nu}\right|} \tag{2.29}$$

The phase shift of the phasors is the slot angle α_{n}, see Eq. (2.12)

$$\alpha_{\mathrm{n}} = 2\pi \frac{p}{Q} = \frac{\pi}{mq} \tag{2.30}$$

The winding factor can be written by a more convenient form

$$\xi_{\nu} = \sin\left(\nu \frac{\pi}{2}\right) \frac{\sin\left(q\nu \frac{\alpha_{\mathrm{n}}}{2}\right)}{q \sin\left(\nu \frac{\alpha_{\mathrm{n}}}{2}\right)} = \sin\left(\nu \frac{\pi}{2}\right) \frac{\sin\left(\nu \frac{\pi}{2m}\right)}{q \sin\left(\nu \frac{\pi}{2mq}\right)} \tag{2.31}$$

The winding factor of the two-layer integral-slot winding may be defined by using only the EMF phasors in the basic winding: the EMF phasors of one phase in the lower layer and its positive zones. Because the coil pitch W is free of choice, the displacement of the upper layer will be taken into account by a correction factor, the pitch factor. The winding factor of the lower layer has to be multiplied by the pitch factor

$$\xi_{\mathrm{chord},\nu} = \sin\left(\nu \frac{W}{\tau_{\mathrm{p}}} \frac{\pi}{2}\right) \tag{2.32}$$

Example: Define the coil placements for the 3-phase winding with 36 slots and 10 poles. The number of slots per pole and per phase of the winding is $q = 1.2 = 1\ 1/5$.

The average value of the number of coils in a coil group is 1 1/5. This means five coil groups (n = 5), all the pole pairs of the winding. The number of coils in the coil groups will be named as q_1, q_2, q_3, q_4, and q_5.

The coil groups are

- 1 coil group (z' = 1) having two coils $\quad q_1 = q_a = g + 1 = 2$,
  and
- 4 coil groups (n - z' = 4) having one coil $\quad q_2 = q_3 = q_4 = q_5 = q_b = g = 1$.

The average number of coils in a coil group is

$$q_{\text{ave}} = \frac{1}{5}\sum_{i=1}^{5} q_i = \frac{1}{5}\times[2+1+1+1+1] = 1\frac{1}{5} = q$$

The coil-group schema of the winding is presented as a basic construction in Fig. 2.10 a. The schema has three repetitions of the series of 5 coil groups. This schema is the basic construction tool for any one-layer fractional-slot winding having n = 5. As an example, a coil-group schema for a winding with $q = 4/5$ has been defined in Fig. 2.10 c. In the schema there are zeroes on the row of the q-values. This means that on those coil groups there are no coils at all. The zone of the phase is there but in the slot (if any) there is a coil of another phase.

The design of the winding with $q = 1\ 1/5$ continues in Fig. 2.11. There are presented different layout formats available to use in design of the one-layer winding. The coil-group schema defines the number of coils in the coil groups of the phase, Fig. 2.11 a.

a)

A	**B**	**C**	A	B	C	A	B	C	A	B	C	A	B	C
q_1	q_2	q_3	q_4	q_5	q_1	q_2	q_3	q_4	q_5	q_1	q_2	q_3	q_4	q_5

b)

A	**B**	**C**	A	B	C	A	B	C	A	B	C	A	B	C
2	**1**	**1**	**1**	**1**	2	1	1	1	1	2	1	1	1	1

c)

A	**B**	**C**	A	B	C	A	B	C	A	B	C	A	B	C
1	**1**	**1**	**1**	**0**	1	1	1	1	0	1	1	1	1	0

Fig. 2.10 Schema of coil groups in one-layer fractional slot winding.
a) Basic schema for all windings having n = 5.
b) The coil-group schema of the winding with $q = 1\ 1/5$.
c) The coil-group schema of the winding with $q = 4/5$.

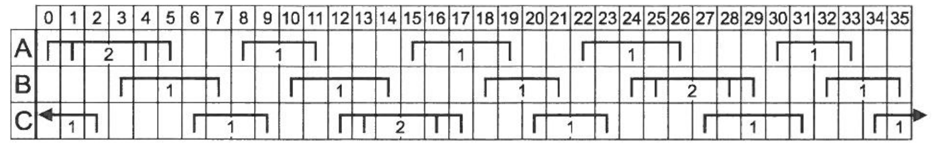
The coil group presentation by using the end-windings gives the information of the related slots, Fig. 2.11 b. It is possible to construct from this information a table where the coil-sides are placed on the rows of the phases and in the columns of the slots, Fig. 2.11 c. A cross-mark in Fig. 2.11 c means an increment to the MMF distribution. For the phase A, the increment is the fictive current "+1", for phases B and C "-0.5". A circle-mark means a decrement, for the phase A the fictive current "-1", for phases B and C "+0.5". The MMF distribution of the phase A is presented in Fig. 2.11 d.

The winding plan can be realised by two kinds of windings: a diamond winding or a concentric winding. The phase A from these windings is presented in Figs. 2.11 e and 2.11 f.

a)

phase	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
q	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1

b)



c)

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
A	X	X			O	O			X			O				X				O			X				O				X			O		
B				X				O			X				O				X			O			X	X			O	O			X			O
C			O				X			O			X	X			O	O			X			O				X				O			X	

d) e) f)

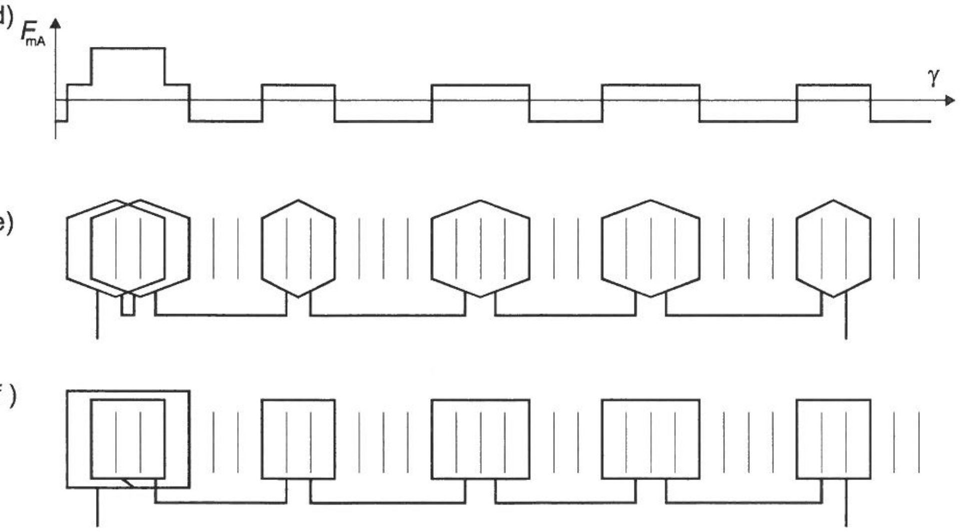


Fig. 2.11 Design of a one-layer winding with $q = 1\ 1/5$.
a) The coil-group schema
b) The coils are presented as parts of the end-winding.
c) Table of phase conductors in order to specify the coil-sides and the slot, and the current flow.
d) Distribution of MMF for phase A.
e) Winding construction of phase A using diamond winding.
f) Winding construction of phase A using concentric winding.

2.12 Two-layer fractional-slot winding

A two-layer fractional-slot winding should be designed by using the phasor star. The coil-group schema used in the previous chapter for the one-layer winding reserves the slots for the right coil-sides. In a two-layer winding, this reservation is not necessary because of the two layers.

It is more convenient to construct the phasor star as a planar presentation then as an angular phasor presentation. The planar presentation uses the equivalent chart of pole pairs. As an example, the planar phasor-star is presented for a two-layer fractional-slot winding with 36 slots, 10 poles, and q = 1.2 = 1 1/5, Fig. 2.12. The base chart presents the upper layer and its zones, Fig. 2.12 a. The second chart presents the lower layer, Fig. 2.12 b. In general, it is enough to define the chart only for the basic winding, and only for the upper layer. When it is necessary to define the MMF distribution of the winding, also the chart of the lower layer is needed. The charts differ from each other:

