



Aalto University
School of Electrical
Engineering

ELEC-E8126: Robotic Manipulation

Friction and grasping

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1.3.2021

Learning goals

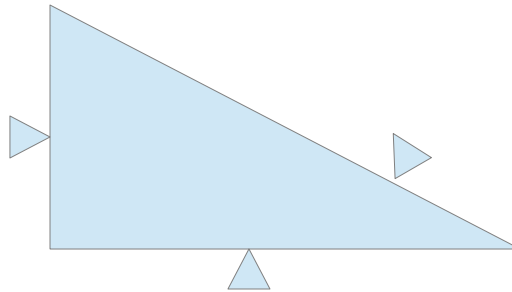
- Understand analytical models of contact with friction.
- Use those to define quality measures for grasps.

Recap: Grasp planning

- Where an object needs to be grasped in order to perform a particular task?
 - Where to place contacts on the object to immobilize it?
 - Or where to place the hand?

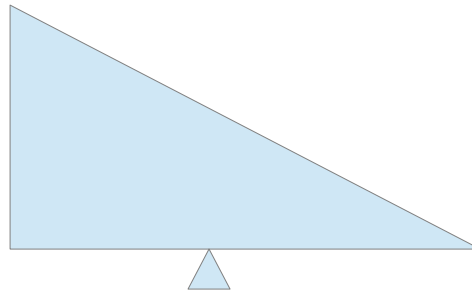
Several contacts in plane

- Can the object move? Around which point?
- What about if there's friction?



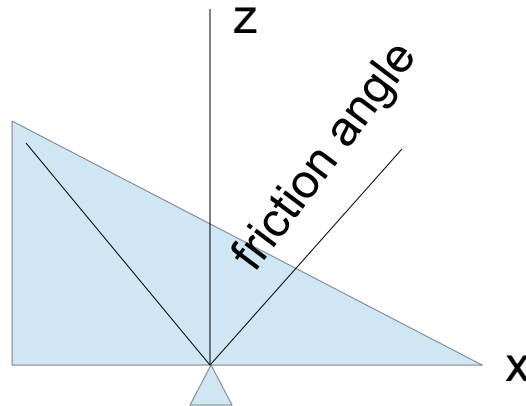
Contact with friction

- What's the range of possible forces caused by the contact?



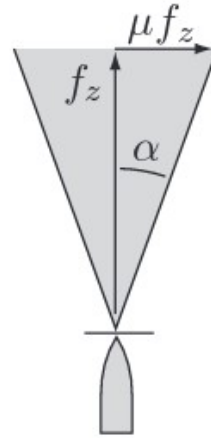
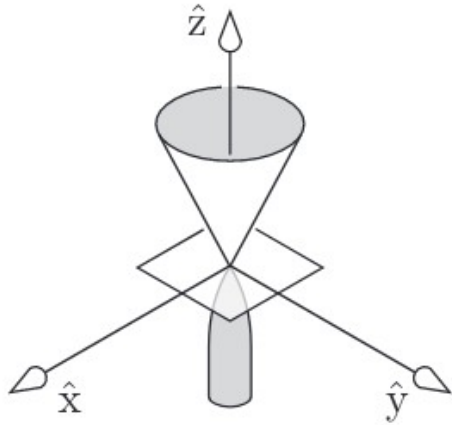
Contact with friction

- What's the range of possible forces caused by the contact?



