5.6 *The limits of my language* mean the limits of my world.

5.61 Logic fills the world: the limits of the world are also its limits.

We cannot therefore say in logic: This and this there is in the world, that there is not.

For that would apparently presuppose that we exclude certain possibilities, and this cannot be the case since otherwise logic must get outside the limits of the world: that is, if it could consider these limits from the other side also.

What we cannot think, that we cannot think: we cannot therefore *say* what we cannot think.

5.62 This remark provides a key to the question, to what extent solipsism is a truth.

In fact what solipsism *means*, is quite correct, only it cannot be *said*, but it shows itself.

That the world is *my* world, shows itself in the fact that the limits of the language (the language which only I understand) mean the limits of *my* world.

5.621 The world and life are one.

5.63 I am my world. (The microcosm.)

5.631 The thinking, presenting subject; there is no such thing.

If I wrote a book "The world as I found it", I should also have therein to report on my body and say which members obey my will and which do not, etc. This then would be a method of isolating the subject or rather of showing that in an important sense there is no subject: that is to say, of it alone in this book mention could *not* be made.

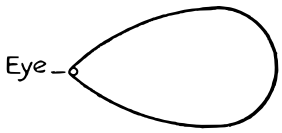
5.632 The subject does not belong to the world but it is a limit of the world.

5.633 *Where in* the world is a metaphysical subject to be noted?

You say that this case is altogether like that of the eye and the field of sight. But you do *not* really see the eye.

And from nothing *in the field of sight* can it be concluded that it is seen from an eye.

5.6331 For the field of sight has not a form like this:



5.634 This is connected with the fact that no part of our experience is also a priori.

Everything we see could also be otherwise.

Everything we can describe at all could also be otherwise.

There is no order of things a priori.

5.64 Here we see that solipsism strictly carried out coincides with pure realism. The I in solipsism shrinks to an extensionless point and there remains the reality co-ordinated with it.

5.641 There is therefore really a sense in which in philosophy we can talk of a non-psychological I.

The I occurs in philosophy through the fact that the "world is my world".

The philosophical I is not the man, not the human body or the human soul of which psychology treats, but the metaphysical subject, the limit—not a part of the world.

6 The general form of truth-function is: $[\overline{p}, \overline{\xi}, N(\overline{\xi})]$.

This is the general form of proposition.

6.001 This says nothing else than that every proposition is the result of successive applications of the operation $N'(\overline{\xi})$ to the elementary propositions.

6.002 If we are given the general form of the way in which a proposition is constructed, then thereby we are also given the general form of the way in which by an operation out of one proposition another can be created.

6.01 The general form of the operation $\Omega'(\overline{\eta})$ is therefore: $[\overline{\xi}, N(\overline{\xi})]'(\overline{\eta})$ $(= [\overline{\eta}, \overline{\xi}, N(\overline{\xi})])$.

This is the most general form of transition from one proposition to another.

6.02 And thus we come to numbers: I define

$$x = \Omega^{0\prime}x \text{ Def. and}$$
$$\Omega'\Omega^{\nu\prime}x = \Omega^{\nu+1\prime}x \text{ Def.}$$

According, then, to these symbolic rules we write the series x, $\Omega'x$, $\Omega'\Omega'x$, $\Omega'\Omega'\Omega'x \ldots\ldots$

$$\text{as: } \Omega^{0\prime}x, \Omega^{0+1\prime}x, \Omega^{0+1+1\prime}x, \Omega^{0+1+1+1\prime}x \ldots\ldots$$

Therefore I write in place of "$[x, \xi, \Omega'\xi]$",

$$\text{"}[\Omega^{0\prime}x, \Omega^{\nu\prime}x, \Omega^{\nu+1\prime}x]\text{"}.$$

And I define:

$$0 + 1 = 1 \text{ Def.}$$
$$0 + 1 + 1 = 2 \text{ Def.}$$
$$0 + 1 + 1 + 1 = 3 \text{ Def.}$$
and so on.

6.021 A number is the exponent of an operation.

6.022 The concept number is nothing else than that which is common to all numbers, the general form of number.

The concept number is the variable number.

And the concept of equality of numbers is the general form of all special equalities of numbers.