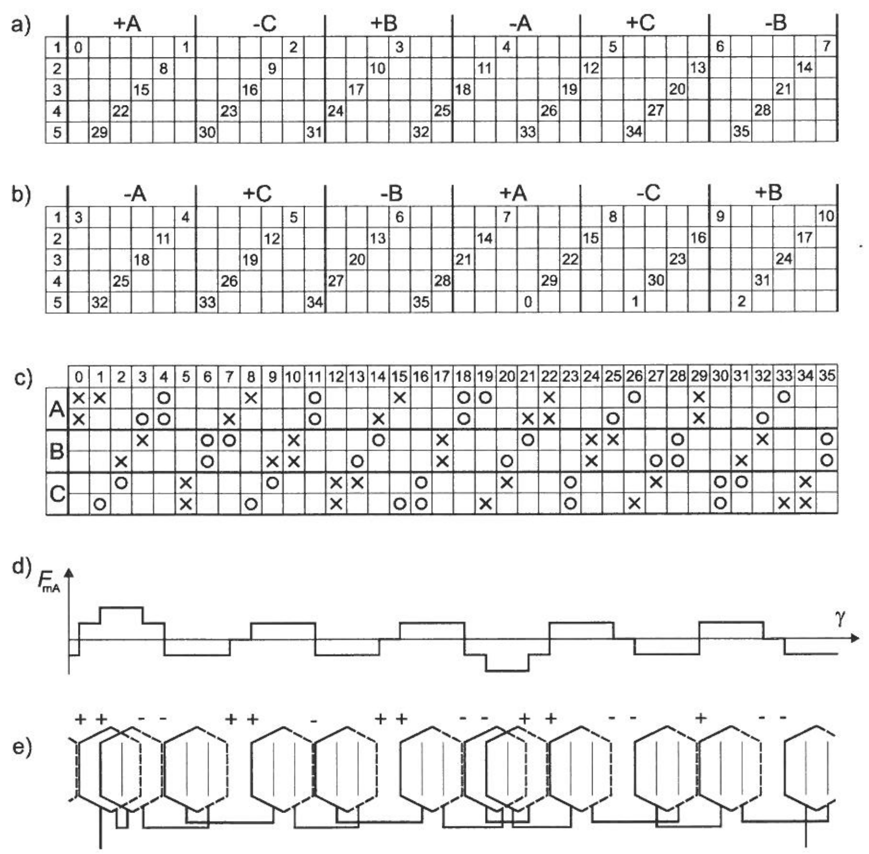


Fig. 2.12 Design of a two-layer winding with q = 1 1/5.
a) Planar phasor star of upper layer by using the slot numbers, the columns indicate the phasor angles in electrical degrees.
b) Planar phasor star of upper layer by using the slot numbers.
c) Schema of zones for phase A.
d) Distribution of MMF for phase A.
e) Lap winding, phase A. (Naming bases on the layout of the coils.)

- The names of the zone columns in the lower and upper layers have different signs.
- The numbering of the slots in the lower layer is shifted from the numbering of slots in upper layer by the value of coil pitch in slot pitches. In this example, the coil pitch is W = 3 slot pitches.

Construction of the planar phasor-star for the two-layer fractional-slot winding

In the following instructions, the constant "n" is the divisor and "z" the dividend in $q = \frac{z}{n}$.

1. The chart of the upper layer is a squared map with Q columns or Q squared in a row.
2. The number of rows is the pole pair number p of the basic winding.
3. The Q columns are organised to $2m$ zones. Every zone has a width of z columns.

   The zones are in order "+A", "-C", "+B", "-A", "+C" and "-B".

5. The filling of charts starts from the chart of upper layer and from the first row and the first column, Fig. 2.12 a. There comes the reference slot number "0".
6. The slot number "1" comes to the same row into the column n + 1. The distance between the slots "0" and "1" is n columns.
7. The next slot numbers come after each other having the distance of n columns. When the row ends the numbering continues to the next row.

   When a slot number comes to the first column of any row, before all the slots are filled in the table, the basic winding smaller than the air-gap periphery has been found. It is not necessary to continue the numbering. The numbering of the next slots is identical to the numbering starting from the slot "0".

In the planar phasor-star, the phase of the zone defines the home phase of the slot. So, all the slots in the columns of the positive zone "+A" and of negative zone "-A" belong to the phase A. By using the planar phasor-stars of both the layers, it is possible to construct a placement table for the coil-sides of a phase where the coil-sides are placed on the rows of the layers and in the columns of the slots. As an example, only the coil-sides of phase A are filled by cross- and circle-marks in the table presented in Fig. 2.12 c. A cross-mark means that the coil-side belongs to a positive zone and a circle-mark that the coil-side belongs to a negative zone. The coil-side on a positive zone causes an increment to the MMF distribution. For the phase A, the increment is the fictive current "+0.5", for phases B and C "-0.25". The coil-side on a negative zone causes a decrement to the MMF distribution. For the phase A, the decrement is the fictive current "-0.5", for phases B and C "+0.25". The MMF distribution of phase A is presented in Fig. 2.12 d.

An alternative winding construction available for the winding plan is presented in Fig. 2.12 e: a lap winding. Another winding construction, a wave winding, is presented in Fig. 2.13 b.

2.12 Two-layer fractional-slot winding

A two-layer fractional-slot winding should be designed by using the phasor star. The coil-group schema used in the previous chapter for the one-layer winding reserves the slots for the right coil-sides. In a two-layer winding, this reservation is not necessary because of the two layers.

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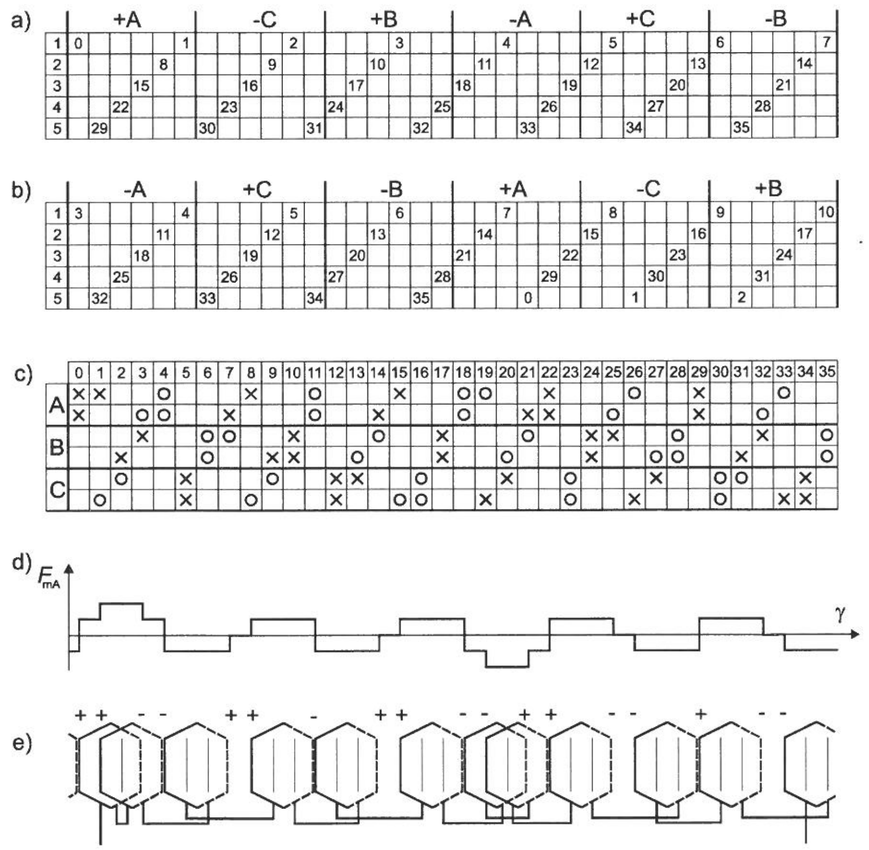

Fig. 2.12 Design of a two-layer winding with q = 1 1/5.
a) Planar phasor star of upper layer by using the slot numbers, the columns indicate the phasor angles in electrical degrees.
b) Planar phasor star of upper layer by using the slot numbers.
c) Schema of zones for phase A.
d) Distribution of MMF for phase A.
e) Lap winding, phase A. (Naming bases on the layout of the coils.)

- The names of the zone columns in the lower and upper layers have different signs.
- The numbering of the slots in the lower layer is shifted from the numbering of slots in upper layer by the value of coil pitch in slot pitches. In this example, the coil pitch is W = 3 slot pitches.

Construction of the planar phasor-star for the two-layer fractional-slot winding

In the following instructions, the constant "n" is the divisor and "z" the dividend in $q = \frac{z}{n}$.

1. The chart of the upper layer is a squared map with Q columns or Q squared in a row.
2. The number of rows is the pole pair number p of the basic winding.
3. The Q columns are organised to $2m$ zones. Every zone has a width of z columns.

   The zones are in order "+A", "-C", "+B", "-A", "+C" and "-B".

5. The filling of charts starts from the chart of upper layer and from the first row and the first column, Fig. 2.12 a. There comes the reference slot number "0".
6. The slot number "1" comes to the same row into the column n + 1. The distance between the slots "0" and "1" is n columns.
7. The next slot numbers come after each other having the distance of n columns. When the row ends the numbering continues to the next row.

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In the planar phasor-star, the phase of the zone defines the home phase of the slot. So, all the slots in the columns of the positive zone "+A" and of negative zone "-A" belong to the phase A. By using the planar phasor-stars of both the layers, it is possible to construct a placement table for the coil-sides of a phase where the coil-sides are placed on the rows of the layers and in the columns of the slots. As an example, only the coil-sides of phase A are filled by cross- and circle-marks in the table presented in Fig. 2.12 c. A cross-mark means that the coil-side belongs to a positive zone and a circle-mark that the coil-side belongs to a negative zone. The coil-side on a positive zone causes an increment to the MMF distribution. For the phase A, the increment is the fictive current "+0.5", for phases B and C "-0.25". The coil-side on a negative zone causes a decrement to the MMF distribution. For the phase A, the decrement is the fictive current "-0.5", for phases B and C "+0.25". The MMF distribution of phase A is presented in Fig. 2.12 d.

