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Theory of the magnetomechanical effect

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Abstract. This study investigated a model theory of the changes in magnetization that a ferromagnetic material undergoes when subjected to an applied uniaxial stress. The description of these effects is shown to be totally different from the description of the changes in the hysteresis curve under a series of constant applied stresses. The main mechanism in the proposed model theory is the unpinning of domain walls by the application of stress, which allows the walls to move and causes a change in the magnetization. This change in magnetization reduces the displacement from the anhysteretic magnetization. In addition, the anhysteretic magnetization itself is changed by the application of stress via the magnetoelastic coupling. It is shown that the effect can be described by an equation in which the rate of change of magnetization with elastic energy is proportional to the displacement of the magnetization from the anhysteretic magnetization. This is termed the 'law of approach'. This law seems to apply when the starting condition of the material is on a major hysteresis loop.

1. Introduction

In recent years there has been a resurgence of interest in an old problem that was never adequately explained. The magnetomechanical effect, that is the change of magnetization of a magnetic material resulting from the application of stress, has attracted attention because of its relevance to several technological problems, including the tendency of previous unmagnetized large structures to become magnetized when stressed in the presence of the earth's magnetic field, the use of magnetic materials in sensors, the tendency of magnetized materials to have their magnetization reduced after stressing and applications of magnetic methods to the non-destructive evaluation of stress in materials. In this paper a phenomenological theory is developed, which can explain previous observations and has been used to develop a predictive computer model for determining how a material behaves under a wide range of conditions of magnetic field and stress.

The original approach to the problem of the magnetomechanical effect was to assume that the process is reversible. In this approach it was argued that, since a magnetic material changes its length when it is magnetized, it is reasonable to expect that its magnetization will change when it is strained. This idea was discussed by Bozorth [1]. Cullity [2] even discussed these effects in terms of Le Chatelier's principle. For small reversible changes a thermodynamic relation does exist, namely

$$\left(\frac{\mathrm{d}\lambda}{\mathrm{d}H}\right)_\sigma = \left(\frac{\mathrm{d}B}{\mathrm{d}\sigma}\right)_H \qquad (1)$$

where $(\mathrm{d}\lambda/\mathrm{d}H)_\sigma$ is the rate of change of magnetostriction with magnetic field at constant stress and $(\mathrm{d}B/\mathrm{d}\sigma)_H$ is the change of magnetic induction with stress at constant field. (In the SI system, the units of B are kg A^{-1} s^{-2} and the units of σ are kg m^{-1} s^{-2}, so that $(\mathrm{d}B/\mathrm{d}\sigma)$ has units of m A^{-1}.) Since $B = \mu_0(H + M)$, and H is not a function of stress, this latter expression is equal to $\mu_0(\mathrm{d}M/\mathrm{d}\sigma)_H$, which is the derivative of interest in the magnetomechanical effect. Equation (1) merely shows that, for reversible processes, a large magnetomechanical effect $(\mathrm{d}B/\mathrm{d}\sigma)_H$ should be observed in materials with a large magnetostrictive strain derivative $(\mathrm{d}\lambda/\mathrm{d}H)_\sigma$.

In fact, the above equation is quite misleading as a description of the magnetomechanical effect in ferromagnetic materials because the magnetization process is hysteretic and therefore inherently irreversible in nature, although reversible changes in magnetization are superposed on the irreversible changes. Therefore, a description of the process must be intimately connected with a description of irreversibility and hysteresis.

Previous work on the development of model theories of the magnetization processes in ferromagnetic materials have concentrated on the description of hysteresis [3, 4] and the changes in hysteresis curves that result from constant applied stress [5]. The magnetomechanical effect, which is defined as the change in magnetization of a magnetic material resulting from a change in applied stress under a constant applied field, has been reported occasionally [6, 7], but the effects have appeared to be very complex.

For example, in the closely related works of Craik and Wood [8] and of Birss, Faunce and Isaac [9], the experimental results were obtained by applying stresses to various polycrystalline magnetic materials in the presence of a small constant magnetic field. It was noted that there were many features in the results that could not be reconciled with the previous theory of Brown [10]. In particular it was noticed that very clear differences between the effects caused by tension and compression were not interpretable in terms of the existing theory.

In Brown's theory it was assumed that the change in magnetization due to domain wall motion obeyed Rayleigh's law [11] at low magnetizations. From this a theoretical curve of magnetization versus stress was derived, based on the idea that both magnetic field and stress are thermodynamically equivalent to a pressure on the magnetic domain walls. In this approach, stress was treated as equivalent to a magnetic field, whereas in fact the effect of stress is actually equivalent to the imposition of an additional anisotropy energy. This approach has some fundamental problems. Stress is a tensor and magnetic field is a vector, and therefore they have different symmetries. However, even if this limitation is overlooked, a significant problem of the theory developed by Brown was that exactly the same changes in magnetization were predicted under both tension and compression.

This was explained by considering equal densities of two types of 90° domain walls: those for which coaxial field and stress cause motion in the same direction as the field and those for which the motion is in the opposite direction. According to conventional understanding, applied stress affects only non-180° domain walls (that is, 90° domain walls in iron and steels). Consequently, changing the sign of the applied stress merely reverses the roles of the two types of 90° domain walls, leading to equivalent changes in magnetization under tension and compression. This does not occur in practice. Even at quite low fields of 27 A m^{-1} (0.366 Oe), a region in which close agreement with Brown's theory would be expected, the data of Lliboutry showed clear differences between tension and compression. The data reported by Brugel and Rimet [12] showed only the effects of tension, and therefore this disagreement between theory and observation did not emerge clearly from their work.

Craik and Wood [8], Birss [12], Schneider and Charlesworth [14] and Finbow [15] have also mentioned the prediction that the changes in magnetization should be independent of the sign of the stress (that is, symmetric with stress). This prediction was shown to be contrary to the experimental results presented in these papers. The 'wall pressure' theory developed by Brown, and later by Brugel and Rimet, also predicted that the magnetization should remain constant as the stress was reduced from its maximum amplitude. This was termed the 'horizontal fly-back' by Birss [9]. This prediction is also known to be at variance with experimental observations, as shown by Schneider and Richardson (figure 5 of [16]) and Schneider and Semcken (figure 4 of [17]), as well as in the results of Craik and Wood [8], Birss *et al* [9] and Jiles and Atherton [18].