$$\sqrt{f_x^2 + f_y^2} \leq \mu f_z, f_z \geq 0$$

Friction cones



$$\sqrt{f_x^2 + f_y^2} \leq \mu f_z, f_z \geq 0$$

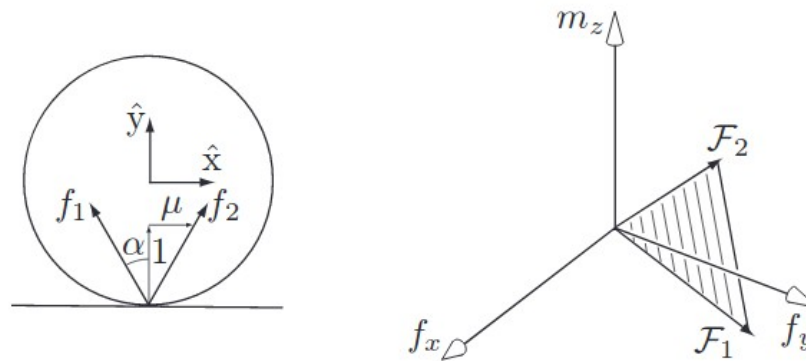
- In 2-D: positive span of edges

$$FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} -\mu \\ 1 \end{pmatrix} \mid k_1, k_2 \geq 0 \right\}$$

Wrench cone

contact point contact normal

- Remember wrench for single contact $F = (\mathbf{p} \times \mathbf{n}, \mathbf{n})$

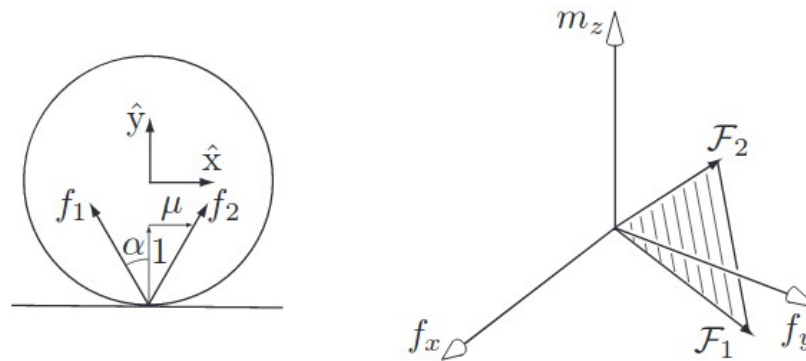


- What's now the wrench cone corresponding to the friction cone $FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} \mu \\ 1 \end{pmatrix} \mid k_1, k_2 \geq 0 \right\}$

Wrench cone

contact point contact normal

- Remember wrench for single contact $F = (\mathbf{p} \times \mathbf{n}, \mathbf{n})$

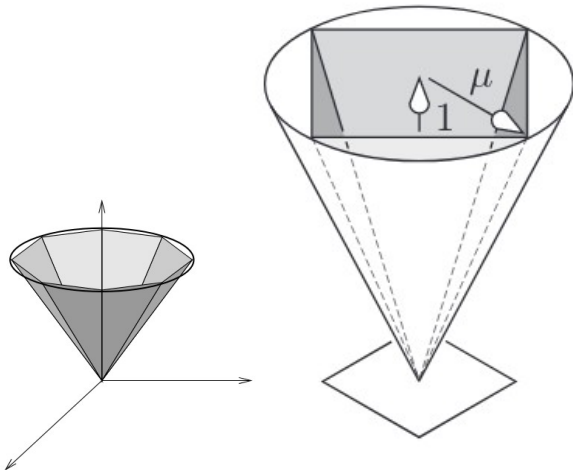


- What's now the wrench cone corresponding to the friction cone $FC = \left\{ k_1 \begin{pmatrix} \mu \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} \mu \\ 1 \end{pmatrix} \mid k_1, k_2 \geq 0 \right\}$
- Wrench cone $WC = \left\{ k_1 \mathbf{F}_1 + k_2 \mathbf{F}_2 \mid k_1, k_2 \geq 0 \right\}$

3-D: Friction cone approximation

- In 3-D, friction cone is usually approximated, because analytical solution is difficult.
- Conservative polyhedral approximation (different number of basis forces can be used)

$$FC = \left\{ \sum_i k_i \mathbf{n}_i \mid k_i \geq 0 \right\}$$



Example: With 4 basis forces,

$$\mathbf{n}_1 = \begin{pmatrix} \mu \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{n}_2 = \begin{pmatrix} -\mu \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{n}_3 = \begin{pmatrix} 0 \\ \mu \\ 1 \end{pmatrix} \quad \mathbf{n}_4 = \begin{pmatrix} 0 \\ -\mu \\ 1 \end{pmatrix}$$