6.03 The general form of the cardinal number is: $[0, \xi, \xi + 1]$.

6.031 The theory of classes is altogether superfluous in mathematics.

This is connected with the fact that the generality which we need in mathematics is not the *accidental* one.

6.1 The propositions of logic are tautologies.

6.11 The propositions of logic therefore say nothing. (They are the analytical propositions.)

6.111 Theories which make a proposition of logic appear substantial are always false. One could *e.g.* believe that the words "true" and "false" signify two properties among other properties, and then it would appear as a remarkable fact that every proposition possesses one of these properties. This now by no means appears self-evident, no more so than the proposition "All roses are either yellow or red" would sound even if it were true. Indeed our proposition now gets quite the character of a proposition of natural science and this is a certain symptom of its being falsely understood.

6.112 The correct explanation of logical propositions must give them a peculiar position among all propositions.

6.113 It is the characteristic mark of logical propositions that one can perceive in the symbol alone that they are true; and this fact contains in itself the whole philosophy of logic. And so also it is one of the most important facts that the truth or falsehood of non-logical propositions can *not* be recognized from the propositions alone.

6.12 The fact that the propositions of logic are tautologies *shows* the formal—logical—properties of language, of the world.

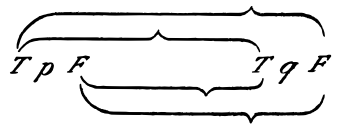
That its constituent parts connected together *in this way* give a tautology characterizes the logic of its constituent parts.

In order that propositions connected together in a definite way may give a tautology they must have definite properties of structure. That they give a tautology when *so* connected shows therefore that they possess these properties of structure.

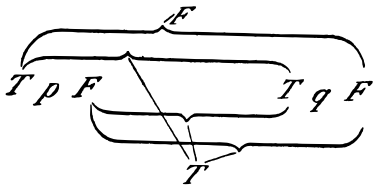
6.1201 That *e.g.* the propositions "p" and "$\sim p$" in the connexion "$\sim(p\,.\,\sim p)$" give a tautology shows that they contradict one another. That the propositions "$p \supset q$", "p" and "q" connected together in the form "$(p \supset q)\,.\,(p) :\supset: (q)$" give a tautology shows that q follows from p and $p \supset q$. That "$(x)\,.\,fx :\supset: fa$" is a tautology shows that fa follows from $(x)\,.\,fx$, etc. etc.

6.1202 It is clear that we could have used for this purpose contradictions instead of tautologies.

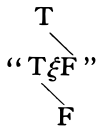
6.1203 In order to recognize a tautology as such, we can, in cases in which no sign of generality occurs in the tautology, make use of the following intuitive method: I write instead of "p", "q", "r", etc., "TpF", "TqF", "TrF", etc. The truth-combinations I express by brackets, *e.g.*:



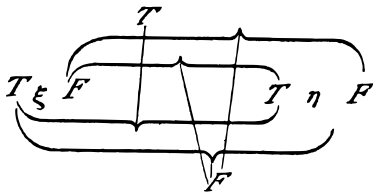
and the co-ordination of the truth or falsity of the whole proposition with the truth-combinations of the truth-arguments by lines in the following way:



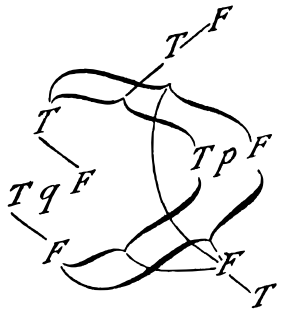
This sign, for example, would therefore present the proposition $p \supset q$. Now I will proceed to inquire whether such a proposition as $\sim(p \,.\, \sim p)$ (The Law of Contradiction) is a tautology. The form "$\sim\xi$" is written in our notation



the form "$\xi \,.\, \eta$" thus:—



Hence the proposition $\sim(p\,.\,\sim q)$ runs thus:—



If here we put "p" instead of "q" and examine the combination of the outermost T and F with the innermost, it is seen that the truth of the whole proposition is co-ordinated with *all* the truth-combinations of its argument, its falsity with none of the truth-combinations.

6.121 The propositions of logic demonstrate the logical properties of propositions, by combining them into propositions which say nothing.