Therefore an improved model theory is needed, which can take into account these differences under tension and compression. The very thorough work conducted on the magnetomechanical effect by Craik and Wood [8] on a number of specimens, including nickel, which has a negative magnetostriction, concluded with the statement that 'the results caused by stress cannot be reconciled with any theory based simply on the movement of existing domain walls ... it seems fairly certain that discontinuous changes in domain structure occur under stress ... and any theory of magnetization under stress must take them into account'.

Birss, Faunce and Isaac [9] also observed that, in general, the dependence of magnetization on stress was asymmetric with respect to tension and compression. They concluded that theories of stress-induced pressure on 90° domain walls and large-scale changes in domain structure due to stress were insufficient to account for the observed results. In a later discussion, Birss [13] gave some important insights into the main mechanisms of the process. These were reduced to three main processes: (i) stress-induced pressure on 90° domain walls, which leads to domain wall motion, (ii) changes in the domain wall pinning energies and (iii) irreversible changes in domain structure, caused by stress-induced preferential occupancy of one of the three orthogonal easy axes of magnetization. As indicated by Birss, only the first of these processes can be described by the theory of Brown.

Although the magnetomechanical effect is now receiving increased attention as a subject for scientific study, as shown by the recent work of Pitman [19], Ruuskanen and Kettunen [20], Schneider, Cannell and Watts [21], Maylin and Squire [22, 23], Makar and Atherton [24, 25] and Jiles and Devine [26, 27], the most comprehensive sets of published data still remain those of Craik and Wood [8] and Birss *et al* [9]. Despite the time that has elapsed since these results were first published, there has been no adequate explanation of the form of the curves. Recent work has therefore concentrated on empirical observations of these effects in different materials. In this paper an explanation of these earlier results is presented, based on the proposed model.

2. The law of approach to the anhysteretic magnetization

Following the observation by Bozorth and Williams that the magnetization curve of permalloy obtained after application of a magnetic field, and subsequently a stress of 39 MPa (4 kg mm^{-2}) was 'as closely as it was possible to tell, identical to the anhysteretic magnetization curve', Jiles and Atherton [18] suggested that the main effect on the magnetization of a magnetic material caused by cycling the applied stress was an irreversible change in the prevailing magnetization towards the anhysteretic magnetization. No quantitative theory was given in the paper, however, other than a brief suggestion that the change in magnetic induction might be proportional to the displacement of the initial magnetic induction from the anhysteretic magnetic induction.

The concept of the law of approach was tested by Pitman [19] and later by Maylin and Squire [22, 23].

The work of Pitman investigated the departure of the magnetization from a major or symmetric hysteresis loop as a result of the application of compressive stress. This work was unique in that it tested the derivative $(\mathrm{d}B/\mathrm{d}\sigma)_H$ at three identical field strengths but different magnetic inductions. These were at 80 A m^{-1} close to the positive remanence, at 80 A m^{-1} close to negative remanence and at 80 A m^{-1} on the initial magnetization curve close to the demagnetized state. These results, according to Pitman, seemed to confirm the law of approach suggested by Jiles and Atherton. The results from close to positive and negative remanence were approximate mirror images of each other, while the amplitude of the change in magnetization was found to be much reduced when the initial magnetization was close to the anhysteretic state.

The results of Maylin and Squire substantiated these results for locations beginning from the major loop. However, according to these authors, for excursions beginning on a minor (asymmetric) loop, the law of approach, were it operative, did not seem to pertain to the principal anhysteretic magnetization. Therefore, Maylin and Squire concluded that, under the action of stress while on a minor hysteresis loop, the magnetization changed so that it approached an equilibrium value, which did not coincide with the principal anhysteretic magnetization.

There are probably three factors that determine the magnitude and sign of the magnetomechanical coefficient $(\mathrm{d}B/\mathrm{d}\sigma)_H$. These are: (i) how far the magnetization is above or below the anhysteretic (the displacement), (ii) how sensitive this displacement is to stress (the rate of approach) and (iii) how the anhysteretic changes with stress. The analysis begins with the last of these because it is the simplest to discuss theoretically.

3. The stress-dependence of the anhysteretic

As described in previous work [28], an applied uniaxial stress σ acts in some respects like an applied magnetic field operating through the magnetostriction λ. This additional 'field' H_σ can be described by considering the energy A of the system along the reversible anhysteretic magnetization curve, namely

$$A = \mu_0 HM + \frac{\mu_0}{2}\alpha M^2 + \frac{3}{2}\sigma\lambda + TS \tag{2}$$

where T is temperature, S is entropy and $\mu_0\alpha M^2/2$ is the self-coupling energy. The dimensionless term α has been defined previously [3] and represents the strength of the coupling of the individual magnetic moments to the magnetization M. The effective magnetic field causes a change in magnetization, and therefore is determined by the derivative of this energy with respect to magnetization M. The derivative of entropy with respect to bulk magnetization M in a ferromagnet will be negligible in the cases under consideration because the fields applied here do not increase the ordering within the domain, although they do lead to a change in the bulk magnetization M. Therefore the effective field is given by

$$\begin{aligned} H_{\text{eff}} &= \frac{1}{\mu_0}\frac{\mathrm{d}A}{\mathrm{d}M} \\ &= H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\frac{\mathrm{d}\lambda}{\mathrm{d}M}. \end{aligned} \tag{3}$$

This means that a correction needs to be made to the anhysteretic magnetization as a result of the application of stress. Surprisingly, this is sufficient to correct the magnetic properties for the effects of a constant applied stress.