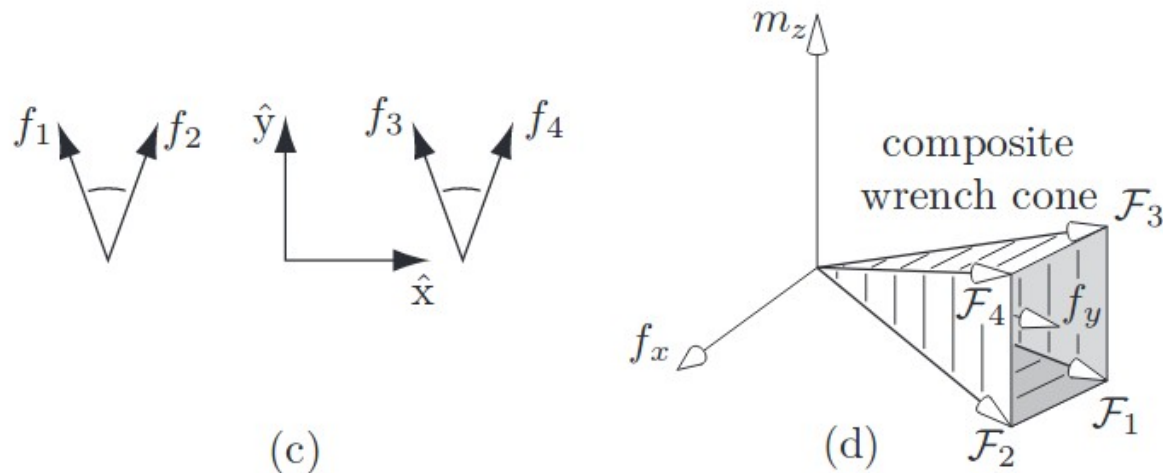
Wrench cone

$$F = (\mathbf{p} \times \mathbf{n}, n)$$

- WC again positive span of individual wrenches

$$WC = \left\{ \sum_i k_i \mathbf{F}_i \mid k_i \geq 0 \right\}$$

- Same also across multiple contacts.



Force closure

- A grasp is *force closure* if for any external wrench there exist contact wrenches that cancel it.
- In other words, if we apply sufficient force at each contact, any external wrench can be compensated for.

e.g. gravity, contact with environment

- This is equivalent to first-order form closure.
 - Contact wrenches span positively entire space

$$\left\{ \sum_i k_i \mathbf{F}_i \mid k_i \geq 0 \right\} = \mathbb{R}^6$$

- Is there a force closure grasp for any object?

Existence of force closure grasps

- Theorem: For any bounded shape that is not a surface of revolution, a force closure grasp exists (Mishra et al., 1987).
- Any non-exceptional surface requires at least $p+1$ contacts without friction (wrench space dimension p)
- Any non-exceptional surface can be grasped by choosing at most $2p$ contacts without friction.

$$p = 3 \text{ (planar)} \implies 4 \leq k \leq 6$$

$$p = 6 \text{ (spatial)} \implies 7 \leq k \leq 12.$$

Determining force closure

- Force-closure is equivalent to
 - Contact wrenches positively span \mathbb{R}^p
 - Convex hull of contact wrenches contains a neighborhood of the origin



Consider this without friction:
What are the wrenches?

How about with friction?
Draw to illustrate.

How could you check the convex hull condition?
What are the boundaries of the convex hull?

