

**Exam question begins**

Consider a design scenario where the AHTS vessel from the previous question (waterline opening angle  $35^\circ$ , vertical angle  $25^\circ$  from horizontal) experiences a glancing impact in the bow. Your structural engineer has calculated the design ice load for different ice classes according to the IACS Polar Class rules. In addition, he has calculated how much extra steel needs to be added to the ice-strengthened regions. If the vessel is designed to ice class PC 6 (baseline), its design displacement would be 10000 tons.

Ice class	Steel weight increase	Design ice load
PC 6	-	6000 kN
PC 5	+ 750 tons	7500 kN
PC 4	+1500 tons	9500 kN
PC 3	+3000 tons	12000 kN

The shipowner is ready to accept an operational speed limit of 9 knots for ice management operations in the marginal ice zone where there is a possibility of encountering multi-year ice floes.

- What is the lowest ice class that could be considered acceptable for the AHTS based on the operational speed limitation proposed by the shipowner? (3 points)
- What kind of operational speed limitation(s) could be considered for the lower ice class(es)? (3 points)

Use the energy-based ice forces method and assume a simple glancing impact to a right-angle edge where all kinetic energy is expended in crushing. Mass reduction coefficient for this impact scenario is 2.0, ice crushing strength for  $1 \text{ m}^2$  reference area can be taken as 2500 kPa, and force-area relationship exponent as  $-0.5$ .

**Exam question ends**

Model answer

The intention is to use the energy-based ice forces method to determine the correct ice class for the vessel. The student can choose different approaches to end up with the final answer; one example is given below.

The following hull angles are given in the exercise paper:

$$\alpha = 35^\circ$$

$$\phi = 25^\circ$$

$$\psi = \text{atan}\left(\frac{\tan \phi}{\sin \alpha}\right) = \text{atan}\left(\frac{\tan 25^\circ}{\sin 35^\circ}\right) \approx 39.1^\circ \rightarrow \beta' = 90^\circ - \psi \approx 50.9^\circ$$

While the equations may look complicated at first, they can be simplified by inserting  $ex = 0.5$  and a trigonometric shorthand  $trig$ :

$$F_n = p_0 \times \left(\frac{1}{\sin \alpha \times \cos \alpha \times \sin \beta' \times \cos^2 \beta'}\right)^{1+ex} \times \zeta_n^{2+2 \times ex} = p_0 \times \sqrt{\frac{1}{trig}} \times \zeta_n$$

$$IE = \frac{p_0}{3+2 \times ex} \times \left(\frac{1}{\sin \alpha \times \cos \alpha \times \sin \beta' \times \cos^2 \beta'}\right)^{1+ex} \times \zeta_n^{3+2 \times ex} = \frac{p_0}{2} \times \sqrt{\frac{1}{trig}} \times \zeta_n^2$$

where the trigonometric shorthand  $trig$  can be calculated as follows:

$$trig = \sin \alpha \times \cos \alpha \times \sin \beta' \times \cos^2 \beta' = \sin 35^\circ \times \cos 35^\circ \times \sin 50.9^\circ \times \cos^2 50.9^\circ \approx 0.15$$

With the above, we can solve a simple equation for indentation energy as a function of normal force as follows:

$$F_n = p_0 \times \sqrt{\frac{1}{trig}} \times \zeta_n \rightarrow \zeta_n = \frac{F_n \times \sqrt{trig}}{p_0}$$