In cases in which the applied stress σ_0 is not co-axial with the direction along which λ and M are measured, the stress σ used in equation (3) is simply the component of applied stress along this direction. For isotropic materials this is given by

$$\sigma = \sigma_0(\cos^2\theta - \upsilon\sin^2\theta) \tag{4}$$

where θ is the angle between the axis of the applied stress σ_0 and the axis of the magnetic field H and υ is Poisson's ratio. Consequently H_σ, the component of the effective field due to stress, is

$$H_\sigma = \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}M}\right)_\sigma = \frac{3}{2}\frac{\sigma_0}{\mu_0}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}M}\right)_\sigma (\cos^2\theta - \upsilon\sin^2\theta). \tag{5}$$

Therefore, if the magnetostriction λ can be described as a function of magnetization and stress, then H_σ can be determined. The anhysteretic magnetization at field H and stress σ is identical to the anhysteretic at field $H + H_\sigma$ and zero stress, that is

$$\begin{aligned} M_{\text{an}}(H,\sigma) &= M_{\text{an}}(H + \alpha M + H_\sigma, 0) \\ &= M_{\text{an}}\left[H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}M}\right)_\sigma, 0\right] \end{aligned} \tag{6}$$

where the effects of stress have been incorporated into the equivalent effective field. It is therefore implicit in this description of the theory that the anhysteretic magnetization under field H and stress σ, is identical to the anhysteretic magnetization under an equivalent effective magnetic field

$$H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}M}\right)_\sigma.$$

In other words, the change in energy of the magnetization in a particular direction can be described either in terms of the stress or, equivalently, in terms of the effective magnetic field that causes the same change in energy.

This requires a description of the bulk magnetostriction, which depends on the domain configuration throughout the material. Theoretically, if a certain domain configuration were assumed, then this relationship could be determined via the known magnetostriction coefficients λ_{100} and λ_{111}. However, in practice this domain configuration in a material cannot be known in advance. It is therefore necessary to develop an empirical model to describe the relation between bulk magnetostriction and bulk magnetization. Since the magnetostriction must be symmetric about $M = 0$, a simple series expansion gives

$$\lambda = \sum_{i=0}^{\infty} \gamma_i M^{2i}. \tag{7}$$

A reasonable first approximation to the magnetostriction of iron can be obtained by including the terms up to $i = 2$. Ignoring the constant term, which is simply the elastic strain and does not play an active role in the magnetomechanical effect, this gives

$$\lambda = \gamma_1 M^2 + \gamma_2 M^4. \tag{8}$$

In a material such as iron, in which $\gamma_1 > 0$ and $\gamma_2 < 0$, this gives a reversal of the sign of the magnetostriction at $M = (-\gamma_1/\gamma_2)^{1/2}$ and a reversal of the slope of the magnetostriction curve at $M = [-\gamma_1/(2\gamma_2)]^{1/2}$. This latter change in slope leads to the well-known Villari effect [29] in the magnetization curves of iron under different levels of stress, in which the magnetization curves under co-axial field and tension lie above the unstressed magnetization curves at low field strengths, but below them at high field strengths. The converse effect is observed under co-axial field and compression.

A more sophisticated approach to describing the magnetostriction curve, which includes hysteresis, has been given by Sablik and Jiles [30], but that approach will not be utilized in the present calculations. Improvements to the description of the magnetostriction as a function of magnetization can also be achieved by the inclusion of higher order terms in equation (8).

4. The stress-dependence of magnetostriction

The stress-dependence of the magnetostriction curve $\lambda(M, \sigma)$ can be described in terms of the stress dependence of γ_1 and γ_2 using a Taylor series expansion,

$$\gamma_i(\sigma) = \gamma_i(0) + \sum_{n=1}^{\infty} \frac{\sigma^n}{n!} \gamma_i^n(0) \tag{9}$$

where $\gamma_i^n(0)$ is the nth derivative of γ_i with respect to stress at $\sigma = 0$. Using only the terms up to $n = 1$, and applying the above equation to the magnetostriction data of Kuruzar and Cullity [31], gave $\gamma_1(0) = 7 \times 10^{-18}$ A^{-2} m^2, $\gamma_1'(0) = -1 \times 10^{-25}$ A^{-2} m^2 Pa^{-1}, $\gamma_2(0) = -3.3 \times 10^{-30}$ A^{-4} m^2 and $\gamma_2'(0) = 2.1 \times 10^{-38}$ A^{-4} m^4 Pa^{-1}. The magnetostriction is then given by

$$\lambda = \sum_{i=0}^{\infty} \gamma_i(\sigma) M^{2i} \tag{10}$$

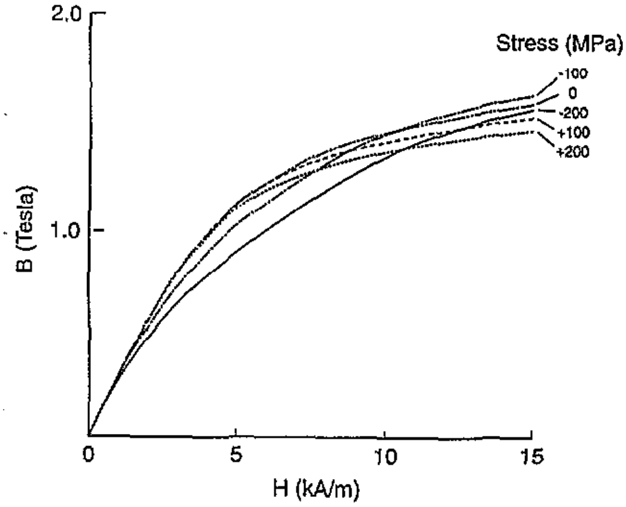

Figure 1. The measured variation in the anhysteretic magnetization with stress, as reported by Jiles and Atherton [17].

and the resulting effective field is obtained by substituting this into equation (3),

$$H_{\text{eff}} = H + \alpha M + \frac{3\sigma}{\mu_0} \sum_{i=0}^{\infty} i \gamma_i(\sigma) M^{2i-1} \tag{11}$$

$$= H + \alpha M + \frac{3\sigma}{\mu_0} \sum_{i=0}^{\infty} \left(i M^{2i-1} \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_i^n(0) \right). \tag{12}$$