Grasp quality metrics

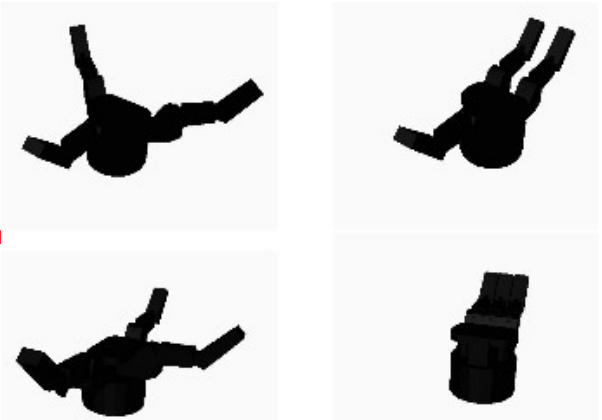
Without friction

- Reminder: Contact forces $CF = \left\{ \sum_i k_i \mathbf{F}_i \mid 0 \leq k_i \leq f_{max} \right\}$
- Approach:
 - Since contact forces at each point not known, normalize wrench basis vectors \mathbf{F} to unit length.
 - First scale moments (torques) by characteristic length of object.
 - Origin at object CoM.
 - Construct convex hull and its supporting hyperplanes.
 - Find shortest distance from origin to any of the hyperplanes.
 - Assumes sum of forces is bounded.

$$CF = \left\{ \sum_i k_i \mathbf{F}_i \mid k_i \geq 0, \sum_i k_i \leq 1 \right\}$$

Sampling based grasp planning revisited

- Sampling approach
 - Choose candidate contacts.
 - Evaluate resulting grasp.
- Instead of choosing contact locations, sample location to place *preshaped* hand, and simulate where contacts happen after closing fingers.
 - Preshapes for prototypical grasps, e.g. pinch grasp, power grasp, cylindrical grasp.
 - Miller et al. 2003.



Caveats

- Quality measures are based on assumptions that are not necessarily entirely true.
- Many quality measures (with different assumptions) have been proposed.
 - Address different issues such as minimizing finger forces, contact placement accuracy, hand configuration (manipulability, joint limits), task compatibility.
 - Recent review: Roa & Suarez, “Grasp quality measures: review and performance”, 2015.

Summary



- Force closure means that a grasp is able to cancel any external wrench if contact forces are sufficiently high.
- Grasps can be planned by maximizing grasp quality metrics.

Extra topics

- Soft contacts
- Complex hands
 - Posture subspaces
- Data-driven grasping
 - Grasp databases
 - Grasping as learning problem

Modeling friction of soft fingertips

















- Soft fingertips can be modeled with friction “cones” that include torsional friction.

Point contact with friction		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{aligned} \sqrt{f_1^2 + f_2^2} &\leq \mu f_3 \\ f_3 &\geq 0 \end{aligned}$
Soft-finger		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} \sqrt{f_1^2 + f_2^2} &\leq \mu f_3 \\ f_3 &\geq 0 \\ f_4 &\leq \gamma f_3 \end{aligned}$

