1) $N_A = 350$ turns, $N_B = 150$ turns and the material of the core is cast steel.

a) We will use Ampere's law to calculate the current that can produce desired magnetic flux density $B = 0.5T$ in the airgap.

$$\int H \cdot dl = N \cdot I$$
$$H \cdot l = \sum N \cdot I = F$$

Where, B is the flux density, H is magnetic field strength and $N.I$ is the mmf.

In our case we can write ;

$$H_c \cdot l_c + H_g \cdot l_g = N \cdot I$$

Since there is no flux leakage and fringing effect ;

$$B_c = B_g = 0.5T$$

Using the graph given in page 6, Fig.1.7 for cast steel magnetic field strength to generate $0.5T$ is found as $H_c = 350 A/m$

From the material equation $B = \mu.H$ and

$$\mu_c = \frac{B_c}{H_c} = \frac{0.5}{H_c} = 1.439 \cdot 10^{-3} H/m$$

Now from the given figure of core we can find the magnetic path lengths l_g and l_c ;

$$l_g = 0.0015m$$

$$l_c = 4 \cdot (0.06 + 0.01 + 0.01) - 0.0015 = 0.3185\text{m}$$

Now magnetic field strength in the airgap is;

$$H_g = \frac{B_g}{\mu_0} = \frac{0.5}{4\pi \cdot 10^{-7}} = 397887.36 A/m$$

From the Ampere's law;

$$F = N \cdot I = H_c \cdot l_c + H_g \cdot l_g = 111.47 + 596.83 = 708.3\ Aturns$$

- For the case where coils are wound and supplied to generate flux in the same direction;

$$F = N_A I_1 + N_B I_1$$
$$I_1 = \frac{F}{N_A + N_B} = \frac{708.3}{350 + 150} = 1.416A$$

- For the case where coils are wound and supplied to generate flux in the opposite direction;

$$F = N_A I_2 - N_B I_2$$

$$I_2 = \frac{F}{N_A - N_B} = \frac{708.3}{350 - 150} = 3.54A$$

b) Self Inductance can be found by using;

$$L = \frac{\phi}{I} = \frac{N^2}{R}$$

Where R is the reluctance of the magnetic path and given by ;

$$R = \frac{l}{\mu \cdot A}$$

$$A_g = A_c = 4 \cdot 10^{-4} m^2$$

Reluctance for the airgap;

$$R_g = \frac{l}{\mu_0 \cdot A_g} = \frac{0.0015}{4\pi \cdot 10^{-7} \cdot 4 \cdot 10^{-4}} = 2984155 \; Aturns / Wb$$

Similarly for the core;

$$R_c = \frac{l}{\mu_c \cdot A_c} = \frac{0.3185}{1.439 \cdot 10^{-3} \cdot 4 \cdot 10^{-7}} = 556818 \; Aturns / Wb$$

So the inductances;

$$L_A = \frac{N_A^2}{R_c + R_g} = \frac{350^2}{2984155 + 556818} = 34.6 \; mH$$

$$L_B = \frac{N_B^2}{R_c + R_g} = \frac{150^2}{2984155 + 556818} = 6.35 \; mH$$

c) $I_A = 2A$ is given and coil B is disconnected. Using the Ampere's Law ;

$$F = N_A \cdot I_A = H_c \cdot l_c + H_g \cdot l_g$$

$$350 \cdot 2 = \frac{B_c}{\mu_c} \cdot l_c + \frac{B_g}{\mu_0} \cdot l_g$$

Since there is no leakage flux and fringing $B_c = B_g$

$$700 = B_g \left[\frac{l_c}{\mu_c} + \frac{l_g}{\mu_0} \right]$$

$$B_g = 0.49T$$

2) Given parameters:

$N_1 = 200$ turns, $N_2 = 400$ turns

$B = 1.2 \cdot \sin(377t)$

$A_c = 25cm^2 = 25 \cdot 10^{-4} m^2,\ l_c = 90cm = 0.9m$

Stacking factor $sf = 0.95$

$\mu_r = 10000$

a) Faraday's law of induction :

$$e(\mathrm{t}) = N \cdot \frac{d\phi}{dt}$$

$$\phi = B \cdot \mathrm{A}_{eff} = B \cdot A_c \cdot sf$$

$$\phi = 25 \cdot 10^{-4} \cdot 0.95 \cdot 1.2 \cdot \sin(377t) = 2.85 \cdot 10^{-3} \cdot \sin(377t)$$

Placing flux to the Faraday's law we obtain;

$$e_1(\mathrm{t}) = 200 \cdot 2.85 \cdot 10^{-3} \cdot 377 \cdot \cos(377t) = 215\cos(377t)$$