This method could be called a zero-method. In a logical proposition propositions are brought into equilibrium with one another, and the state of equilibrium then shows how these propositions must be logically constructed.

6.122 Whence it follows that we can get on without logical propositions, for we can recognize in an adequate notation the formal properties of the propositions by mere inspection.

6.1221 If for example two propositions "p" and "q" give a tautology in the connexion "$p \supset q$", then it is clear that q follows from p.

E.g. that "q" follows from "$p \supset q\,.\,p$" we see from these two propositions themselves, but we can also show it by combining them to "$p \supset q.p :\supset: q$" and then showing that this is a tautology.

6.1222 This throws light on the question why logical propositions can no more be empirically established than they can be empirically refuted. Not only must a proposition of logic be incapable of being contradicted by any possible experience, but it must also be incapable of being established by any such.

6.1223 It now becomes clear why we often feel as though "logical truths"

must be "*postulated*" by us. We can in fact postulate them in so far as we can postulate an adequate notation.

6.1224 It also becomes clear why logic has been called the theory of forms and of inference.

6.123 It is clear that the laws of logic cannot themselves obey further logical laws.

(There is not, as Russell supposed, for every "type" a special law of contradiction; but one is sufficient, since it is not applied to itself.)

6.1231 The mark of logical propositions is not their general validity.

To be general is only to be accidentally valid for all things. An ungeneralized proposition can be tautologous just as well as a generalized one.

6.1232 Logical general validity, we could call essential as opposed to accidental general validity, *e.g.* of the proposition "all men are mortal". Propositions like Russell's "axiom of reducibility" are not logical propositions, and this explains our feeling that, if true, they can only be true by a happy chance.

6.1233 We can imagine a world in which the axiom of reducibility is not valid. But it is clear that logic has nothing to do with the question whether our world is really of this kind or not.

6.124 The logical propositions describe the scaffolding of the world, or rather they present it. They "treat" of nothing. They presuppose that names have meaning, and that elementary propositions have sense. And this is their connexion with the world. It is clear that it must show something about the world that certain combinations of symbols—which essentially have a definite character—are tautologies. Herein lies the decisive point. We said that in the symbols which we use much is arbitrary, much not. In logic only this expresses: but this means that in logic it is not *we* who express, by means of signs, what we want, but in logic the nature of the essentially necessary signs itself asserts. That is to say, if we know the logical syntax of any sign language, then all the propositions of logic are already given.

6.125 It is possible, even in the old logic, to give at the outset a description of all "true" logical propositions.

6.1251 Hence there can *never* be surprises in logic.

6.126 Whether a proposition belongs to logic can be determined by determining the logical properties of the *symbol*.

And this we do when we prove a logical proposition. For without troubling ourselves about a sense and a meaning, we form the logical propositions out of others by mere *symbolic rules*.

We prove a logical proposition by creating it out of other logical propositions by applying in succession certain operations, which again generate tautologies out of the first. (And from a tautology only tautologies *follow*.)

Naturally this way of showing that its propositions are tautologies is quite unessential to logic. Because the propositions, from which the proof starts, must show without proof that they are tautologies.

6.1261 In logic process and result are equivalent. (Therefore no surprises.)

6.1262 Proof in logic is only a mechanical expedient to facilitate the recognition of tautology, where it is complicated.

6.1263 It would be too remarkable, if one could prove a significant proposition *logically* from another, and a logical proposition *also*. It is clear from the beginning that the logical proof of a significant proposition and the proof *in* logic must be two quite different things.

6.1264 The significant proposition asserts something, and its proof shows that it is so; in logic every proposition is the form of a proof.

Every proposition of logic is a modus ponens presented in signs. (And the modus ponens can not be expressed by a proposition.)

6.1265 Logic can always be conceived to be such that every proposition is its own proof.

6.127 All propositions of logic are of equal rank; there are not some which are essentially primitive and others deduced from these.

Every tautology itself shows that it is a tautology.

6.1271 It is clear that the number of "primitive propositions of logic" is arbitrary, for we could deduce logic from one primitive proposition by simply forming, for example, the logical product of Frege's primitive propositions. (Frege would perhaps say that this would no longer be immediately self-evident. But it is remarkable that so exact a thinker as Frege should have appealed to the degree of self-evidence as the criterion of a logical proposition.)