Torsional friction

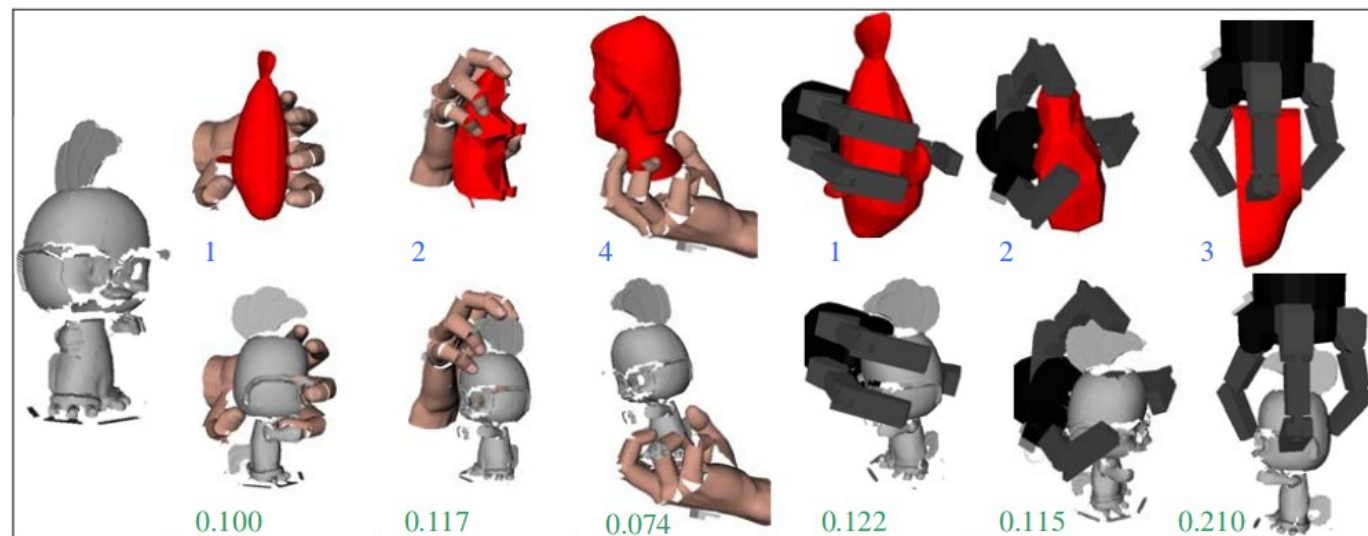
Grasping and complex hands

- How to plan with complex hands beyond preshapes?
- Idea: Use low-dimensional posture space. Correlated joints

Model	DOFs	Eigengrasp 1			Eigengrasp 2		
		Description	min	max	Description	min	max
Barrett	4	Spread angle opening			Finger flexion		
DLR	12	Prox. joints flexion Finger abduction Thumb flexion			Dist. joints flexion Prox. joints extension Thumb flexion		
Robonaut	14	Thumb flexion MCP flexion Index abduction			Thumb flexion MCP extension PIP flexion		
Human	20	Thumb rotation Thumb flexion MCP flexion Index abduction			Thumb flexion MCP extension PIP flexion		

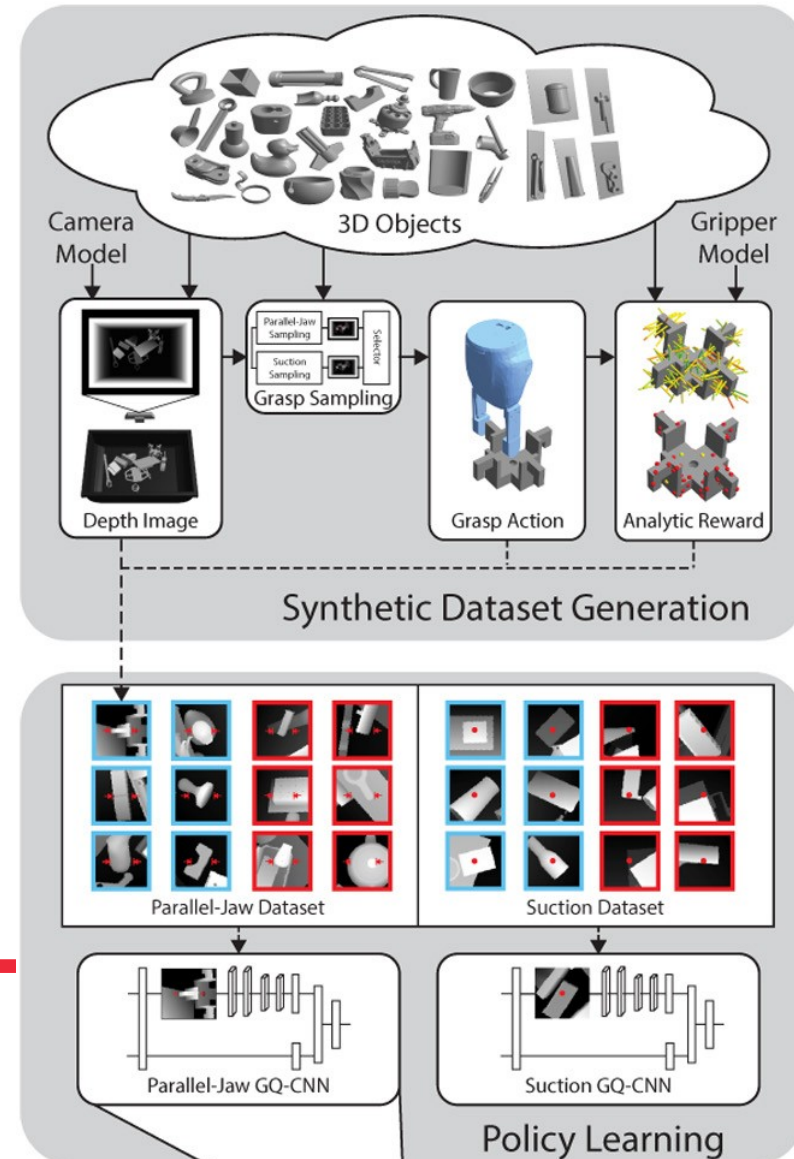
Data-driven grasping for unknown objects