An alternative winding construction available for the winding plan is presented in Fig. 2.12 e: a lap winding. Another winding construction, a wave winding, is presented in Fig. 2.13 b.

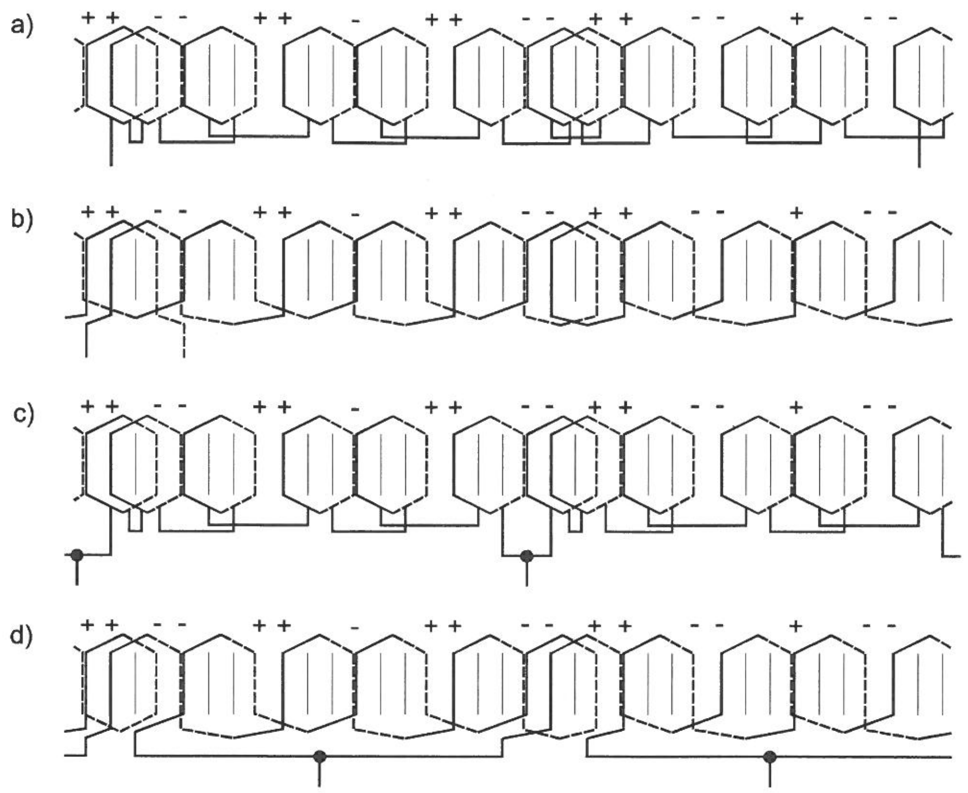

Fig. 2.13 Alternative constructions and connections of the two-layer winding with $q = 1\ 1/5$, phase A.
a) Lap winding, all the coils are connected in series.
b) Wave winding, all the coils are connected in series.
c) Lap winding. The winding is divided into two basic windings. The coils inside the basic winding are connected in series. The basic windings are connected in parallel but by reversing the connections of the other basic winding.
d) Wave winding. The winding is divided into two basic windings. The coils inside the basic winding are connected in series. The basic windings are connected in parallel but by reversing the connections of the other basic winding.

Remarks

The serial connection of all the coils in the basic winding is not the only choice to build a two-layer fractional-slot winding.

The MMF distribution of phase A in Fig. 2.12 d has a mirror image between the two winding parts of 18 slots. When the placing of all the phases is symmetrical, also the MMF distribution of the total winding has the same mirror image. These two 18-slot winding-parts form the basic winding of the first-order fractional-slot winding. The serial connection of the 18-slots windings is presented in Figs. 2.13 a and 2.13 b. A parallel connection of the winding parts is in Fig. 2.13 c for a lap winding and for a wave winding in Fig. 2.13 d. In parallel connections, the basic windings are connected by reversing the connections of the second basic winding to keep the defined pole number of the winding.

2.13 Winding factor of fractional slot windings

In the first order one-layer fractional slot winding, the number of voltage phasors of the phase is even, Q^* / m = even. The phase shift between the phasors of the phasor star is

$$\alpha_z = \frac{\alpha_n}{p^*} = \frac{\alpha_n}{n} \tag{2.39}$$

In the second order one-layer fractional slot winding, the number of voltage phasors of the phase in the basic winding is odd, Q^* / m = odd. The phase shift between the phasors of the phasor star is

$$\alpha_z = \frac{\alpha_n}{p^*} = 2\frac{\alpha_n}{n} \tag{2.40}$$

For the winding factor definition, the phase-winding voltage-phasors of both the one-layer windings shall be sorted into two groups of phasors. The original groups of positive and negative zones have either different number of phasors or the phasors are not placed evenly. The sorted phasors have to form:

- two phasor groups of equal number of phasors, in Figs. 2.14 b and 2.14 d, and
- exactly the same layout for the phasor groups, in Figs. 2.14 c and 2.14 e.

The two sorted phasor groups form a phasor star equivalent to a two-layer integer-slot winding. The sorted phasor groups differ from each other by a phase shift of α, Figs. 2.14 c and 2.14 e. The total winding factor of the one-layer fractional slot winding is

$$\xi_v = \sin\left(v\frac{\pi}{2}\right)\frac{\sin\left(v\dfrac{\pi}{2m}\right)}{nq\sin\left(v\dfrac{\pi}{2mnq}\right)}\cos\left(v\frac{\alpha}{2}\right) \tag{2.41}$$

In the first order two-layer fractional slot winding, the number of voltage phasors of the phase in the basic-winding layer is even. The number of positive and negative voltage phasors is equal. The phase shift between the phasors of the phasor star is

$$\alpha_z = \frac{\alpha_n}{p^*} = \frac{\alpha_n}{n} \tag{2.42}$$

In the second order two-layer fractional slot winding, the number of voltage phasors of the phase in the basic-winding layer is odd. The number of positive and negative voltage phasors is different. Two basic windings are necessary for the total winding. The number of phasors is $2p^*q$. The phase shift between the phasors of the phasor star is

$$\alpha_z = \frac{\alpha_n}{p^*} = 2\frac{\alpha_n}{n} \tag{2.43}$$

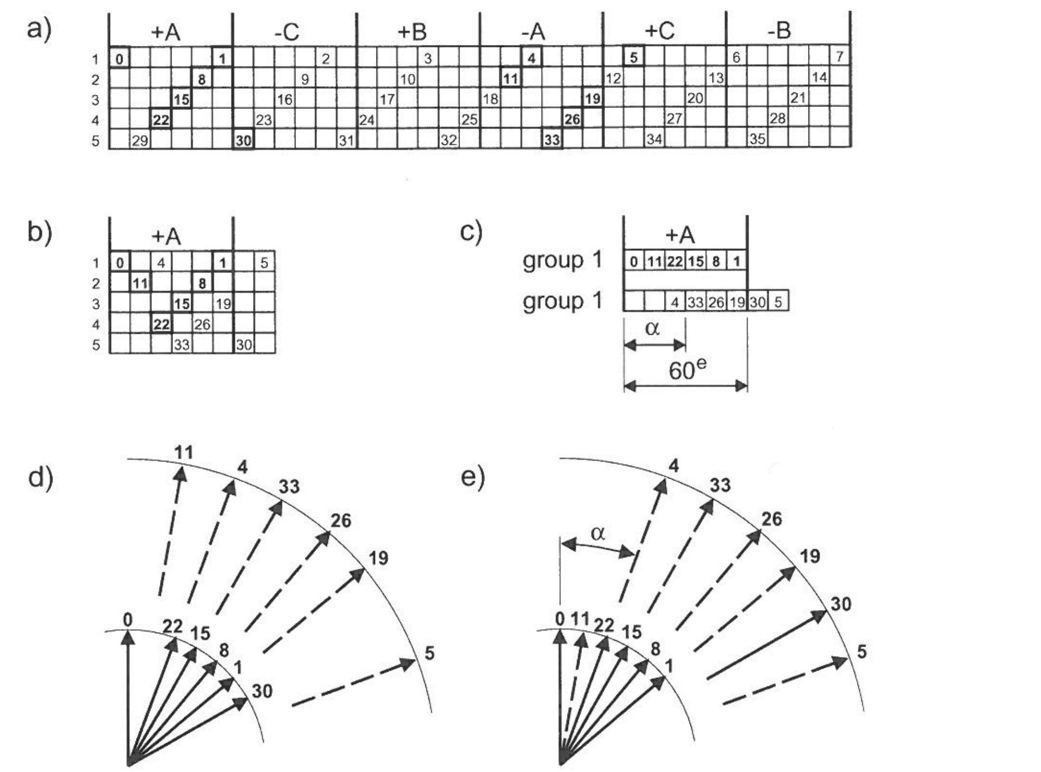

Fig. 2.14 Phasor groups of one-layer winding, $q = 1\ 1/5$

a) Planar phasor-presentation by using the slot numbers, the columns indicate the phasor angles in electrical degrees. The phasors of phase A are marked by rectangles.
b) The phasors of phase A. The phasors of negative zone are moved by 180^e.
c) The phasors of phase A are sorted to two equivalent groups to define the phase shift of the groups, α. Members of Group 1 are marked also in Fig. 2.14 b by bold faces.
d) The phasors of phase A are grouped by using the angular phasor-presentation. The phasors of negative zones, marked by dashed lines, are placed on the outer arc.
e) The phasors of arcs are filled to define the phase shift of the groups, α.