In the isotropic limit, the stress-dependence of the anhysteretic magnetization curve can be determined from the equation

$$M_{\text{an}}(H, \sigma) = M_s \left[\coth\left(\frac{H + H_\sigma + \alpha M}{a} \right) - \frac{a}{H + H_\sigma + \alpha M} \right] \tag{13}$$

where $a = k_B T / \mu_0 M$ [3]. Stress-dependent anhysteretic magnetization curves from the measurement data of Jiles and Atherton [18] are shown in figure 1. An important point to note is that the anhysteretic curves at various stress levels cross at different points. This is a direct result of the stress-dependent magnetostriction curve of iron $\lambda(M, \sigma)$, which leads to a stress-dependence of the magnetization at which the sign of the differential magnetostriction changes ($\mathrm{d}\lambda/\mathrm{d}M = 0$). Calculations using a stress-independent magnetostriction curve (that is, one with $\gamma_1'(0) = 0$ and $\gamma_2'(0) = 0$) have shown that all anhysteretics cross at the same location on the M–H plane. The predictions of the present model equation for the stress-dependent anhysteretic are shown in figure 2 for selected values of the model parameters.

5. The stress-dependence of the magnetization

The effect of changing stress on the magnetization of a magnetic material leads to behaviour in which the magnetization has been observed to increase or

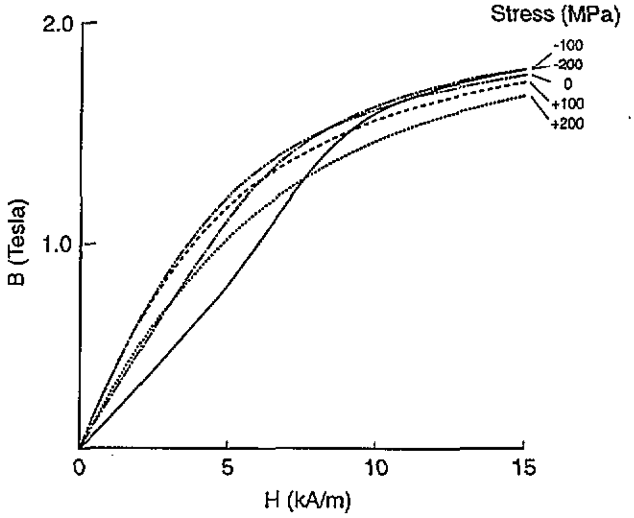

Figure 2. The modelled variation in the anhysteretic magnetization curve for various levels of stress using equations (12) and (13) together with the following values of the coefficients: $M_s = 1.7 \times 10^6$ A m^{-1}, $a = 1000$ A m^{-1}, $k = 1000$ A m^{-1}, $\alpha = 0.001$, $c = 0.1$, $\gamma_1 = 4 \times 10^{-18} - (2 \times 10^{-26})\sigma$ A^{-2} m^2 and $\gamma_2 = 2 \times 10^{-30} - (5 \times 10^{-39})\sigma$ A^{-4} m^4.

decrease under exposure to the same stress under the same external applied field. This indicates that the phenomenon is dependent on more than simply the external influences of stress σ and magnetic field H. In fact, the behaviour depends on the magnetization history of the specimen, which for major (that is, symmetric) hysteresis loops can be expressed in terms of the displacement from the anhysteretic $M_{an} - M$. This, together with the field H and stress σ, specifies the state of the material on a major hysteresis loop.

Given these conditions, it has been found in previous studies [8, 9, 18] that the direction of the change in magnetization with applied stress is independent of the sign of the stress for small stresses when the magnetization is sufficiently distant from the anhysteretic. This means that the direction of change is not directly dependent on the stress, but rather on some other related quantity, which is independent of the sign of the stress. A reasonable next hypothesis is to consider the elastic energy per unit volume W supplied to the material by the changing applied stress. This depends on the square of the stress:

$$W = \sigma^2/(2E) \tag{14}$$

where E is the relevant elastic modulus. It may reasonably be anticipated that some of this elastic energy causes unpinning of domain walls.

We now have two factors to consider: the displacement of the prevailing magnetization from the anhysteretic magnetization and the change in elastic energy. A law of approach to the anhysteretic state, in which the rate of change of magnetization with elastic energy is proportional to the displacement of the magnetization from the anhysteretic, can be used to explain the magnetomechanical effect. It will be shown in the subsequent development that this law, with suitable generalization to account for the stress-dependence of the anhysteretic magnetization, describes both quantitatively and qualitatively the behaviour of magnetization under stress.

6. The reversible component of magnetization

In previous work [4] it has been shown that the reversible component of magnetization M_{rev} is given by

$$M_{rev} = c(M_{an} - M_{irr}) \tag{15}$$

where M_{an} is the anhysteretic magnetization and M_{irr} is the irreversible magnetization, which is achieved when all domain walls are returned to their planar condition and all reversible rotations of domain magnetizations are relaxed back to zero. The coefficient c, which has been defined previously [4], describes the flexibility of the magnetic domain walls. This equation can then be differentiated with respect to the elastic energy W supplied to the material as a result of applied stress:

$$\frac{dM_{rev}}{dW} = c\left(\frac{dM_{an}}{dW} - \frac{dM_{irr}}{dW}\right). \tag{16}$$

