**Problem set 1**

Due March 8, 10:15 AM



# Key concepts from quantum mechanics (Lecture 1 )

(1) Show the following properties of Hermitian operators, where $\alpha \in \mathbb{C}$

(a) $(A + \alpha B)^\dagger = A + \alpha^* B$

(b) $(AB)^\dagger = B^\dagger A^\dagger$

(2) For any two operators $A$, $B$ the above result also holds. Show that $(ABCD)^\dagger = D^\dagger C^\dagger B^\dagger A^\dagger$.

(3) Show that for any three operators $A$, $B$, $C$, the Jacobi identity holds $[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$.

(4) Show that the inner product of any two vectors $|\psi\rangle$, $|\phi\rangle \in \mathcal{H}$ is invariant under unitary transformations $UU^\dagger = U^\dagger U = 1$.

(5) Show that in a Hilbert space $||\lambda |\psi\rangle|| = |\lambda|||\psi\rangle||$, where $\lambda \in \mathbb{C}$.

(6) Consider applying the following transformation to the Pauli basis vectors $|0\rangle \to (|0\rangle + |1\rangle)/\sqrt{2}$, $|1\rangle \to (|0\rangle - |1\rangle)/\sqrt{2}$.

(a) Write down the matrix representations of the Pauli-spin operators in the new basis.

(b) Show that the Pauli-spin operators anti-commute, where the anti-commutator is defined as $[A, B]_+ = AB + BA$.

(7) An electron can be in one of two potential wells that are so close that it can "tunnel" from one to the other. Its state vector can be written as $\alpha |0\rangle + \beta |1\rangle$ where $|0\rangle$ is the state of being in the first well and $|1\rangle$ is the state of being in the second well and all kets are correctly normalised. What is the probability of finding the particle in the first well given that: (a) $\alpha = i/2$ (b) $\beta = e^{i\pi}$ (c) $\beta = 1/3 + i/\sqrt{2}$?

(8) A particle is trapped in a potential such that its allowed eigen-energies are $E = n^2\mathcal{E}$, where $n = 1, 2, 3, ...$ is an integer, and $\mathcal{E}$ is a positive constant. The eigen-vectors of the system are the complete number basis. At $t = 0$ the state of the particle is $|\psi(0)\rangle = 0.2\,|1\rangle + 0.3\,|2\rangle + 0.4\,|3\rangle + 0.843\,|4\rangle$

(a) Suppose that at $t = 0$ the particle's energy was measured, what is the probability of getting an energy value less than $6\mathcal{E}$?

(b) Calculate the mean and the square root of the second moment (rms value) of the particle's energy in the state $|\psi(0)\rangle$.

(c) Calculate the state of the system after time $t$. Do the quantities calculated in (b) remain the same in this new state?

(d) When the energy is measured it turns out to be $16\mathcal{E}$. After the measurement, what is the state of the system? What result is obtained if the energy is measured again?

(9) Given that the wavefunction of a particle is defined as

$$\Psi(x) = \begin{cases} 2A \sin \frac{\pi x}{a} & -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

Determine the value of $A$, the expectation value of $x$, $x^2$, $p$, and $p^2$. What is the value of the uncertainty in the position-momentum?

# Circuit elements (Lecture 2)

(1) Calculate the average power absorbed by a $20\Omega$ resistor when a current of $I(t) = 3/4\sin(3t)A$ is applied.

(2) A $10\mu H$ inductor is driven by a current $I(t) = 30\cos(50t)mA$. Calculate the power supplied to the inductor.

(3) Suppose that a current of $-3A$ flows through a surface S, how many electrons pass through S when $t$ equals (a) $1sec$ (b)$3\mu sec$ (c)$53.4fsec$.

(4) A resistance of $5\Omega$ has a current $I(t) = 510^3A$ in the interval $0 \leq t \leq 2msec$. Obtain the instantaneous and average power.

(5) An inductance of $4mH$ has a voltage$V(t) = 2e^{-10^3t}V$. Obtain the maximum stored energy. Assume initially that the current is zero.