The total winding factor of the two-layer fractional-slot winding is

$$\xi_\nu = \sin\left(\nu\frac{\pi}{2}\right)\frac{\sin\left(nq\nu\frac{\alpha_z}{2}\right)}{nq\sin\left(\nu\frac{\alpha_z}{2}\right)}\sin\left(\nu\frac{W}{\tau}\frac{\pi}{2}\right) = \sin\left(\nu\frac{\pi}{2}\right)\frac{\sin\left(\nu\frac{\pi}{2m}\right)}{nq\sin\left(\nu\frac{\pi}{2mnq}\right)}\sin\left(\nu\frac{W}{\tau}\frac{\pi}{2}\right) \tag{2.44}$$

2.14 Slotting harmonics

The winding factor of a one-layer integral-slot winding is, see Eq. (2.31)

$$\xi_\nu = \sin\left(\nu\frac{\pi}{2}\right)\frac{\sin\left(\nu\frac{\pi}{2m}\right)}{q\sin\left(\nu\frac{\pi}{2mq}\right)} \tag{2.45}$$

Because the sinus-function is periodic, the winding factor may get same values for different harmonics. The most important values for the winding factor are the values equal to the winding factor of fundamental wave $\nu = 1$. This happens when

$$\begin{cases}\nu\frac{\pi}{2mq} = \mathrm{k}\pi + \frac{\pi}{2mq}\\ \nu\frac{\pi}{2mq} = \mathrm{k}\pi - \frac{\pi}{2mq}\end{cases} \Rightarrow \begin{cases}\nu = 2\mathrm{k}mq + 1\\ \nu = 2\mathrm{k}mq - 1\end{cases} \tag{2.46}$$

where k is any integer.

These harmonics ν are called slotting harmonics because their index is the number of slots in the basic winding or its multiple incremented or decremented by one

$$\begin{cases}\nu = 2\mathrm{k}mq + 1 = \mathrm{k}\frac{Q}{p} + 1\\ \nu = 2\mathrm{k}mq - 1 = \mathrm{k}\frac{Q}{p} - 1\end{cases} \tag{2.47}$$

The winding factor of a one-layer fractional-slot winding, see Eq. (2.41)

$$\xi_\nu = \sin\left(\nu\frac{\pi}{2}\right)\frac{\sin\left(\nu\frac{\pi}{2m}\right)}{nq\sin\left(\nu\frac{\pi}{2mnq}\right)}\cos\left(\nu\frac{\alpha}{2}\right) \tag{2.48}$$

Studying the middle part of the equation, the periodicity of sinus-function gives the definition to the flux-density harmonic inducing at the same effectiveness as the fundamental harmonic

$$\begin{cases}\nu\frac{\pi}{2mnq} = \mathrm{k}\pi + \frac{\pi}{2mnq}\\ \nu\frac{\pi}{2mnq} = \mathrm{k}\pi - \frac{\pi}{2mnq}\end{cases} \Rightarrow \begin{cases}\nu = 2\mathrm{k}\,m\,\mathrm{n}q + 1\\ \nu = 2\mathrm{k}\,m\,\mathrm{n}q - 1\end{cases} \tag{2.49}$$

where k is any integer and n the divider of q.

The order of these slotting harmonics, ν, may be defined as the harmonics in Eq.(2.46) by the number of slots in the basic winding or its multiple incremented or decremented by one

$$\begin{cases} \nu = 2\mathrm{k}\, m\mathrm{n}q + 1 = 2\mathrm{k}\, m\, p^* q + 1 = \mathrm{k}Q^* + 1 = \mathrm{k}\dfrac{p^*}{p}Q + 1 \\ \nu = 2\mathrm{k}\, m\mathrm{n}q - 1 = 2\mathrm{k}\, m\, p^* q - 1 = \mathrm{k}Q^* - 1 = \mathrm{k}\dfrac{p^*}{p}Q - 1 \end{cases} \tag{2.50}$$

Example

The order of the slot harmonics starts from the number of slots in the basic winding.

The first slot harmonic pairs for q = 3, 3.25, 3.5, and 4 are

- for a integral slot winding	$q = 3$	ν = 17 and 19.
for a fractional slot winding	$q = 3.25$	ν = 77 and 79.
for a fractional slot winding	$q = 3.5$	ν = 41 and 43.
- for a integral slot winding	$q = 4$	ν = 23 and 25.

The fractional slot winding brings the slotting harmonics apart from the fundamental harmonic. Also the interval between the slotting harmonic pairs comes wider.

2.15 Skewing

A skewing of the slots is a method to affect the harmonics in the coil. Either the stator slotting or rotor slotting is skewed from the axial direction by an angle, the skewing pitch τ_{skew}, Fig. 2.15. Usually the skewing is equal to the slot pitch. In squirrel-cage motors the skewing is about the slot pitch of the stator winding, $\tau_{\mathrm{skew}} = \tau_{\mathrm{nS}}$.

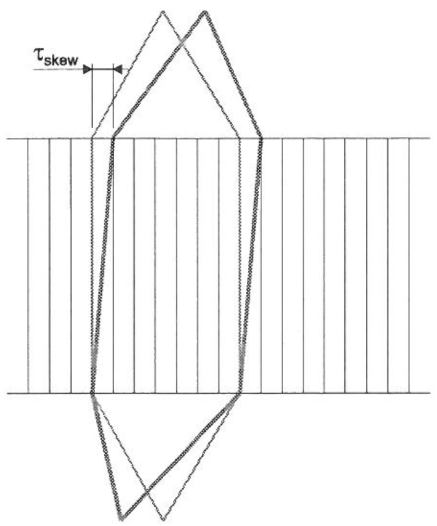

Fig. 2.15 Skewing of stator or rotor slotting from axial direction.

To define the change into the winding factor, the skewed coil side may be modelled as a unlimited tight coil group with infinite number of coils on the peripheral area of the skewing sector. The axial length of the skewing sector is the core length and the width the skewing pitch τ_{skew}. The skewing factor, a correction factor to the winding factor, is defined by

$$\xi_{\mathrm{skew},\nu} = \frac{\sin\left(q\nu\frac{\alpha_{\mathrm{n}}}{2}\right)}{q\sin\left(\nu\frac{\alpha_{\mathrm{n}}}{2}\right)} = \frac{\sin\left(\frac{\nu}{2}q\alpha_{\mathrm{n}}\right)}{q\sin\left(\nu\frac{\alpha_{\mathrm{n}}}{2}\right)} = \frac{\sin\left(\frac{\nu}{2}\frac{\tau_{\mathrm{skew}}}{\tau_{\mathrm{p}}}\right)}{q\sin\left(\frac{\nu}{2}\frac{\tau_{\mathrm{skew}}}{q\tau_{\mathrm{p}}}\right)} = \left.\frac{\sin\left(\frac{\nu}{2}\frac{\tau_{\mathrm{skew}}}{\tau_{\mathrm{p}}}\right)}{\frac{\nu}{2}\frac{\tau_{\mathrm{skew}}}{\tau_{\mathrm{p}}}}\right|_{q\to\infty} \tag{2.51}$$

3 Winding data

3.1 Effective turns of winding

The winding design starts from the main flux and its fundamental harmonic ϕ_{h}, meaning the harmonic having the wave length equal to two pole pitches.

In general, the EMF in a coil, the fundamental harmonic, is induced by the flux component ϕ_{h} flowing through the whole coil

$$E = \frac{\omega}{\sqrt{2}} N_{\mathrm{coil}}\hat{\phi}_{\mathrm{h}} \tag{3.1}$$

where

ω is the fundamental angular frequency and
N_{coil} the number of turns in the coil.

In a slot winding, the phase winding bases on the winding zones of pole pitches. The division of the coils inside the zone, a distributed winding, causes that the flux component induces different voltage value to every coil in the zone. This effect is taken into account by the winding factor, for the fundamental harmonic ξ_1.

In a one-layer integral-slot winding, the coils of the zone in a pole pair form a possible branch for parallel connection. The number of parallel branches, a, has to fulfil the equation

$$\frac{p}{a} = \mathrm{k} \tag{3.2}$$

where k is any integer number.