7. The irreversible component of magnetization

Returning to the observation by Craik and Wood that 'discontinuous changes in domain structure occur under stress, and any theory of magnetization under stress must take them into account', we need to develop a model theory for irreversible changes in magnetization. The proposition that we examine here is the law of approach as applied to the irreversible component of magnetization. This law can be expressed as

$$\frac{dM_{irr}}{dW} = \frac{1}{\xi}(M_{an} - M_{irr}) \tag{17}$$

where ξ is a coefficient with dimensions of energy per unit volume, which relates the derivative of irreversible magnetization with respect to elastic energy to the displacement of the irreversible magnetization from the anhysteretic magnetization. The derivative of the total magnetization with respect to the elastic energy is then obtained by summing the irreversible and reversible components from equations (16) and (17):

$$\frac{dM}{dW} = \frac{1}{\xi}(M_{an} - M_{irr}) + c\frac{d}{dW}(M_{an} - M_{irr}) \tag{18}$$

$$= \frac{(1-c)}{\xi}(M_{an} - M_{irr}) + c\frac{dM_{an}}{dW}. \tag{19}$$

This last equation can be transformed into a derivative with respect to stress σ. From equation (14) the differential of the elastic energy dW is given by

$$dW = \left(\frac{\sigma}{E}\right) d\sigma \tag{20}$$

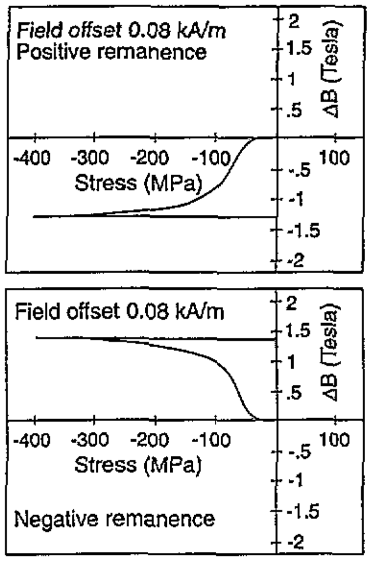

Figure 3. The variation in magnetic induction B with compressive applied stress under an applied field of $H = 80\ \mathrm{A\ m^{-1}}$ after Pitman [19]: (*a*) above the anhysteretic and (*b*) below the anhysteretic.

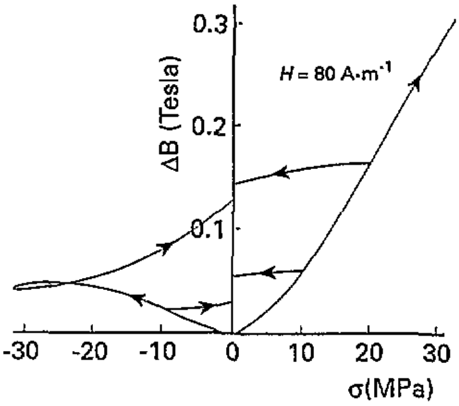

Figure 4. The variation in magnetic induction B with stress for a specimen of mild steel, after Craik and Wood [8]. At low stress amplitudes the change in magnetization with stress has the same sign, irrespective of whether the stress is compressive or tensile. This indicates that $M_{an}(H,\sigma) - M(H,0)$ dominates the process at low stress. At compressive stresses of magnitude exceeding −30 MPa the stress derivative $\mathrm{d}B/\mathrm{d}\sigma$ changes sign, indicating that the magnetization has crossed the anhysteretic.

and therefore equation (19) becomes

$$\frac{\mathrm{d}M}{\mathrm{d}\sigma} = \frac{1}{\epsilon^2}\sigma(1-c)(M_{an} - M_{irr}) + \frac{\mathrm{d}M_{an}}{\mathrm{d}\sigma} \tag{21}$$

where $\epsilon = (E\xi)^{1/2}$ is a coefficient that has dimensions of stress.

Alternatively, using equation (15) and the expression $M = M_{irr} + M_{rev}$, equation (19) can be shown to be equivalent to

$$\frac{\mathrm{d}M}{\mathrm{d}W} = \frac{1}{\xi}(M_{an} - M) + c\frac{\mathrm{d}M_{an}}{\mathrm{d}W} \tag{22}$$

which conveniently expresses the law in terms of the directly measurable quantities M and M_{an}. Solutions of this equation can be obtained under a variety of conditions of applied stress and magnetic field. The changes in magnetic induction B can then be determined by substituting $B = \mu_0(H+M)$ and $B_{an} = \mu_0(H+M_{an})$ in equation (22).

8. Results of previous investigations

Experimental results of Pitman [19] are shown in figure 3. These exhibit the principal feature of interest which is the ΔB versus $\Delta\sigma$ locus under compression. Positive or negative changes in B were observed with the same compressive stress, depending on whether the magnetization began well below, or well above, the anhysteretic.

The results of Craik and Wood, although they did not show that the sign of the change in magnetization could be the same under apparently identical external conditions, were more diverse in other respects than those of Pitman. In particular, their results showed the essential asymmetry of the dependence of magnetization on stress at higher stress levels, depending on whether tension or compression was applied. A representative example is shown in figure 4 (which is taken from figure 5 of [8]). At small stress amplitudes of up to about ±5 MPa, the change in magnetization with stress was almost symmetric so that the result did not depend on whether the stress was tensile or compressive. Even up to ±20 MPa, the sign of the change was positive under both tension and compression. However, beyond ±30 MPa, the derivative of magnetization with stress was negative under compression but positive under tension. A vast range of different behaviour of magnetization under stress was reported by Craik and Wood on different materials, all showing asymmetry under tension or compression, and in which the amplitude of the changes was dependent on the strength of the applied field. Some of these are shown in figure 5. However, because Craik and Wood did not measure the anhysteretic magnetization, the significance of the observed changes was not apparent.

In the work of Birss *et al* it was also found that, for small changes in magnetization, the magnetization–stress curves were symmetric with respect to stress, as shown in figure 6. For larger changes in magnetization, Birss *et al* reported similar findings to Craik and Wood, namely a change in sign of the stress derivative of the magnetic induction $\mathrm{d}B/\mathrm{d}\sigma$ in iron and steels under compression, leading to an asymmetry in the response between tension and compression.