6.13 Logic is not a theory but a reflexion of the world.

Logic is transcendental.

6.2 Mathematics is a logical method.

The propositions of mathematics are equations, and therefore pseudo-propositions.

6.21 Mathematical propositions express no thoughts.

6.211 In life it is never a mathematical proposition which we need, but we use mathematical propositions *only* in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics.

(In philosophy the question "Why do we really use that word, that proposition?" constantly leads to valuable results.)

6.22 The logic of the world which the propositions of logic show in tautologies, mathematics shows in equations.

6.23 If two expressions are connected by the sign of equality, this means that they can be substituted for one another. But whether this is the case must show itself in the two expressions themselves.

It characterizes the logical form of two expressions, that they can be substituted for one another.

6.231 It is a property of affirmation that it can be conceived as double denial.

It is a property of "$1 + 1 + 1 + 1$" that it can be conceived as "$(1 + 1) + (1 + 1)$".

6.232 Frege says that these expressions have the same meaning but different senses.

But what is essential about equation is that it is not neces-

sary in order to show that both expressions, which are connected by the sign of equality, have the same meaning: for this can be perceived from the two expressions themselves.

6.2321 And, that the propositions of mathematics can be proved means nothing else than that their correctness can be seen without our having to compare what they express with the facts as regards correctness.

6.2322 The identity of the meaning of two expressions cannot be *asserted*. For in order to be able to assert anything about their meaning, I must know their meaning, and if I know their meaning, I know whether they mean the same or something different.

6.2323 The equation characterizes only the standpoint from which I consider the two expressions, that is to say the standpoint of their equality of meaning.

6.233 To the question whether we need intuition for the solution of mathematical problems it must be answered that language itself here supplies the necessary intuition.

6.2331 The process of calculation brings about just this intuition.

Calculation is not an experiment.

6.234 Mathematics is a method of logic.

6.2341 The essential of mathematical method is working with equations. On this method depends the fact that every proposition of mathematics must be self-intelligible.

6.24 The method by which mathematics arrives at its equations is the method of substitution.

For equations express the substitutability of two expressions, and we proceed from a number of equations to new equations, replacing expressions by others in accordance with the equations.

6.241 Thus the proof of the proposition $2 \times 2 = 4$ runs:

$$(\Omega^{\nu})^{\mu\prime}x = \Omega^{\nu\times\mu\prime}x \text{ Def.}$$

$$\Omega^{2\times 2\prime}x = (\Omega^2)^{2\prime}x = (\Omega^2)^{1+1\prime}x = \Omega^{2\prime}\Omega^{2\prime}x = \Omega^{1+1\prime}\Omega^{1+1\prime}x$$
$$= (\Omega'\Omega)'(\Omega'\Omega)'x = \Omega'\Omega'\Omega'\Omega'x = \Omega^{1+1+1+1\prime}x = \Omega^{4\prime}x.$$

6.3 Logical research means the investigation of *all regularity*. And outside logic all is accident.

6.31 The so-called law of induction cannot in any case be a logical law, for it is obviously a significant proposition.—And therefore it cannot be a law a priori either.

6.32 The law of causality is not a law but the form of a law.*

6.321 "Law of Causality" is a class name. And as in mechanics there are, for instance, minimum-laws, such as that of least action, so in physics there are causal laws, laws of the causality form.

6.3211 Men had indeed an idea that there must be *a* "law of least action", before they knew exactly how it ran. (Here, as always, the a priori certain proves to be something purely logical.)

6.33 We do not *believe* a priori in a law of conservation, but we *know* a priori the possibility of a logical form.

6.34 All propositions, such as the law of causation, the law of continuity in nature, the law of least expenditure in nature, etc. etc., all these are a priori intuitions of possible forms of the propositions of science.

