# Computer Vision

CS-E4850, 5 study credits

Juho Kannala
Aalto University

# Plan for today

- Background
  - What is computer vision?
  - Why to study computer vision?
- Overview of the course
- Lecture 1: Image formation

Credits: Material for slides borrowed from Victor Prisacariu, Andrew Zisserman, Esa Rahtu, James Hays, Derek Hoiem, Svetlana Lazebnik, Steve Seitz, David Forsyth, and others

# Course personnel

- Lecturer:
  **Juho Kannala**
  juho.kannala@aalto.fi

- Main course assistant:
  **Xiaotian Li**
  firstname.lastname@aalto.fi

# A few words about me

Juho Kannala

Assistant Professor of Computer vision

- PhD, University of Oulu 2010
- Professor at Aalto since 2016

- Working with computer vision since 2000
- Recent projects and other info available on my homepage: https://users.aalto.fi/~kannalj1/

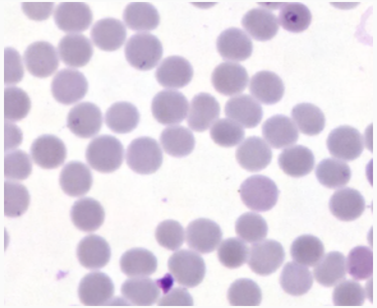
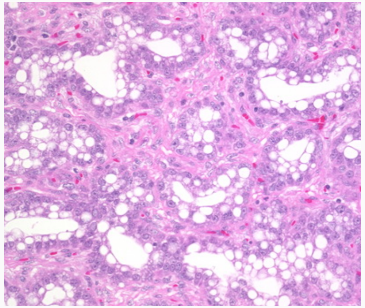
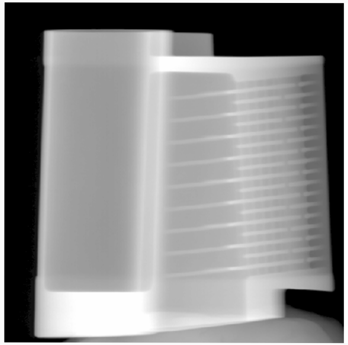
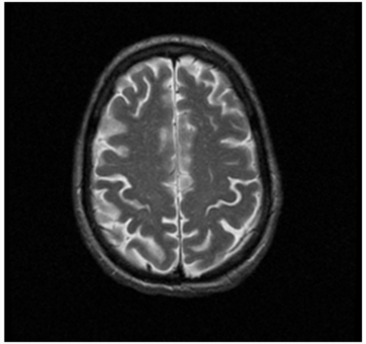
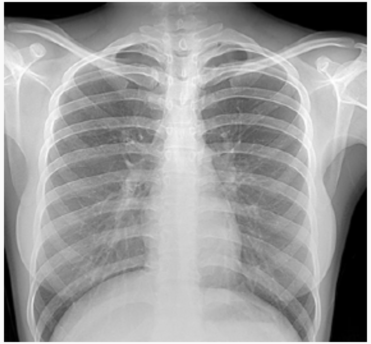
# Motivation - what is computer vision?

# Make computers understand images

- What kind of scene?
- Where are the cars?
- How far are the buildings?
- Where are the cars going?
- …..

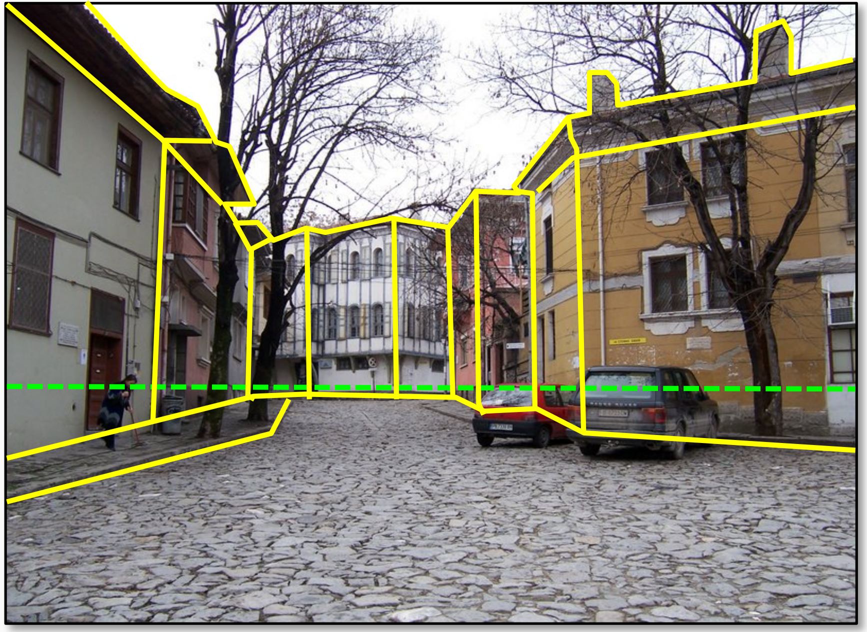
# Many data modalities

- 2D or 3D still images
- Video frames
- X-ray
- Ultra-sound
- Microscope
- ….