9. Results of model calculations

The results of model calculations using equation (22) are shown subsequently. In figures 7 and 8, calculations have been made using parameters that describe the material used by Pitman. The similarity between these theoretical predictions and the experimental

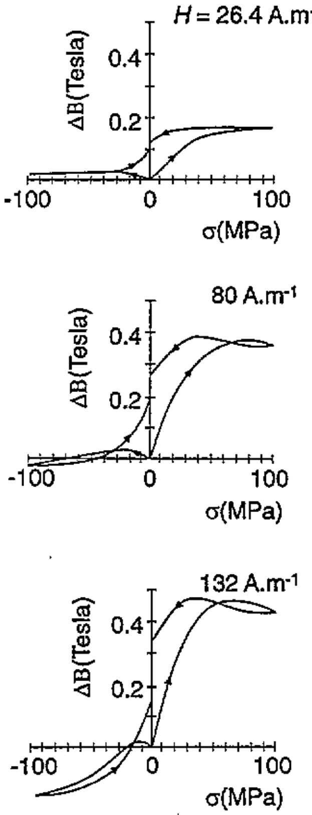

Figure 5. The variation in magnetic induction B with stress for mild steel as reported by Craik and Wood [8] at field strengths of 26, 80 and 132 A m^{-1}.

measurements can be seen by comparing the results with figure 3. The values of the measured and modelled changes in magnetic induction at the maximum stress (ΔB_{max}) and at remanence when the stress has been reduced to zero (ΔB_{rem}) are compared in table 1. These results show good agreement between calculation and measurement both in terms of the shapes of the curves and in terms of the numerical values.

The results show that the model provides theoretical justification for the differences in sign of $dB/d\sigma$ that have been observed by others in the same material under identical external conditions of stress and magnetic field [19]. The reason for the differences in behaviour under apparently identical conditions arises because of differences in the magnetic field exposure of the material giving it a different 'magnetic history' under the same external conditions.

The calculated changes in magnetic induction at three different field strengths under conditions similar to those investigated experimentally by Craik and Wood in mild steel [8] are shown in figure 9. The results show an increasing amplitude of the magnetomechanical effect as the field was increased from 26 to 132 A m^{-1} along the initial magnetization curve. The looping behaviour under tension became more pronounced as the field amplitude was increased. This is in agreement with the experimental observations in figure 4. Furthermore, under compression the amplitude of the magnetomechanical effect was found to be much reduced, with at first an increase, but then a pronounced decrease in magnetic induction as the compressive stress was increased. Although the expected decrease in

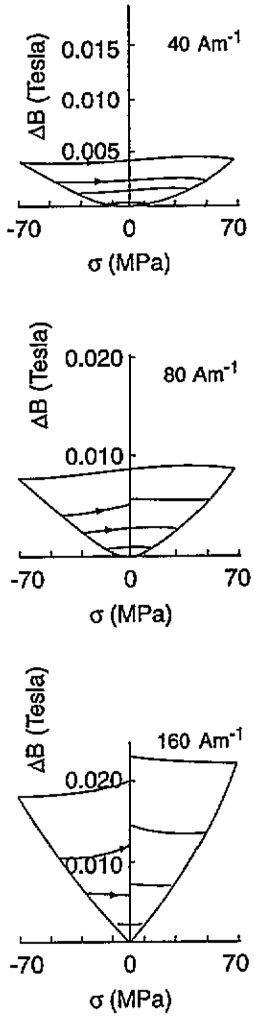

Figure 6. The variation in magnetic induction B with stress for Fe–0.2 wt% C as reported by Birss *et al* [9] at field strengths of 40, 80 and 160 A m^{-1}.

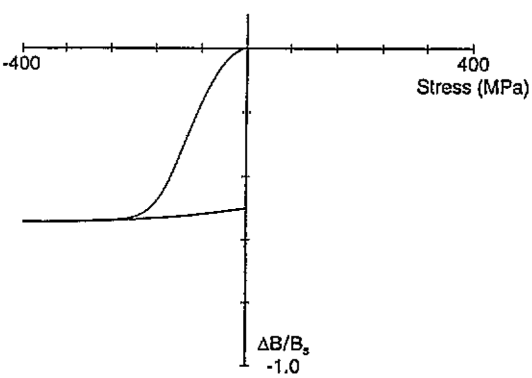

Figure 7. The calculated variation in magnetic induction B with stress at a field of 80 A m^{-1} under conditions similar to those employed by Pitman [19]. The specimen was first magnetized by applying a field of 40 kA m^{-1} and the field was subsequently reduced to 80 A m^{-1}. The specimen was then subjected to stress of up to 400 MPa. Values of the model parameters were $M_s = 1.71 \times 10^6$ A m^{-1}, $a = 955$ A m^{-1}, $k = 2015$ A m^{-1}, $\alpha = 0.8 \times 10^{-3}$, $c = 0.099$, $\gamma_{11} = 2 \times 10^{-18}$ A^{-2} m^2, $\gamma_{12} = 1 \times 10^{-26}$ A^{-2} m^2 Pa^{-1}, $\gamma_{21} = 1 \times 10^{-30}$ A^{-4} m^4, $\gamma_{22} = 5 \times 10^{-39}$ A^{-4} m^4 Pa^{-1}, $\epsilon = 0.7 \times 10^8$ Pa and $\xi = 24.5 \times 10^3$ Pa.

magnetic induction was shown at larger stresses, the

Table 1. A comparison of measured and modelled changes in magnetic induction with stress under various conditions.

Reference	H (A m^{-1})	σ_{max} (MPa)	ΔB_{max} (T) Measured	ΔB_{max} (T) Model	ΔB_{rem} (T) Measured	ΔB_{rem} (T) Model
Pitman [19]	80	−400	−1.25	−1.27	−1.20	−1.17
			1.30	1.24	1.25	1.35
Craik and Wood [8]	26	98	0.16	0.09	0.12	0.07
		−98	0.02	0.02	0.10	0.06
	80	98	0.37	0.25	0.27	0.20
		98	−0.02	0.07	0.20	0.19
	132	98	0.43	0.36	0.34	0.32
		−98	−0.16	0.11	0.15	0.31
Birss *et al* [9]	40	69	0.005	0.006		
		−69	0.005	0.005		
	80	69	0.01	0.012		
		−69	0.01	0.009		
	160	69	0.024	0.024		
		−69	0.020	0.018		
Jiles and Atherton [18]	320	140	0.007	0.009		
	960	140	0.018	0.021		
	1600	140	0.031	0.027		
	3200	140	0.036	0.029		