(6) Calculate the source voltage $V$ in Fig.1 then define its polarity in the following two cases (a)$I = 2A$, (b)$I = -2A$.

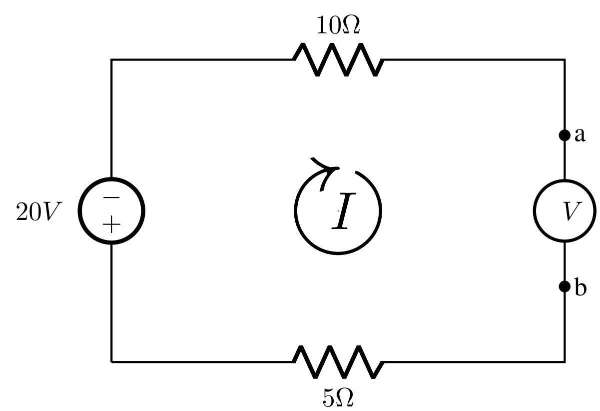


Figure 1

(7) Calculate the equivalent resistance of the circuit shown in Fig.2 when (a)$R_x = \infty$, (b) $R_x = 0$ (c) $R_x = 5$

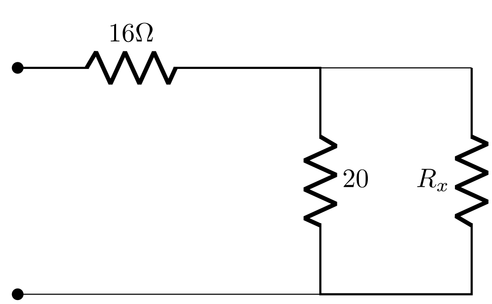


Figure 2

(8) Using the current division method, calculate the source current in Fig.3 and the power delivered to the circuit.

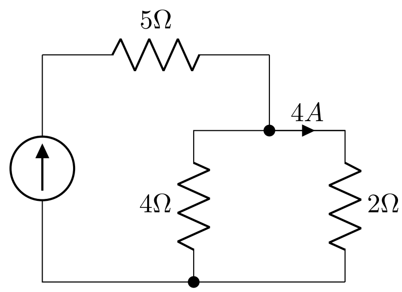


Figure 3

(9) Using voltage division calculate $V_1$ and $V_2$ in the circuit shown in Fig.4

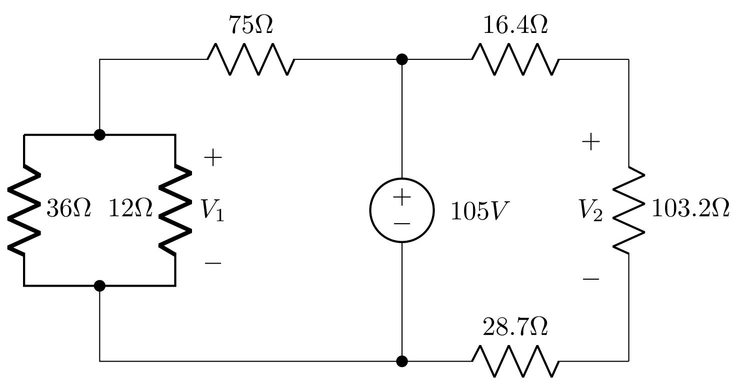


Figure 4

(10) Show that for a four resistors in parallel the current in $R_4$ branch is related to the total current by $I_4 = I_T\left(\frac{R'}{R'+R_4}\right)$, where $R' = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$

(11) A transmission line has the following per-unit-length parameters: $L = 0.5\,\mu H/m$, $C = 200\,pF/m$, $R = 4.0\,\Omega/m$, and $G = 0.02\,S/m$. Calculate the propagation constant and characteristic impedance of this line at 800 *MHz*. If the line is 30 *cm* long, what is the attenuation in dB? Recalculate these quantities in the absence of loss (R = G = 0).