6.341 Newtonian mechanics, for example, brings the description of the universe to a unified form. Let us imagine a white surface with irregular black spots. We now say: Whatever kind of picture these make I can always get as near as I like to its description, if I cover the surface with a sufficiently fine square network and now say of every square that it is white or black. In this way I shall have brought the description of the surface to a unified form. This form is arbitrary, because I could have applied with equal success a net with a triangular or hexagonal mesh. It can happen that the description would have been simpler with the aid of a triangular mesh; that is to say we might have described the surface more accurately with a triangular, and coarser, than with the finer square mesh, or vice versa, and so on. To the different networks correspond different systems of describing the world. Mechanics determine a form of description by saying: All propositions in the description of the world must be obtained in a given way from a number of given propositions—the mechanical axioms. It thus provides the bricks for building the edifice of science, and says: Whatever building thou wouldst erect, thou

I.e. not the form of one particular law, but of any law of a certain sort (B. R.).

shalt construct it in some manner with these bricks and these alone.

(As with the system of numbers one must be able to write down any arbitrary number, so with the system of mechanics one must be able to write down any arbitrary physical proposition.)

6.342 And now we see the relative position of logic and mechanics. (We could construct the network out of figures of different kinds, as out of triangles and hexagons together.) That a picture like that instanced above can be described by a network of a given form asserts *nothing* about the picture. (For this holds of every picture of this kind.) But *this* does characterize the picture, the fact, namely, that it can be *completely* described by a definite net of *definite* fineness.

So too the fact that it can be described by Newtonian mechanics asserts nothing about the world; but *this* asserts something, namely, that it can be described in that particular way in which it is described, as is indeed the case. The fact, too, that it can be described more simply by one system of mechanics than by another says something about the world.

6.343 Mechanics is an attempt to construct according to a single plan all *true* propositions which we need for the description of the world.

6.3431 Through the whole apparatus of logic the physical laws still speak of the objects of the world.

6.3432 We must not forget that the description of the world by mechanics is always quite general. There is, for example, never any mention of *particular* material points in it, but always only of *some points or other*.

6.35 Although the spots in our picture are geometrical figures, geometry can obviously say nothing about their actual form and position. But the network is *purely* geometrical, and all its properties can be given a priori.

Laws, like the law of causation, etc., treat of the network and not of what the network described.

6.36 If there were a law of causality, it might run: "There are natural laws".

But that can clearly not be said: it shows itself.

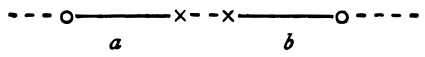
6.361 In the terminology of Hertz we might say: Only *uniform* connexions are *thinkable*.

6.3611 We cannot compare any process with the "passage of time"—there is no such thing—but only with another process (say, with the movement of the chronometer).

Hence the description of the temporal sequence of events is only possible if we support ourselves on another process.

It is exactly analogous for space. When, for example, we say that neither of two events (which mutually exclude one another) can occur, because there is *no cause* why the one should occur rather than the other, it is really a matter of our being unable to describe *one* of the two events unless there is some sort of asymmetry. And if there *is* such an asymmetry, we can regard this as the *cause* of the occurrence of the one and of the non-occurrence of the other.

6.36111 The Kantian problem of the right and left hand which cannot be made to cover one another already exists in the plane, and even in one-dimensional space; where the two congruent figures a and b cannot be made to cover one another without moving them out of this space. The right and left hand are in fact completely congruent. And the fact that they cannot be made to cover one another has nothing to do with it.



A right-hand glove could be put on a left hand if it could be turned round in four-dimensional space.

6.362 What can be described can happen too, and what is excluded by the law of causality cannot be described.

6.363 The process of induction is the process of assuming the *simplest* law that can be made to harmonize with our experience.

6.3631 This process, however, has no logical foundation but only a psychological one.

It is clear that there are no grounds for believing that the simplest course of events will really happen.

6.36311 That the sun will rise to-morrow, is an hypothesis; and that means that we do not *know* whether it will rise.

6.37 A necessity for one thing to happen because another has happened does not exist. There is only *logical* necessity.

6.371 At the basis of the whole modern view of the world lies the illusion that the so-called laws of nature are the explanations of natural phenomena.

6.372 So people stop short at natural laws as at something unassailable, as did the ancients at God and Fate.

And they both are right and wrong. But the ancients were clearer, in so far as they recognized one clear conclusion, whereas in the modern system it should appear as though *everything* were explained.

6.373 The world is independent of my will.

6.374 Even if everything we wished were to happen, this would only be, so to speak, a favour of fate, for there is no *logical* connexion between will and world, which would guarantee this, and the assumed physical connexion itself we could not again will.