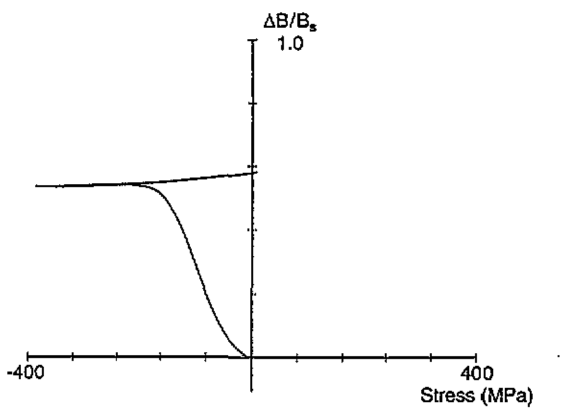

Figure 8. The calculated variation of magnetic induction B with stress at a field of 80 A m^{-1} under conditions similar to those employed by Pitman [19]. The specimen was first magnetized by applying a field of −40 kA m^{-1} and the field was subsequently increased 80 A m^{-1}. The specimen was then subjected to stress of up to 400 MPa. The values of the model parameters were $M_s = 1.71 \times 10^6$ A m^{-1}, $a = 955$ A m^{-1}, $k = 2015$ A m^{-1}, $\alpha = 0.8 \times 10^{-3}$, $c = 0.099$, $\gamma_{11} = 2 \times 10^{-18}$ A^{-2} m^2, $\gamma_{12} = 1 \times 10^{-26}$ A^{-2} m^2 Pa^{-1}, $\gamma_{21} = 1 \times 10^{-30}$ A^{-4} m^4, $\gamma_{22} = 5 \times 10^{-39}$ A^{-4} m^4 Pa^{-1}, $\epsilon = 0.7 \times 10^8$ Pa and $\xi = 24.5 \times 10^3$ Pa.

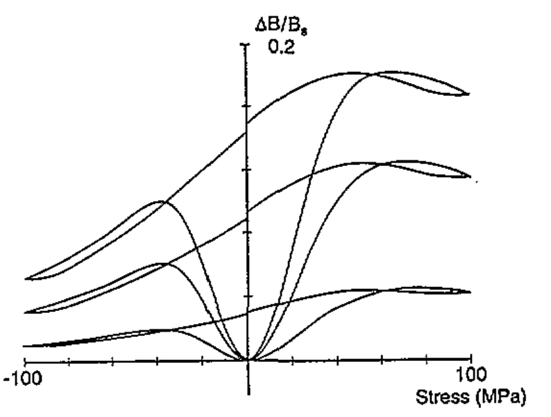

Figure 9. The calculated variation of magnetic induction B with stress at fields of 26, 80 and 132 A m^{-1} under conditions similar to those employed by Craik and Wood [8]. The specimen was demagnetized and then subjected to a field of the given magnitude. It was then subjected to an applied stress of up to 100 MPa, either in tension or compression. The values of the model parameters were $M_s = 1.71 \times 10^6$ A m^{-1}, $a = 900$ A m^{-1}, $k = 2000$ A m^{-1}, $\alpha = 1.1 \times 10^{-3}$, $c = 0.1$, $\gamma_{11} = 2 \times 10^{-18}$ A^{-2} m^2, $\gamma_{12} = 1.5 \times 10^{-26}$ A^{-2} m^2 Pa^{-1}, $\gamma_{21} = 2 \times 10^{-30}$ A^{-4} m^4, $\gamma_{22} = 5 \times 10^{-39}$ A^{-4} m^4 Pa^{-1}, $\epsilon = 1.1 \times 10^7$ Pa and $\xi = 605$ Pa.

actual values of ΔB did not become negative as had been observed by Craik and Wood at higher field amplitudes. A comparison of the values of ΔB_{max} and ΔB_{rem} is given in table 1, showing again good quantitative agreement in most cases.

These results give the first theoretical explanation for the changes in sign of $dB/d\sigma$, which have been observed, as stress is increased monotonically on some materials. This phenomenon has been widely observed in some iron alloys under compressive stress. The reason for this is that while the applied stress causes the prevailing magnetization to approach the anhysteretic magnetization, it also changes the value of the anhysteretic. Therefore, as stress is continually increased, the anhysteretic magnetization can actually cross the prevailing magnetization with a resultant change in sign of $dB/d\sigma$ as the stress increases further. A specific example occurs in materials with positive $d\lambda/dM$ when they are subjected to increasing

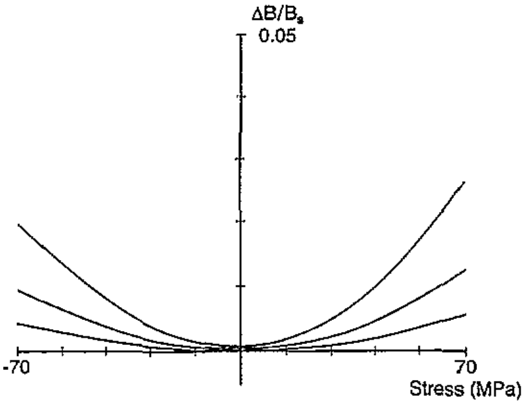

Figure 10. The calculated variation in magnetic induction B with stress at fields of 40, 80 and 160 A m^{-1} under conditions similar to those employed by Birss *et al* [9]. The specimen was demagnetized and then subjected to a field of the given magnitude. It was then subjected to an applied stress of up to 70 MPa, either in tension or compression. The values of the model parameters were $M_s = 1.71 \times 10^6$ A m^{-1}, $a = 1000$ A m^{-1}, $k = 2000$ A m^{-1}, $\alpha = 1 \times 10^{-3}$, $c = 0.1$, $\gamma_{11} = 4 \times 10^{-18}$ A^{-2} m^2, $\gamma_{12} = 3 \times 10^{-26}$ A^{-2} m^2 Pa^{-1}, $\gamma_{21} = 2 \times 10^{-30}$ A^{-4} m^4, $\gamma_{22} = 5 \times 10^{-39}$ A^{-4} m^4 Pa^{-1}, $\epsilon = 1.1 \times 10^8$ Pa and $\xi = 60.5 \times 10^3$ Pa.

