CS-E4850 Computer Vision
Exercise Round 1

The problems should be solved before the exercise session and solutions returned via the MyCourses page in a single PDF file. The time for completing the first round is less than for the later rounds but the problems are relatively easy. Handwritten solutions are fine if they are scanned to PDF format and clear to read. You do not need to solve all tasks as points will be rewarded also to partial solutions.

If you are not familiar with homogeneous coordinates or projective geometry in advance, it will be helpful to first read Sections 2.1-2.5 from Chapter 2 of the book *Multiple View Geometry in Computer Vision* by Hartley and Zisserman. Electronic version of the book is available via university library at `https://web.lib.aalto.fi/en/aaltoreader/`.

Exercise 1. Homogeneous coordinates.
a) The equation of a line in the plane is

$$ax + by + c = 0.$$

Show that by using homogeneous coordinates this can be written as

$$\mathbf{x}^\top \mathbf{l} = 0$$

where $\mathbf{l} = \begin{pmatrix} a & b & c \end{pmatrix}^\top$.
b) Show that the intersection of two lines $\mathbf{l}$ and $\mathbf{l}'$ is the point $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$.
c) Show that the line through two points $\mathbf{x}$ and $\mathbf{x}'$ is $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$.
d) Show that for all $\alpha \in \mathbb{R}$ the point $\mathbf{y} = \alpha\mathbf{x} + (1 - \alpha)\mathbf{x}'$ lies on the line through points $\mathbf{x}$ and $\mathbf{x}'$.
(Hint: In tasks b, c, d above, you can utilize the fact that for three-element vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ the scalar triple product, $(\mathbf{a} \times \mathbf{b})^\top \mathbf{c}$, is zero if any two of the vectors are parallel.)

Exercise 2. Transformations in 2D.
a) Use homogeneous coordinates and give the matrix representations of the following transformation groups: translation, Euclidean transformation (rotation+translation), similarity transformation (scaling+rotation+translation), affine transformation, projective transformation.
b) What is the number of degrees of freedom in these transformations?
c) Why is the number of degrees of freedom in a projective transformation less than the number of elements in a 3×3 matrix?
(Hint: The answers to the first two sub-tasks are directly given in Table 2.1 in Hartley & Zisserman.)

Exercise 3. Planar projective transformation.
The equation of a line on a plane, $ax + by + c = 0$, can be written as $\mathbf{l}^\top \mathbf{x} = 0$, where $\mathbf{l} = [a\ b\ c]^\top$ and $\mathbf{x}$ are homogeneous coordinates for lines and points, respectively. Under a planar projective transformation, represented with an invertible 3×3 matrix $\mathbf{H}$, points transform as

$$\mathbf{x}' = \mathbf{H}\mathbf{x}.$$

a) Given the matrix $\mathbf{H}$ for transforming points, as defined above, define the line transformation (i.e. transformation that gives $\mathbf{l}'$ which is a transformed version of $\mathbf{l}$).

b) A projective invariant is a quantity which does not change its value in the transformation. Using the transformation rules for points and lines, show that two lines, $\mathbf{l}_1, \mathbf{l}_2$, and two points, $\mathbf{x}_1, \mathbf{x}_2$, not lying on the lines have the following invariant under projective transformation:

$$I = \frac{(\mathbf{l}_1^\top \mathbf{x}_1)(\mathbf{l}_2^\top \mathbf{x}_2)}{(\mathbf{l}_1^\top \mathbf{x}_2)(\mathbf{l}_2^\top \mathbf{x}_1)}.$$

Why similar construction does not give projective invariants with fewer number of points or lines? (Hint: Projective invariants defined via homogeneous coordinates must be invariant also to arbitrary scaling of the homogeneous coordinate vectors with a non-zero scaling factor.)

Note: Exercise 3 above is from Chapter 2 of the book by Hartley and Zisserman and that chapter is helpful reference.