6.375 As there is only a *logical* necessity, so there is only a *logical* impossibility.

6.3751 For two colours, *e.g.* to be at one place in the visual field, is impossible, logically impossible, for it is excluded by the logical structure of colour.

Let us consider how this contradiction presents itself in physics. Somewhat as follows: That a particle cannot at the same time have two velocities, *i.e.* that at the same time it cannot be in two places, *i.e.* that particles in different places at the same time cannot be identical.

(It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The assertion that a point in the visual field has two different colours at the same time, is a contradiction.)

6.4 All propositions are of equal value.

6.41 The sense of the world must lie outside the world. In the world everything is as it is and happens as it does happen. *In* it there is no value—and if there were, it would be of no value.

If there is a value which is of value, it must lie outside all happening and being-so. For all happening and being-so is accidental.

What makes it non-accidental cannot lie *in* the world, for otherwise this would again be accidental.

It must lie outside the world.

6.42 Hence also there can be no ethical propositions.

Propositions cannot express anything higher.

6.421 It is clear that ethics cannot be expressed.

Ethics are transcendental.

(Ethics and æsthetics are one.)

6.422 The first thought in setting up an ethical law of the form "thou shalt ..." is: And what if I do not do it. But it is clear that ethics has nothing to do with punishment and reward in the ordinary sense. This question as to the *consequences* of an action must therefore be irrelevant. At least these consequences will not be events. For there must be something right in that formulation of the question. There must be some sort of ethical reward and ethical punishment, but this must lie in the action itself.

(And this is clear also that the reward must be something acceptable, and the punishment something unacceptable.)

6.423 Of the will as the bearer of the ethical we cannot speak.

And the will as a phenomenon is only of interest to psychology.

6.43 If good or bad willing changes the world, it can only change the limits of the world, not the facts; not the things that can be expressed in language.

In brief, the world must thereby become quite another. It must so to speak wax or wane as a whole.

The world of the happy is quite another than that of the unhappy.

6.431 As in death, too, the world does not change, but ceases.

6.4311 Death is not an event of life. Death is not lived through.

If by eternity is understood not endless temporal duration but timelessness, then he lives eternally who lives in the present.

Our life is endless in the way that our visual field is without limit.

6.4312 The temporal immortality of the soul of man, that is to say, its eternal survival also after death, is not only in no way guaranteed, but this assumption in the first place will not do for us what we always tried to make it do. Is a riddle solved by the fact that I survive for ever? Is this eternal life not as enigmatic as our present one? The solution of the riddle of life in space and time lies *outside* space and time.

(It is not problems of natural science which have to be solved.)

6.432 *How* the world is, is completely indifferent for what is higher. God does not reveal himself *in* the world.

6.4321 The facts all belong only to the task and not to its performance.

6.44 Not *how* the world is, is the mystical, but *that* it is.

6.45 The contemplation of the world sub specie aeterni is its contemplation as a limited whole.

The feeling of the world as a limited whole is the mystical feeling.

6.5 For an answer which cannot be expressed the question too cannot be expressed.

The riddle does not exist.

If a question can be put at all, then it *can* also be answered.

6.51 Scepticism is *not* irrefutable, but palpably senseless, if it would doubt where a question cannot be asked.

For doubt can only exist where there is a question; a question only where there is an answer, and this only where something *can* be *said*.

6.52 We feel that even if *all possible* scientific questions be answered, the problems of life have still not been touched at all. Of course there is then no question left, and just this is the answer.

6.521 The solution of the problem of life is seen in the vanishing of this problem.

(Is not this the reason why men to whom after long doubting

the sense of life became clear, could not then say wherein this sense consisted?)

6.522 There is indeed the inexpressible. This *shows* itself; it is the mystical.

6.53 The right method of philosophy would be this. To say nothing except what can be said, *i.e.* the propositions of natural science, *i.e.* something that has nothing to do with philosophy: and then always, when someone else wished to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions. This method would be unsatisfying to the other—he would not have the feeling that we were teaching him philosophy—but it would be the only strictly correct method.

6.54 My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)

He must surmount these propositions; then he sees the world rightly.

7 Whereof one cannot speak, thereof one must be silent.