$e_{1\max} = 215V$ since maximum value of *cos* function is '1'. Therefore rms value is;

$$e_{1rms} = \frac{e_{\max}}{\sqrt{2}} = 152V$$

b) From Ampere's Law:

$$h \cdot l = N \cdot i$$

$$i = \frac{h \cdot l}{N}$$

$$h = \frac{B}{\mu} = \frac{B}{\mu_r \mu_0} = \frac{1.2 \cdot \sin(377t)}{10000 \cdot 4\pi \cdot 10^{-7}}$$

$$h = 95.5\sin(377t)$$

Therefore *i* is:

$$i = \frac{h \cdot l}{N} = \frac{95.5 \cdot \sin(377t) \cdot 0.9}{200}$$

$$i = 0.43 \cdot \sin 377t\ A$$

$$I_{\max} = 0.43 \text{ and } I_{rms} = \frac{I_{\max}}{\sqrt{2}} = 0.3A$$

c) From the Faraday's law:

$$e_2(\mathrm{t}) = N_2 \cdot \frac{d\phi}{dt}$$

We have found ϕ as:

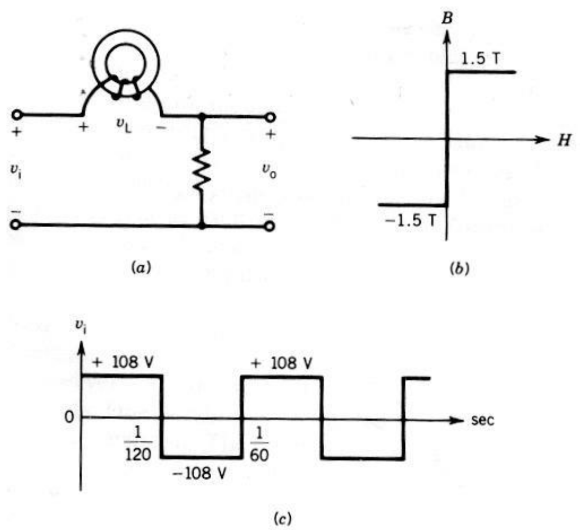
$$\phi = 2.85 \cdot 10^{-3} \cdot \sin(377t)$$

Therefore induced voltage in the second winding is:

$$e_2(t) = 400 \cdot 377 \cdot 2.85 \cdot 10^{-3} \cdot \cos(377t)$$

$$e_{2\max} = 429.8 \text{ and } e_{2rms} = \frac{e_{2\max}}{\sqrt{2}} = 304V$$

3)



Given parameters :

$A = 2cm^2$

$f = 60Hz$

$N = 1000$ turns

From the given circuit and using the Kirchoff's law:

$$U_i = U_L + U_0 \quad (1)$$

And from the Faraday's law:

$$U_L = \frac{d\phi}{dt} = N \cdot A \cdot \frac{dB}{dt} \quad (2)$$

$$U_0 = R \cdot i \quad (3)$$

From the Ampere's law we know that:

$$H \cdot l = N \cdot i$$
$$i = \frac{H \cdot l}{N}$$

Putting this into eq. (2) gives:

$$U_0 = R \cdot \frac{H \cdot l}{N} \quad (4)$$

And if we place this equation into the eq.(1) we obtain:

$$U_i = N \cdot A \cdot \frac{dB}{dt} + \frac{R \cdot H \cdot l}{N}$$

- For $t = 0$:

From given characteristic curve $B = 0$, $H = 0$ since $H = \frac{B}{\mu}$.

So from eq. (4) $U_0 = 0$ and from eq. (1) $U_i = U_0$

Therefore,

$$U_i = N \cdot A \cdot \frac{dB}{dt} \quad (5)$$

At $t = 0$, $U_i = 108V$ (from given waveform). From the eq. (5):

$$108 = \frac{2}{10000} \cdot 1000 \cdot \frac{dB}{dt}$$

$$\frac{dB}{dt} = 540$$

$$B = \int 540 \cdot dt$$

$$B = 540 \cdot t$$

For $B = 1.5T$

$$1.5 = 540 \cdot t$$
$$t_s = \frac{1.5}{540} = \frac{1}{360} s$$

- For $0 \le t \le t_s$

From the B-H curve we can find the values of *B* and *H*:

$$B < 1.5T$$
$$H = 0\ A/m$$
$$U_0 = 0V$$
$$U_L = U_i = 108V$$

- For $t_s \le t \le \frac{1}{120}$

$$B = 1.5T$$
$$\frac{dB}{dt} = 0 \text{ and } U_L = 0$$
$$U_0 = U_i = 108V$$

- For $t \ge \frac{1}{120}$

$$U_i = -108V$$
$$-1.5T < B < 1.5T$$

When B=0;

$$H = 0\ A/m$$
$$U_0 = 0V$$

So,

$$U_L = U_i = -108V$$
$$-108 = N \cdot A \cdot \frac{dB}{dt}$$
$$\frac{dB}{dt} = -\frac{108}{0.2} = -540$$
$$B = \int -540 \cdot dt$$
$$B = -540 \cdot t + B_0$$

Therefore,

$$-1.5 = -540 \cdot t_{sw} + 1.5$$

Where $t_{sw} = 1/180s$.

Total time is : $\frac{1}{120} + \frac{1}{180} = \frac{1}{72}s$

- For $\frac{1}{72} \le t \le \frac{1}{60}$

$$B = -1.5T$$
$$\frac{dB}{dt} = 0$$
$$U_L = 0V$$
$$U_i = U_0 = -108V$$