Algebraic Number Theory, Spring 2021 period V.

- Instructor: Guillermo Mantilla-Soler, TA: Taoufiq Damir
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- Time and Place: Monday 12:15-14:00, Wednesday 12:15 - 14:00 via zoom. From Monday April the 19th until Wednesday May the 26th.
- Exercise sessions/additional lectures as needed: Friday 12:15 - 14:00 via zoom. From Friday April the 23rd until Friday May the 28th.

Course summary. This will be an introductory course on algebraic number theory, intended for advance undergraduates and graduate students. In this course we will learn how tools from group theory, linear algebra, ring theory, Galois theory and basic analysis are usually combined to solve problems in arithmetic. Often we will use different techniques from different areas to solve the same problem and see what is the relation between such areas. For instance in the first lecture we will study the integer solutions to the *Pythagorical equation*

$$x^2 + y^2 = z^2.$$

An integer solution of the above consists of a triple $(a, b, c) \in \mathbb{Z}^3$ that satisfies the above equation, e.g., $(3, 4, 5), (5, 12, 13)$ and $(691, 238740, 238741)$. In the first lecture we will find all the integer solutions to the above using two different methods, one coming from basic geometry, and other from basic arithmetic. These two ideas will help us to motivate the study of the ring of *integers of a number field* and *p-adic fields.* Both of these are central to the study of modern number theory. Throughout the course we will learn different methods to solve several type of equations of the above type know as *Diophantine equations.*

Approximate Schedule. We will cover these and probably, depending on the time, a bit more.

- Introduction: Prehistory and history of the subject. Interesting arithmetic properties of $\mathbb{Z}$ e.g., unique factorization, we know its group of units pretty well, there is an Euclidean algorithm and more. Review of basic concepts from algebra. Introducing the Gaussian integers, its properties and the concept of number field.

- Ring of integers, Dedekind domains, fractional ideals. Ramification and the discriminant. Special attention to quadratic fields and cyclotomic extensions.

- Failure to have unique factorization; the ideal class group. The geometry of numbers and its applications. Finiteness of the class number and Dirichlet's theorem on the unit group.

- The Dedekind zeta function and the class number formula. A quick tour of Galois theory applied to number fields and the reciprocity law.

- The p-adics and the Hasse-Minkowski principle.

Prerequisites: It's assumed that the students a course in Galois theory or equivalent.

Assignments: The final course grade will be based entirely on five homework assignments. There is no exam.

Bibliography. Although I will not follow strictly any text book, Milne's notes are a pretty good reference to keep up with the material from the class. They are available at

· `http://www.jmilne.org/math/CourseNotes/ant.html`

Other good references for the class are:

· P. Samuel, *Théories algébrique des nombres.*

· J. Neukirch, *Algebraic number theory.*