In a one-layer fractional-slot winding, the number of parallel branches has to fulfil the equation

$$\frac{p}{a\,p^*}=\frac{p}{a\,\mathrm{n}}=\mathrm{k} \tag{3.3}$$

where n is the divider of q.

In a two-layer integral-slot winding, the coils of the zone in a pole pair form a possible branch for parallel connection. The number of parallel branches has to fulfil the equation

$$\frac{2p}{a}=\mathrm{k} \tag{3.4}$$

where k is any integer number.

In a two-layer fractional-slot winding, the number of parallel branches has to fulfil the equation

$$\frac{2p}{a\,p^*}=\frac{2p}{a\,\mathrm{n}}=\mathrm{k} \tag{3.5}$$

where n is the divider of q.

The effective number of turns N_{eff} in a phase winding is

$$\begin{cases} N_{\mathrm{eff,one}}=\dfrac{Q\,N_{\mathrm{coil}}}{2am}=pq\dfrac{N_{\mathrm{coil}}}{a}, & \text{for a one-layer winding} \\ N_{\mathrm{eff,two}}=\dfrac{Q\mathrm{N}_{\mathrm{coil}}}{am}=2pq\dfrac{N_{\mathrm{coil}}}{a}, & \text{for a two-layer winding} \end{cases} \tag{3.6}$$

To get the definition of N_{eff} to be independent from the winding type, a better choice for the definition is, instead of the number of turns in the coil, to use the number of turns in a slot

$$\begin{cases} N_{\mathrm{eff,one}}=\dfrac{Q\,N_{\mathrm{coil}}}{2am}=\dfrac{Q\,N_{\mathrm{slot}}}{2am}=pq\dfrac{N_{\mathrm{slot}}}{a}, & \text{for a one-layer winding} \\ N_{\mathrm{eff,two}}=\dfrac{2Q\,N_{\mathrm{coil}}}{am}=\dfrac{Q\,N_{\mathrm{slot}}}{am}=pq\dfrac{N_{\mathrm{slot}}}{a}, & \text{for a two-layer winding} \end{cases} \tag{3.7}$$

The EMF of the phase winding, the fundamental harmonic, induced by the flux component ϕ_{h} flowing through the distributed winding is obtained from equation

$$E=\frac{\omega}{\sqrt{2}}\xi_1 N_{\mathrm{eff}}\,\hat{\phi}_{\mathrm{h}}=\frac{\omega}{\sqrt{2}}\xi_1\frac{pq}{a}N_{\mathrm{slot}}\,\hat{\phi}_{\mathrm{h}} \tag{3.8}$$

The induced voltage may be adjusted mainly by the number of turns N_{eff} and partly by the winding factor ξ_1. The winding factor ξ_1 may have values between 0.8 ... 0.96. Inside the factor N_{eff} the pole-pair number, p, is constant, number of slots per poles per phases, q, has a

limited range about 0.5 to both directions. The main factor in voltage regulation is the factor N_{slot}. It has the minimum value of 1 for a one-layer winding and 2 for a two-layer winding. The high-voltage winding is designed by changing mainly the number of turns in slot, N_{slot}. The low-voltage winding is designed by changing both the number of slots per pole and per phase, q, and the chording of the winding, the winding factor ξ_1.

3.2 Synchronous machine

A synchronous generator has to product a sinusoidal output voltage according to the standards of distortion in the wave form of electrical network. This means that the induced voltage of the winding has to be practically sinusoidal. The demand has to be taken into account already in the design of the winding. In practise, this is made by choosing the winding having the smallest winding factors for higher harmonics.

On land application in Finland, the stator-winding connection of a low voltage generator is a star connection with neutral (YN) when the generator is connected directly to the grid. The generator is loaded by single phase loads. This means that the third harmonic and its multiples may flow in the network. The winding should not produce any third harmonic component in the line-to-neutral voltage, phase voltage. The winding has to have a winding with coil width of 2/3 of the pole pitch.

The high voltage generators are connected to grid via a block transformer, in Finland, so the connection of stator winding may be star (Y). The generator is loaded only by phase-to-phase voltage. This gives the possibility to use a one-layer winding.

The open slots used in high-voltage and high-current machines produce large permeance harmonics in the air gap. The effect shall be damped by using a large radial air-gap length.

The generators having the connection in star may be, if needed, chorded for the 5th and 7th harmonic by chording the coils near to the value 5/6 of the pole pitch. Otherwise, a one-layer winding is more practical.

The slotting harmonics have the same winding factor as the fundamental harmonic. It is possible to get the harmonics far from the fundamental harmonics by using fractional slot windings.

The damper or starting winding on the rotor should be designed with a slot pitch τ_{nRD}

$$\tau_{\mathrm{nRD}}=\left(\mathrm{k}\pm 0.1...0.15\right)\tau_{\mathrm{nS}},\qquad \mathrm{k}=1,2,3,... \tag{3.9}$$

to damp the air-gap harmonics caused by the currents in both the stator and rotor winding.

3.3 Asynchronous machine

In an asynchronous machine, both the stator winding and the rotor winding produce harmonics into the air-gap flux-density distribution. The harmonics may cause problems to the working of the machine: asynchronous and synchronous torque-ripples to the operation.

The winding should be an integral slot winding if possible, mainly $q > 2$. A fractional slot winding with n = 2 may also be possible. The winding design uses mainly a winding chorded for the 5th and 7th harmonic, the coil width near to the value 5/6 of the pole pitch.

There are some successful numbers of slots used in the squirrel cage motors, e.g. $Q = 36$. It is practicable for many pole-pair numbers: $p = 1$ $q = 6$, $p = 2$ $q = 3$, $p = 3$ $q = 2$, $p = 4$ $q = 1.5$.

The air gap in asynchronous machines is small in comparison to synchronous machines. This means that the slot-opening design is important. The slot opening in round-wire windings is narrow but also a magnetic slot wedge may be used to damp the permeance harmonics.

The rotor winding of slip-ring machine is, for lower power, a round-wire two-level or two-layer integral-slot winding, for higher power, mainly profile-copper two-layer integral-slot winding. The slotting is chosen according to the stator slotting

$$q_R = \frac{Q_R}{2pm_R} = q_S \pm 1, (q_S \pm 2) \tag{3.10}$$

where the index R refers to rotor quantities and S stator quantities. The number of rotor phases m_R is mainly $m_R = 3$.

The rotor winding of cage induction machine is, in practice, with either aluminium or copper bars and end-rings. The slotting is dependent on the stator winding

$$q_R = \frac{Q_R}{6p} = q_S \pm \frac{2}{3} \tag{3.11}$$

and, at the same time, the number of slots Q_R has to be an even number. This happens always together with an integral slot winding in the stator. Having a fractional slot winding in the stator and the value of $2q_S$ not an integer, the equation (3.11) is fulfilled only when the value of p is even.

To damp the asynchronous torque-ripples the number of slots in the rotor, Q_R, should be as small as possible, anyway it has to fulfil the rule $Q_R \leq 1.25\,Q_S$.

One list of rules for the forbidden slot numbers in a rotor without skewing is

$$\begin{cases} Q_R \neq Q_S, \quad Q_R \neq \dfrac{Q_S}{2}, \quad Q_R \neq 2\,Q_S \\ Q_R \neq Q_S \pm 2p, \quad Q_R \neq Q_S \pm 2p, \quad Q_R \neq Q_S \pm (2p \pm 1) \end{cases} \tag{3.12}$$

As an example, for the stator slotting with $Q_S = 36$ and $p = 2$, the allowed variants for the number of slots in rotor, Q_R, are

- for slotting without skewing: $Q_R = 10, 14, 22, 26, 30, 42, 46, 50, 54$ and 58.
- for slotting with skewing $\tau_{skew} = \tau_{nS}$ $Q_R = 16, 30, 40, 42, 44$ and 60.

4 Leakage reactance

A conductor having a current produces around itself a magnetic field. The energy associated with the magnetic field is, in general, presented in the electrical equivalent circuit as a reactance. If the magnetic field causes something useful in an electrical machine, like causes a part of the main flux, we are calling the reactance a main-flux reactance or a portion of it. If the magnetic field has no such a useful meaning, e.g. the flux flows only outside the main-flux path, we are calling it a leakage reactance or a portion of it.

4.1 Leakage flux through the slot

When the conductor is in a slot the conductor always produces a flux component going through the slot, a leakage flux.

The reactance component $X_{\sigma,slot}$ from the leakage flux shall be calculated using the magnetic energy distribution inside the slot. The distribution depends on the distribution of the conductors inside the slot and on the currents of the conductors. If there are parallel wires in the slot the distribution of currents between the parallel wires shall also be taken into account.

In practise, the magnetic energy distribution may be defined by reluctance network. The slot area will be divided into regions according to the layout of the slot and of the conductor distribution, Fig. 4.1. Every region in the slot division shall have its own equivalent branch in the reluctance network including the MMF source of the current inside the region and the reluctance of the region to the direction of the leakage flux. The main divisions are the conductor area, the distance between the layers of windings, the wedge area, and the slot opening area.