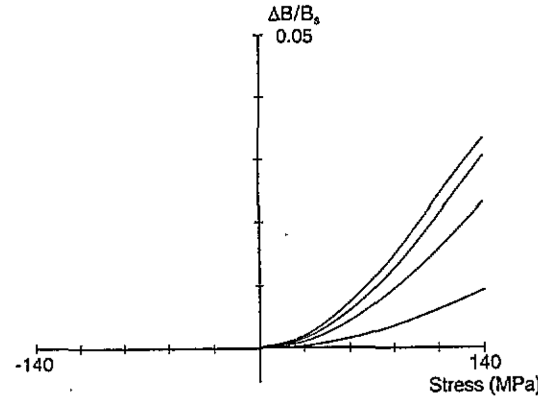

Figure 11. The calculated variation in magnetic induction with stress at fields of 0.32, 0.96, 1.6 and 3.2 kA m^{-1} under conditions similar to those employed by Jiles and Atherton [18]. The specimen was demagnetized and then subjected to a field of the given magnitude. It was then subjected to an applied stress of up to 140 MPa in tension. The values of the model parameters were $M_s = 1.67 \times 10^6$ A m^{-1}, $a = 5000$ A m^{-1}, $k = 1300$ A m^{-1}, $\alpha = 1 \times 10^{-3}$, $c = 0.1$, $\gamma_{11} = 1 \times 10^{-18}$ A^{-2} m^2, $\gamma_{12} = 3 \times 10^{-26}$ A^{-2} m^2 Pa^{-1}, $\gamma_{21} = 1 \times 10^{-30}$ A^{-4} m^4, $\gamma_{22} = 5 \times 10^{-39}$ A^{-4} m^4 Pa^{-1}, $\epsilon = 1.8 \times 10^8$ Pa and $\xi = 162 \times 10^3$ Pa.

compressive stress.

The calculated changes in magnetic induction for values of parameters close to those of Birss *et al* [9] are shown in figure 10. In these cases the starting value of the magnetic induction was along the initial magnetization curve far from the anhysteretic. Therefore, the dependence of magnetic induction on stress according to the model is approximately quadratic under these conditions, with the rate of change dependent on the applied field strength. The form of the modelled curves is very similar to that observed by Birss *et al* and the numerical values of ΔB_{max} and ΔB_{rem}, as shown in table 1, are also in good agreement.

The calculated change in magnetic induction with stress under conditions similar to those investigated by Jiles and Atherton [18] in high-strength steel is shown in figure 11. These calculations show a monotonic increase in the maximum change in induction ΔB_{max} at 140 MPa under field strengths of 0.32, 0.96, 1.6 and 3.2 kA m^{-1}. It can be seen that the increment in ΔB_{max} began to decline at the higher field strength (that is, ΔB_{max} (3.2 kA m^{-1})$-\Delta B_{max}$ (1.6 kA m^{-1}) was smaller than ΔB_{max} (1.6 kA m^{-1})$-\Delta B_{max}$(0.96 kA m^{-1}). This is in agreement with experimental observations. A comparison of the numerical values is also given in table 1, which again shows good quantitative agreement between the calculations and experimental observations.

10. Conclusions

The model theory described in this paper has been developed to explain the apparently disparate range of observations of the magnetomechanical effect that have been reported. The equations have been derived based on the concept that, under a changing applied stress at constant magnetic field, the magnetization changes so that it approaches the anhysteretic magnetization. This concept has been developed to include a quantitative description of stress-dependent magnetostriction and anhysteretic magnetization curves, and the mechanism by which the change in elastic energy supplied to the material causes a reduction in the displacement of the magnetization from the anhysteretic magnetization.

The underlying equation describing the phenomenon has been derived (equation (22)), and this provides a description, not only of the reduction in displacement of the magnetization from the anhysteretic, but also of the asymmetry in response under tension or compression, which occurs under certain circumstances as a result of the stress-dependence of the anhysteretic magnetization. Furthermore, the change in sign of $\mathrm{d}B/\mathrm{d}\sigma$ reported by earlier investigators is explained by the theory. As a result, some of the apparently very complex dependence of magnetization on stress that has been reported previously can be seen to be the result of this law applied under a variety of conditions.

If the magnetization approaches the anhysteretic magnetization as a result of the application of stress, then, for small stress amplitudes, it may be expected that the size of the change will be the same, independent of whether the stress is compressive or tensile, because the anhysteretic magnetization will lie initially above or below the magnetization (assuming that these are not by chance identical), and the derivative $\mathrm{d}B/\mathrm{d}\sigma$ will be determined principally by the displacement $M_{an} - M$.

The anhysteretic magnetization itself is stress-dependent, and in this case the effective field H_σ does depend on the sign of the stress. This means that, at any point on the anhysteretic curve, if the anhysteretic magnetization increases with tension, it will necessarily decrease with compression, and vice versa.

The anhysteretic magnetization represents a reversible magnetization state, and therefore equation (1) applies to it.

Since the anhysteretic magnetization is stress-sensitive in this way, when larger amplitude stresses are applied the displacement $M_{\mathrm{an}} - M_{\mathrm{irr}}$ will decrease or increase depending on the sign of the stress σ and the derivative $(\mathrm{d}\lambda/\mathrm{d}M)$. For sufficiently large stress amplitudes the difference $M_{\mathrm{an}} - M_{\mathrm{irr}}$ can even change sign as the stress is increased. This can lead to a change in sign of $\mathrm{d}M/\mathrm{d}\sigma$, which explains some of the behaviour observed in iron and steels under compressive stress, in which the magnetization at first increases and then decreases with monotonically increasing stress.

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