- Idea: Database-driven grasping
 - Pre-compute (as above) grasps for many objects of different sizes and shapes, store in a database.
 - Measure shape of object.
 - Find closest corresponding object (or its part) in database and use its grasps.

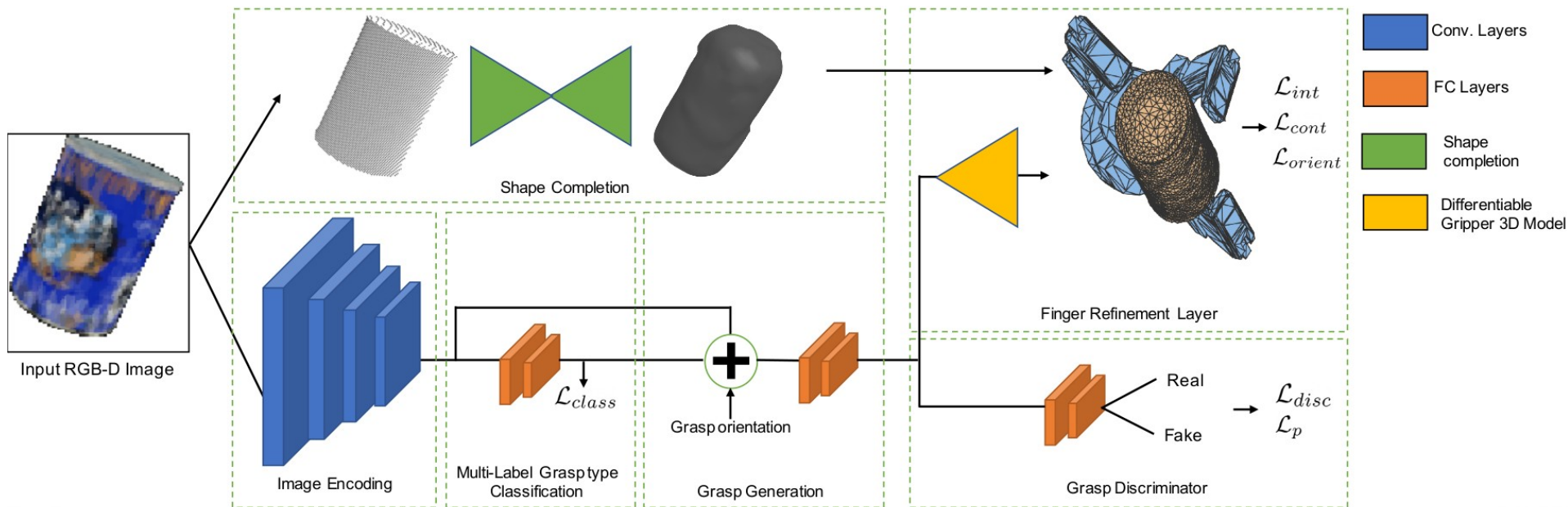


Grasping as a learning problem

- Optimization of metrics slow and difficult from visual information.
- Idea: Learn mapping from images to grasps.
 - Generate synthetic “good” grasps using existing approaches.
 - Train a neural network to predict grasp success.
- Training data metrics may still limit.



Example: GanHand



Next time: Closed kinematic chains

- Readings:
 - Lynch & Park, Chapter 7-7.1.3