The permeance of a region i is defined from the dimensions of the flux path

$$\Lambda_i = \mu_0 \frac{h_i l_i}{b_i} \tag{4.1}$$

where

h_i	is the height of the region, perpendicular to the flux flow,
l_i	the ideal length of the core, and
b_i	the width of the region, parallel to the flux flow.

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where the index R refers to rotor quantities and S stator quantities. The number of rotor phases m_R is mainly $m_R = 3$.

The rotor winding of cage induction machine is, in practice, with either aluminium or copper bars and end-rings. The slotting is dependent on the stator winding

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and, at the same time, the number of slots Q_R has to be an even number. This happens always together with an integral slot winding in the stator. Having a fractional slot winding in the stator and the value of $2q_S$ not an integer, the equation (3.11) is fulfilled only when the value of p is even.

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4.1 Leakage flux through the slot

When the conductor is in a slot the conductor always produces a flux component going through the slot, a leakage flux.

The reactance component $X_{\sigma,slot}$ from the leakage flux shall be calculated using the magnetic energy distribution inside the slot. The distribution depends on the distribution of the conductors inside the slot and on the currents of the conductors. If there are parallel wires in the slot the distribution of currents between the parallel wires shall also be taken into account.

In practise, the magnetic energy distribution may be defined by reluctance network. The slot area will be divided into regions according to the layout of the slot and of the conductor distribution, Fig. 4.1. Every region in the slot division shall have its own equivalent branch in the reluctance network including the MMF source of the current inside the region and the reluctance of the region to the direction of the leakage flux. The main divisions are the conductor area, the distance between the layers of windings, the wedge area, and the slot opening area.

The permeance of a region i is defined from the dimensions of the flux path

$$\Lambda_i = \mu_0 \frac{h_i l_i}{b_i} \tag{4.1}$$

where

h_i is the height of the region, perpendicular to the flux flow,
l_i the ideal length of the core, and
b_i the width of the region, parallel to the flux flow.

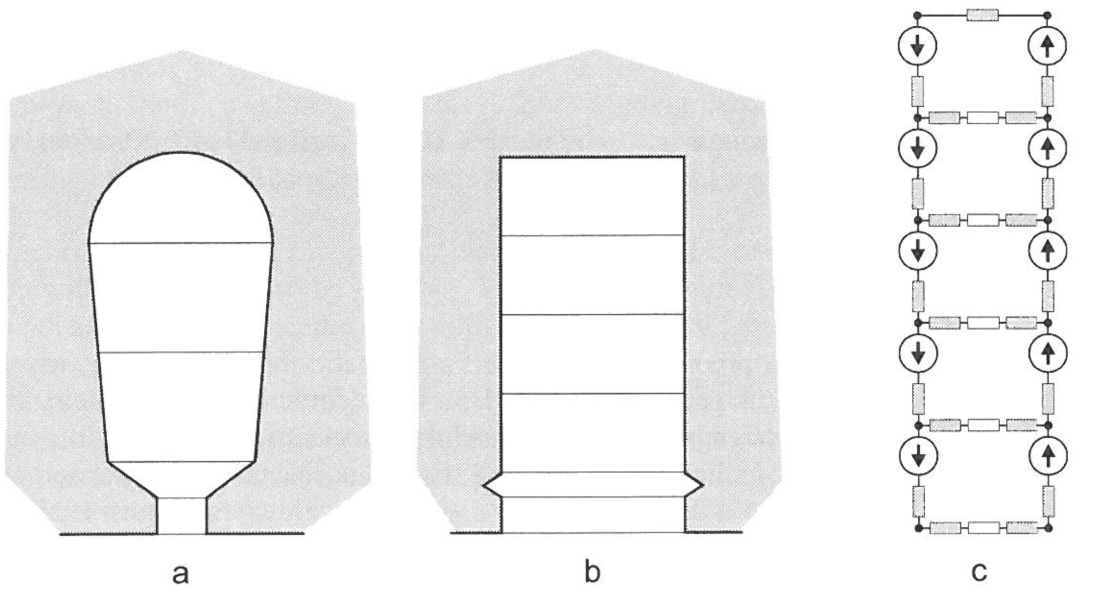

Fig. 4.1 Leakage flux through the slot.
a) A round-wire slot divided into areas for modelling the magnetic field.
b) A profile-copper slot divided into areas for modelling the magnetic field.
c) A reluctance model for slot divisions presented in Figs. 4.1a and b.

The reactance of the leakage flux is calculated from the energy distribution by

$$X_{\sigma,\text{slot}} = \omega L_{\sigma,\text{slot}} = 2\omega pq\left(\frac{N_{\text{slot}}}{a}\right)^2 \mu_0 l_i \, \Lambda_{\text{slot}}(\phi_{\sigma,\text{slot}}) \tag{4.2}$$

where $\Lambda_{\text{slot}}(\phi_{\sigma,\text{slot}})$ is the total permeance of the slot calculated using the magnetic energy distribution inside the slot.

If a current-carrying conductor is placed in a slot and consists of parallel wires, the current density of the conductor is not even. The current density of the parallel wires is a function of the distance of the parallel wire from the bottom of the slot. The skin effect of the conductor changes the current distribution between the parallel wires. The value of skin effect is possible to define using the reluctance model of the slot in Fig. 4.1. The MMF sources of the model are the currents in the parallel wires. If the height of the parallel wire is large in relation to the width of the parallel wire, every parallel wire has to be divided to parts in the reluctance model. For a round-wire coil, the conductor may be studied by areas of conducting material. The method is chosen for this study taking into account the manufacturing process of the coil.

To fulfil the validity of Eq. (4.2), the permeance calculation of one slot requires that the winding construction is correctly distributed along the periphery of the slotting. This means that every conductor, a parallel wire or an effective turn, shall have equivalent positions inside the region of all the slots. This is guaranteed in practice by transposition of the conductors.

A round-wire winding is practically always transposed by itself by assumption of unorganised nature of wires in a coil. The coil construction of a profile-copper coil and the number of parallel wires has to be planned so that the full transposition of the conductors is possible.

The full transposition of the parallel conductors means:

- For a concentric winding
  In general, a coil of a concentric winding has all the conductors always on the same level inside the coil. As an example, a coil of one-layer concentric-winding is presented in Fig. 4.2 a. The connection between two coils has to be transposed outside the slot. This is done by special transposing connections at the end-winding region, Fig. 4.2 b.

- For a lap winding
  In general, a coil of a lap winding has a transposition made on the construction of coil. All the conductors are transposed in order upside done on the other coil side of the coil. As an example, a coil of two-layer lap- or diamond-winding is presented in Fig. 4.2 c. The connection between two coils has to be transposed outside the slot. This is done by special transposing connections at the end-winding region, Fig. 4.2 d.

- In overall
  To fulfil the definition of reactance in Eq. (4.2), the number of parallel wires is limited according to the connection of the winding and according to the slots in a zone in spite of the winding type.

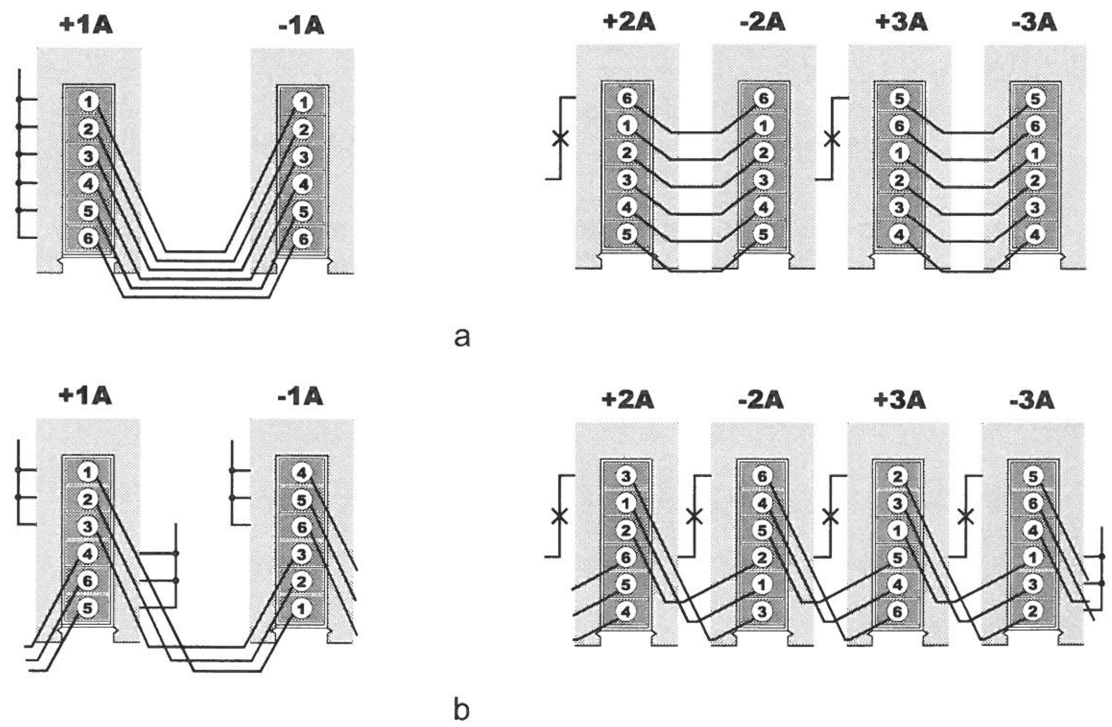

Fig. 4.2 Transposition of parallel conductors of coil.
a) A coil of a one-layer concentric winding set into the slots. On the right the next serial coils: Transposition of parallel conductors at the end-winding region of connection end of machine.
b) A coil of a two-layer lap winding set into the slots, transposition of parallel conductors by the construction of the coil. On the right the next serial coils: Transposition of parallel conductors at the end-winding region of connection end of machine.