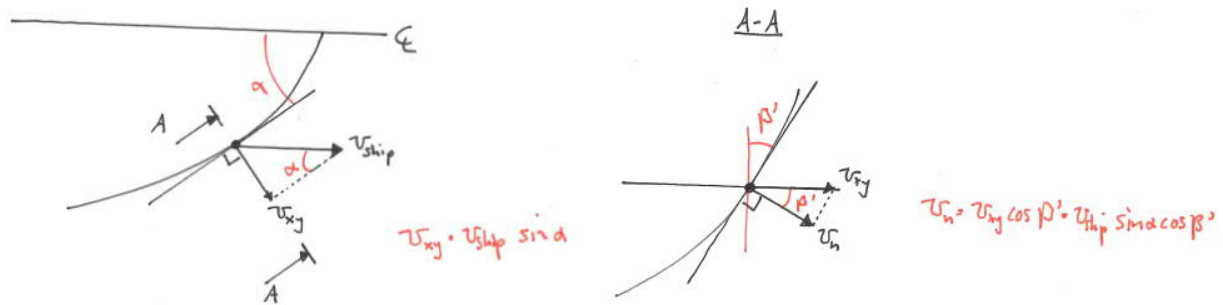
$$IE = \frac{p_0}{2} \times \sqrt{\frac{1}{trig}} \times \zeta_n^2 = \frac{p_0}{2} \times \sqrt{\frac{1}{trig}} \times \frac{F_n^2 \times trig}{p_0^2} = \frac{1}{2} \times \sqrt{trig} \times \frac{F_n^2}{p_0} \approx 0.194 \times \frac{F_n^2}{p_0}$$

The result are shown in the table below:

Ice class	Normal force	Indentation energy
PC 6	6000 kN	2794 kJ
PC 5	7500 kN	4365 kJ
PC 4	9500 kN	7003 kJ
PC 3	12000 kN	11174 kJ

As we need to determine the velocity component normal to the hull surface  $V_n$  when calculating the kinetic energy  $KE_e$ , we have to establish a connection between it and the speed of the ship through hull angles. This is basic trigonometry and can be solved either using the "Lindqvist angles" or "Daley's angles":

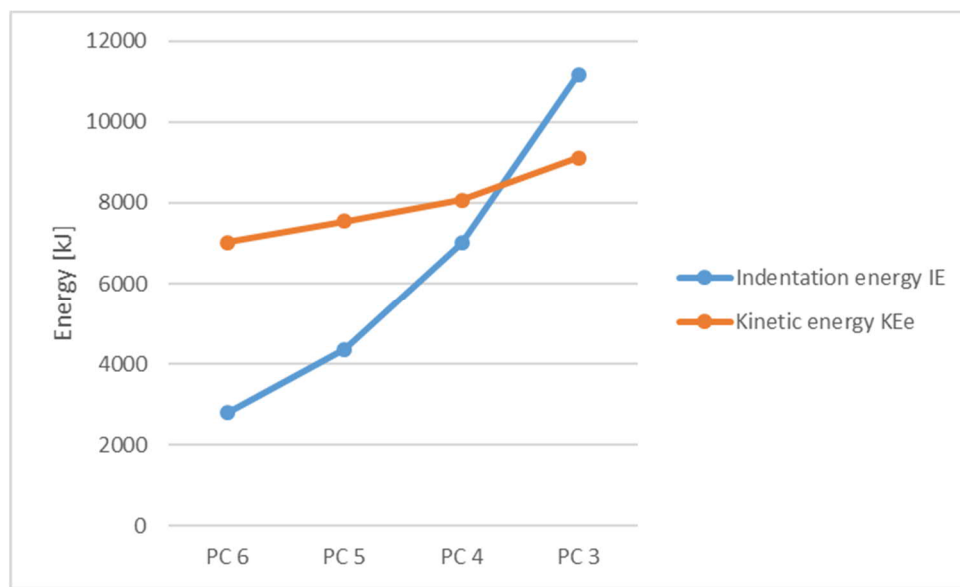
$$V_n = V_{ship} \times \sin \alpha \times \sin \psi = V_{ship} \times \sin \alpha \times \cos \beta'$$



Since the owner is willing to accept an operational speed limitation of 9 knots (4.63 m/s), we can calculate the kinetic energy for each ice class (noting that  $V_n \approx 1.68 \frac{m}{s}$ ).

