

Assignment 1: Hysteresis model and magnetostriction

1. Objective of the assignment

The aim of the assignment is to implement a hysteresis model in Matlab. The learning objective is to be able to apply knowledge from the literature to solve real life problems.

2. Theory

One of the most commonly used hysteresis model for soft magnetic materials is the so-called Jiles-Atherton hysteresis model. The model is based on the following equations for the magnetization \mathbf{M} and the magnetic field strength \mathbf{H} . In general, the model is written in vector form, but for this assignment, we will deal with the scalar case only.

Model equations

The magnetic field strength inside the material is computed from the applied magnetic field and the material magnetization as

$$H_e = H + \alpha M \quad (1)$$

where α is a model parameter. The effective field H_e can be understood as the field seen by the material, which is in general different from the applied field H . In this assignment, we will consider that the applied field and the effective field are equal and thus set $H_e=H$ in equation (1).

The total magnetization M in the model is given as the sum of contributions from the anhysteretic M_{an} and the irreversible M_{irev} magnetization:

$$M = cM_{an} + (1 - c)M_{irev} \quad (2)$$

where c is a material parameter.

The anhysteretic magnetization M_{an} can be described by any appropriate function, but in the JA-model the Langevin-function is used for this purpose

$$M_{an} = M_s \left(\coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right) \quad (3)$$

where a is a model parameter and M_s is the saturation magnetization, which is a material parameter.

In the JA-model, the irreversible magnetization is defined by a differential equation:

$$\frac{dM_{irev}}{dH_e} = \frac{M_{an} - M_{irev}}{k\delta - \alpha(M_{an} - M_{irev})} \quad (4)$$

where k is another model parameter and δ is equal to 1 if H increases and -1 if H decreases.

Note that the hysteresis model is rate-independent, which means that the response of the model is the same whatever is the frequency. In the following, we will use the frequency only for the generation of H-values and it does not affect the model. You can thus set the frequency to 1 Hz in the simulations below.

Solution of the differential equation

From equation (4) it is necessary to integrate the magnetization as function of the magnetic field strength. This kind of problem can be solved in different ways. One way to cope with it is to use the time as a variable and the Euler time-integration scheme to integrate the differential equation. The Euler method is described as follows:

We seek a discretized solution for the time evolution of the equation:

$$\begin{aligned}\frac{dy}{dt} &= g(t, y) \\ y(t_0) &= y_0\end{aligned}\tag{5}$$

The method consists of defining a suitable time step h and starting from the initial solution $y_0 = y(t_0)$, the discretized time at each step is given as:

$$t_{n+1} = t_n + h\tag{6}$$

Which results in the y -function value as:

$$y_{n+1} = y_n + hg(t_n, y_n)\tag{7}$$

The method is direct and does not need any iteration.

The Euler method can be applied to Eq. (4) if we replace h in Eq. (7) by:

$$\Delta H = H(t_{n+1}) - H(t_n)\tag{8}$$

Remember to compute

$$\delta = \text{sign}\left(\frac{dH}{dt}\right)\tag{9}$$

As it is needed in eq. (4).

3. Tasks

The specific tasks are as follows:

1. Make a Matlab function that returns the instantaneous value of a sinusoidal magnetic field strength with respect to time at each time step:

$$\text{function [H dH]=HH(f,t)}$$

Inside the function define $H = H_{\max} \sin(2\pi ft)$ and $dH = 2\pi fH_{\max} \cos(2\pi ft)$

2. Make the main program, where you set the model parameters (given in the table below) and run a time stepping for-loop that computes the values of the magnetization at each time step. Start the time stepping loop from $i=2$. Before the loop, compute the values for $i=1$, so that M_{rev} in (4) is equal to zero.
3. After the end of the for-loop compute the magnetic flux density from the equation $B = \mu_0(H + M)$ and plot the HB-curves.

4. Compute the magnetostriction from the equation below and plot it as function of B and as function of M :

$$\lambda_{ms} = \lambda_s \left(\frac{M}{M_s} \right)^2 \quad (10)$$

5. Draft a report with the plots and your own comments on the validity of the model, and attach the Matlab code to your report.

Table 1: Model parameters

Parameter	Value
a	512
c	0.75
k	570
M_s	$1.7e^6$
l_s	$10e^{-6}$
α	0.032
μ_0	$4\pi 10^{-7}$
H_{\max}	8000