4.2 Leakage flux round the end-winding

When the conductor is in air like in the end-winding the conductor always produces a flux component going around itself, a leakage flux. The whole leakage-flux concentration depends on all the phase currents in the winding but also on all the phase currents in the other windings on the same end-winding region. So the currents in the rotor winding affect to the leakage flux distribution in the stator winding.

The reactance, reduced to stator, of the leakage flux is calculated by

$$X_{\sigma,\text{end-winding}} = \omega L_{\sigma,\text{end-winding}} = 2\omega p\left(\frac{q\mathrm{N}_{\text{slot}}}{\mathrm{a}}\right)^2 \Lambda_{\text{end-winding}}(\phi_{\sigma,\text{end-winding}}) \tag{4.3}$$

where $\Lambda_{\text{end-winding}}(\phi_{\sigma,\text{end-winding}})$ is the permeance of the coil outside the slot on one end-region.

The permeance $\lambda_{\text{coil-end}}$ is defined in Table 4.1 as a function of the winding types in stator and rotor and as a function of the coil length outside the core on the end-winding in stator. These values are specified with model windings. To solve the values of Table 4.1 in the target end-winding the dimensions of the target end-winding cause changes to the value of the leakage permeance of the model winding. This effect is taken into account by correction factors of each winding. In the case of 2-level-concentric, diamond or evolvent 3-phase windings in the stator and a cage winding in the rotor the total permeance $\Lambda_{\text{end-winding}}$ of the winding pair may be defined by

$$\Lambda_{\sigma,\text{end-winding}} = \chi_S\, \rho\, \iota\, \sigma\, \lambda_{\text{coil-end}} \tag{4.4}$$

where χ_S, ρ, ι and σ are correction factors of the target windings. The values for the correction factors χ, ρ, ι and σ are presented for different windings in Tables 4.2 to 4.5.

The correction factor χ of the target end-winding construction bases on the relation of the circumference of a coil group in the end-winding, u, to the pole pitch τ_p. The circumference u is defined by

$$u = 2\left[(q-1)\tau_{n,\text{coil}} + b_{\text{coil}} + h_{\text{coil}}\right] \tag{4.5}$$

where

$\tau_{n,\text{coil}}$ is the distance between the centres of two coils,
b_{coil} the width of the coil side, and
h_{coil} the height of the coil side.

For a two-layer winding, the circumference should be taken only for one layer. Every winding has its own correction factor χ_S for the stator winding and χ_R for the rotor winding. The correction factor of the winding pair χ is the average of these values.

Table 4.1 Permeance factors for model winding (Richter 1936, Table 3, p. 162)
Cage winding in rotor

3-phase 2-level concentric winding in stator			
$l_{\text{coil-end}} / \tau_p$	1.61	1.92	2.23
$\lambda_{\text{coil-end}}$	0.324	0.359	0.367
3-phase diamond winding in stator			
$l_{\text{coil-end}} / \tau_p$	1.46	1.66	1.86
$\lambda_{\text{coil-end}}$	0.298	0.325	0.343
3-phase two-layer evolvent winding in stator			
$l_{\text{coil-end}} / \tau_p$	2.15	2.28	2.40
$\lambda_{\text{coil-end}}$	0.280	0.269	0.258
3-phase one-layer evolvent winding in stator			
$l_{\text{coil-end}} / \tau_p$	2.25	2.40	2.55
$\lambda_{\text{coil-end}}$	0.344	0.318	0.289

The correction factors ρ and σ bases on the relation of the coil pitch in the target winding, W, to the pole pitch, τ_p. In addition, the correction factor σ bases on the relation of a to τ_p.

The correction factor ι bases on the relations s to τ_p and a to τ_p. The quantity s is the distance between the centre points of the coil sides of the target stator and rotor windings. The quantity a is in the cage winding the distance between the centre point of the end ring and the surface of the rotor core, a_R. In the case of stator winding, the quantity a is the length of the straight part of the stator coil outside the stator core, a_S.

Table 4.2 Correction factor χ for Table 4.1 as a function of winding dimensions

u / τ_p	0.45	0.5	0.6	0.7	0.8	0.9	0.95
χ	1.20	1.13	1.05	1.00	0.92	0.89	0.86

Table 4.3 Correction factors ρ, ι and σ for Table 4.1 as a function of winding dimensions
The values are for a 3-phase 2-level concentric winding in stator and a cage winding in rotor. Correction factor σ = 1. For a 3-phase 2-level concentric winding in stator and a pole winding in rotor, the values are at relation of $a_R / \tau_p = 0$.

a_R / τ_p	0.05	0.10	0.15	0.20	0.25	0.30
ρ	1.01	1.00	0.97	0.92	0.87	0.80

	s / τ_p	0.10	0.15	0.20	0.25	0.30
a_R / τ_p						
0.20	ι	0.81	0.90	1.00	1.10	1.19
0.10	ι	0.90	0.95	1.00	1.05	1.10
0.05	ι	0.98	0.99	1.00	1.01	1.02

Table 4.4 Correction factors σ, ι and ρ for Table 4.1 as a function of winding dimensions
The values are for a 3-phase diamond winding in stator and a cage winding in rotor. For a 3-phase diamond winding in stator and a pole winding in rotor, the values are at relation of a_R / τ_p = 0.

a_S / τ_p	a_R / τ_p	0.05	0.10	0.15	0.20	0.25
0.30	ρ	1.00	0.98	0.94	0.87	0.78
0.10	ρ	0.99	0.960	0.86	0.72	0.56

a_R / τ_p	s / τ_p	0.10	0.15	0.20	0.25	0.30
0.20	ι	0.88	0.94	1.00	1.06	1.12
0.10	ι	0.92	0.96	1.00	1.04	1.08
~0	ι	1.00	1.009	1.00	1.00	1.00

a_S / τ_p	W / τ_p	0.65	0.70	0.75	0.80	0.85	0.90	1.00
0.30	σ	0.89	0.92	0.95	0.97	0.98	0.99	1.00
0.20	σ	0.83	0.87	0.90	0.94	0.96	0.98	1.00
0.10	σ	0.71	0.78	0.83	0.89	0.93	0.96	1.00
0.00	σ	0.50	0.61	0.71	0.80	0.88	0.93	1.00

Table 4.5 Correction factors ρ, ι and σ for Table 4.1 as a function of winding dimensions
The values are for a 3-phase evolvent winding in stator and a cage winding in rotor. For a 3-phase evolvent winding in stator and a pole winding in rotor, the values are at relation of a_R / τ_p = 0.

	a_R / τ_p	0.05	0.10	0.15	0.20	0.25	0.30
two-layer	ρ	1.04	1.06	1.04	1.02	0.98	0.94
one-layer	ρ	1.02	1.03	1.01	0.97	0.94	0.89

a_R / τ_p	s / τ_p	0.10	0.15	0.20	0.25	0.30
0.20	ι	0.81	0.90	1.00	1.10	1.19
0.10	ι	0.90	0.95	1.00	1.05	1.10
0.05	ι	0.98	0.99	1.00	1.01	1.02

a_S / τ_p	W / τ_p	0.65	0.70	0.75	0.80	0.85	0.90	1.00
0.30	σ	0.89	0.92	0.95	0.97	0.98	0.99	1.00
0.20	σ	0.83	0.87	0.90	0.94	0.96	0.98	1.00
0.10	σ	0.71	0.78	0.83	0.89	0.93	0.96	1.00
0.00	σ	0.50	0.61	0.71	0.80	0.88	0.93	1.00

Errata

Corrected parts in the text are in bold.
Corrected equations have their number in parenthesis written in bold.
Corrected figures and tables have their number written in bold.