Ice class	Displacement	Kinetic energy
PC 6	10000 tons	7016 kJ
PC 5	10750 tons	7542 kJ
PC 4	11500 tons	8068 kJ
PC 3	13000 tons	9121 kJ

Excessive hull loads are avoided if  $KE_e \leq IE$ , meaning that the final solution can be evaluated as follows:



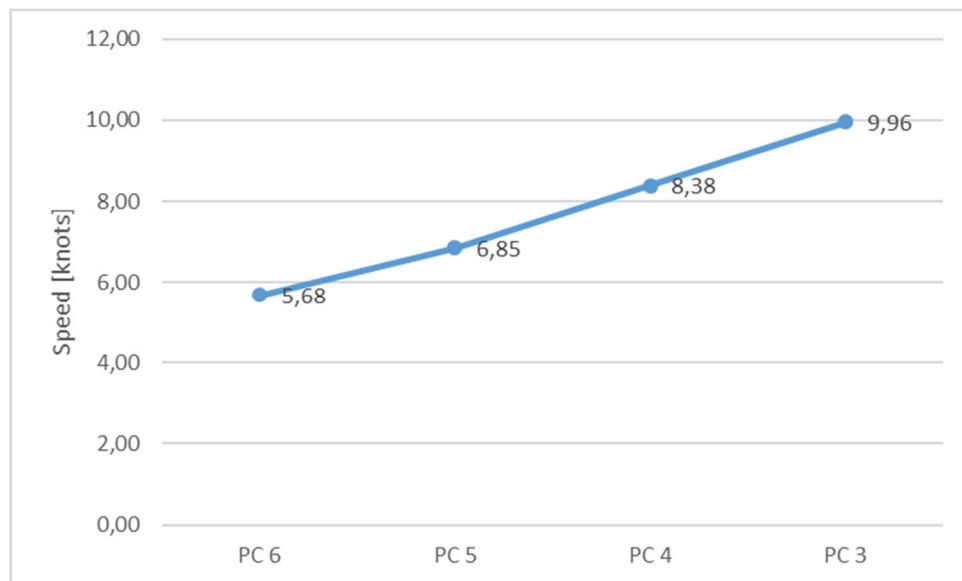
The graph indicates that if the operational speed limitation is 9 knots, the ice class must be PC 3 (or higher).

The second question can be answered by selecting

$$KE_e = IE$$

and solving for  $V_{ship}$ :

Ice class	Indentation energy = Kinetic energy	Normal velocity	Ship velocity (operational speed limitation)	
PC 6	2794 kJ	1.06 m/s	2.92 m/s	5.68 knots
PC 5	4365 kJ	1.27 m/s	3.52 m/s	6.85 knots
PC 4	7003 kJ	1.56 m/s	4.31 m/s	8.38 knots
PC 3	11174 kJ	1.85 m/s	5.12 m/s	9.96 knots



About grading

Both a) and b) are worth 50% of the total score (3+3=6 points).

The student may get full score by using any approach that utilizes the energy-based method, including but not necessarily limited to the following:

- compare indentation energies:  $IE(F_n) \geq KE_e(9 \text{ knots})$ ;
- compare maximum allowed speeds:  $V_{ship}(F_n) \geq 9 \text{ knots}$
- compare normal forces:  $F_n(9 \text{ knots}) \leq F_n(\text{ice class})$

However, the student must show understanding of the energy-based ice forces method and some calculations to justify the selection of ice class; just picking the highest one will not yield any points.

The student should note the varying steel weight between the ice classes; the intention is to vary the displacement, but if (and only if) the student notes in writing that the additional steel weight is taken from the deadweight and because of that the displacement remains constant, that can be accepted as well (the result will then be either PC 4 or PC 3 depending on the accuracy of the student's calculations).

Minor variations in the numerical results can be accepted due to rounding of values during calculation, but the results should be of the same magnitude.

Partial score may still be awarded if the student demonstrates understanding of the Daley's method but has either not performed the numerical calculations or has multiple errors.