On page 11

The design of windings bases on the pole pitches. One pole pitch is 180 electrical degrees, 180^e. The pole pitch consists of zones of phases. The width of a zone τ_{zone} is the pole pitch τ_p divided by m, the number of phases of the winding

$$\tau_{zone} = \frac{\tau_p}{m} = \frac{\pi D_i}{2pm} \qquad \mathbf{(2.2)}$$

The zones are named sequentially and symmetrically for the phases. Every phase has both a positive and a negative zone. The angle between the positive zones of the phases is the angle between the excitation axes of the phase windings

$$\alpha_{zone} = \frac{360^e}{m} \qquad \mathbf{(2.3)}$$

On page 34

Fig. 2.12 Design of a two-layer winding with q = 1 1/5.
a) Planar phasor star of upper layer by using the slot numbers, the columns indicate the phasor angles in electrical degrees.
b) Planar phasor star of lower layer by using the slot numbers.
c) Schema of zones for phase A.
d) Distribution of MMF for phase A.
e) Lap winding, phase A. (Naming bases on the layout of the coils.)

On page 38

Corrected definition of α in the Fig. 2.14 is in **Fig. 2.14c.**

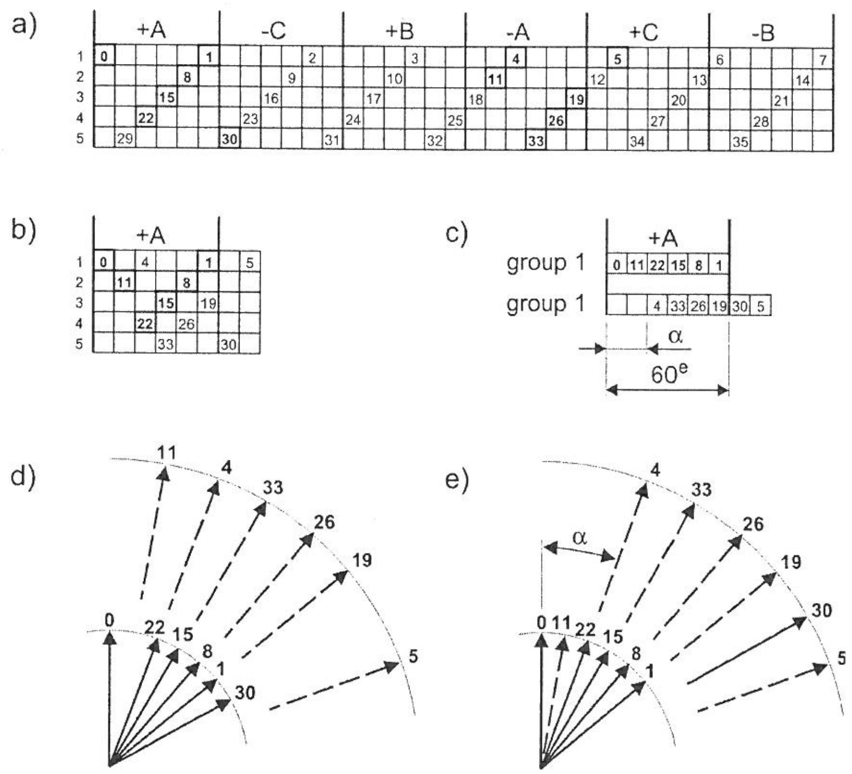

Fig. 2.14 Phasor groups of one-layer winding, $q = 1\ 1/5$

a) Planar phasor-presentation by using the slot numbers, the columns indicate the phasor angles in electrical degrees. The phasors of phase A are marked by rectangles.
b) The phasors of phase A. The phasors of negative zone are moved by 180^{e}.
c) The phasors of phase A are sorted to two equivalent groups to define the phase shift of the groups, α. Members of Group 1 are marked also in Fig. 2.14 b by bold faces.
d) The phasors of phase A are grouped by using the angular phasor-presentation. The phasors of negative zones, marked by dashed lines, are placed on the outer arc.
e) The phasors of arcs are filled to define the phase shift of the groups, α.

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To get the definition of N_{eff} to be independent from the winding type, a better choice for the definition is, instead of the number of turns in the coil, to use the number of turns in a slot

$$\begin{cases} N_{\mathrm{eff,one}} = \dfrac{Q N_{\mathrm{coil}}}{2am} = \dfrac{Q N_{\mathrm{slot}}}{2am} = pq\dfrac{N_{\mathrm{slot}}}{a}, & \text{for a one-layer winding} \\ N_{\mathrm{eff,two}} = \dfrac{Q N_{\mathrm{coil}}}{am} = \dfrac{2Q N_{\mathrm{coil}}}{2am} = \dfrac{Q N_{\mathrm{slot}}}{2am} = pq\dfrac{N_{\mathrm{slot}}}{a}, & \text{for a two-layer winding} \end{cases} \tag{3.7}$$

On page 48, the text is rewritten and the equation corrected

In the case of 2-level-concentric, diamond or evolvent 3-phase windings in the stator and a cage winding in the rotor the total permeance $\Lambda_{\mathrm{end\text{-}winding}}$ of the winding pair may be defined by

$$\Lambda_{\mathrm{end\text{-}winding}}(\phi_{\sigma,\mathrm{end\text{-}winding}}) = \chi_{\mathrm{S}}\,\rho\,\iota\,\sigma\,\lambda_{\mathrm{coil\text{-}end}}\,\mu_0\,l_{\mathrm{coil\text{-}end}} \tag{4.4}$$

where χ_{S}, ρ, ι and σ are correction factors, and $\lambda_{\mathrm{coil\text{-}end}}$ is a permeance factor of an experimental coil (Richter 1936, Table 3, p. 162).

The values for the factor $\lambda_{\mathrm{coil\text{-}end}}$ are experimental and specified by using model windings. The permeance factors $\lambda_{\mathrm{coil\text{-}end}}$ of the modelled end-windings are presented in Table 4.1 as a function of the winding types in stator and in rotor, and as a function of the coil length outside the core on the end-winding in stator, $l_{\mathrm{coil\text{-}end}}$. To solve the values of Table 4.1 in the target end-winding, the dimensions of the target end-winding cause changes to the value of the leakage permeance of the model winding. This effect is taken into account by the correction factors χ_{S}, ρ, ι and σ of each winding type. The values for the correction factors are presented in Tables 4.2 to 4.5. In all these tables, the values for the quantities and factors between the given values shall be taken by linear interpolation.

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Table 4.1 Permeance factors for model winding (Richter 1936, Table 3, p. 162)
Cage winding in rotor

3-phase 2-level concentric winding in stator

$l_{\text{coil-end}} / \tau_p$	1.61	1.92	2.23
a_S / τ_p	**0.08**	**0.20**	**0.32**
$\lambda_{\text{coil-end}}$	0.324	0.359	0.367

3-phase diamond winding in stator

$l_{\text{coil-end}} / \tau_p$	1.46	1.66	1.86
a_S / τ_p	**0.10**	**0.20**	**0.30**
$\lambda_{\text{coil-end}}$	0.298	0.325	0.343

3-phase two-layer evolvent winding in stator

$l_{\text{coil-end}} / \tau_p$	2.15	2.28	2.40
a_S / τ_p	**0.16**	**0.21**	**0.26**
$\lambda_{\text{coil-end}}$	0.280	0.269	0.258

3-phase one-layer evolvent winding in stator

$l_{\text{coil-end}} / \tau_p$	2.25	2.40	2.55
a_S / τ_p	**0.20**	**0.26**	**0.32**
$\lambda_{\text{coil-end}}$	0.344	0.318	0.289

The correction factor ι bases on the relations s to τ_p and a to τ_p. The quantity s is the distance between the centre points of the coil sides of the target stator and rotor windings. The quantity a is in the cage winding the distance between the centre point of the end ring and the surface of the rotor core, a_R. In the case of stator winding, the quantity a is the length of the straight part of the stator coil outside the stator core, a_S. **The values for a_S are in Table 4.1.**

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Table 4.4 Correction factors σ, ι and ρ for Table 4.1 as a function of winding dimensions
The values are for a 3-phase diamond winding in stator and a cage winding in rotor. For a 3-phase diamond winding in stator and a pole winding in rotor, the values are at relation of $a_R / \tau_p = 0$.

a_S / τ_p	a_R / τ_p	0.05	0.10	0.15	0.20	0.25
0.30	ρ	1.00	0.98	0.94	0.87	0.78
0.10	ρ	0.99	**0.96**	0.86	0.72	0.56

a_R / τ_p	s / τ_p	0.10	0.15	0.20	0.25	0.30
0.20	ι	0.88	0.94	1.00	1.06	1.12
0.10	ι	0.92	0.96	1.00	1.04	1.08
~0	ι	1.00	**1.00**	1.00	1.00	1.00

a_S / τ_p	W / τ_p	0.65	0.70	0.75	0.80	0.85	0.90	1.00
0.30	σ	0.89	0.92	0.95	0.97	0.98	0.99	1.00
0.20	σ	0.83	0.87	0.90	0.94	0.96	0.98	1.00
0.10	σ	0.71	0.78	0.83	0.89	0.93	0.96	1.00
0.00	σ	0.50	0.61	0.71	0.80	0.88	0.93	1.00

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