# What kind of information can be extracted?


Semantic information



Geometric information

# What do we have here?

# Wrong! Very hard big data problem…

- Hardware perspective:
  - RGB stereo images with 30 frames per second -> 100s MB/s data stream.
  - Non-trivial processing per each byte.
  - Massive image collections.
- Mathematical perspective
  - Information is highly implicit or lost by perspective projection
  - 2D -> 3D mapping is ill-posed and ill-conditioned -> need to use constraints

# Wrong! Very hard big data problem…

- Artificial intelligence perspective
  - Images have uneven information content
  - Computational visual semantics is hard (what does visual stuff mean exactly?)
  - If we have limited time, what is the important visual stuff right now?

Still a massive challenge - if we want genuine autonomy.

# Natural vision

- Humans see effortlessly

# Natural vision

- Humans see effortlessly, but… it is very hard work for our brains!
  - There are billions of neurons in human brain
  - Years of evolution generated hardwired priors.

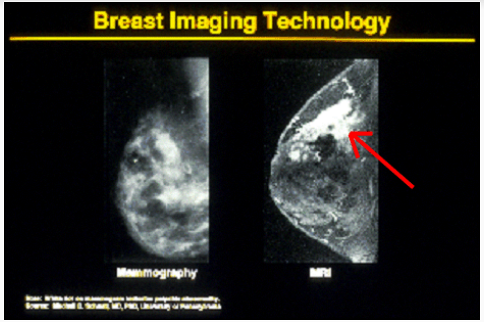
**So why bother?**
**What are the advantages?**

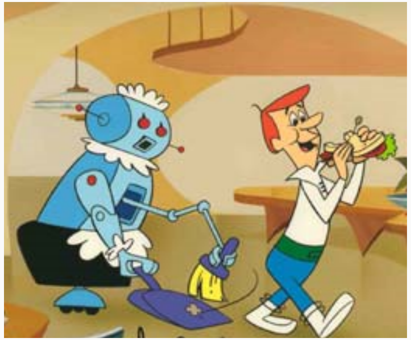
# Why computer vision matters?

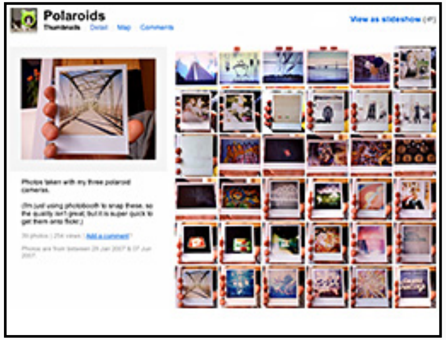
- Engineering point of view - Computer Vision helps to solve many practical problems: business potential
- Scientific point of view - Human kind of visual system is one of the grand challenges of Artificial Intelligence (AI)
    - AI itself is a grand challenge of computing

# Why computer vision matters?

- Safety
- Health
- Security
- Fun
- Access
- ….





# Computer vision is already here

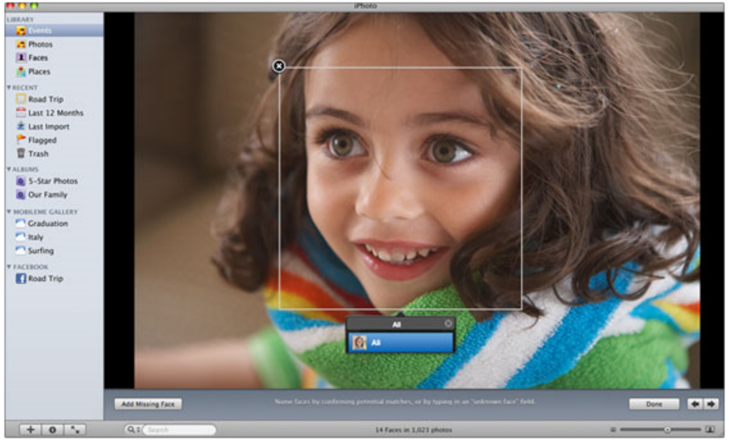
- You are surrounded by devices using computer vision
- Imagine what can be done with already installed cameras!

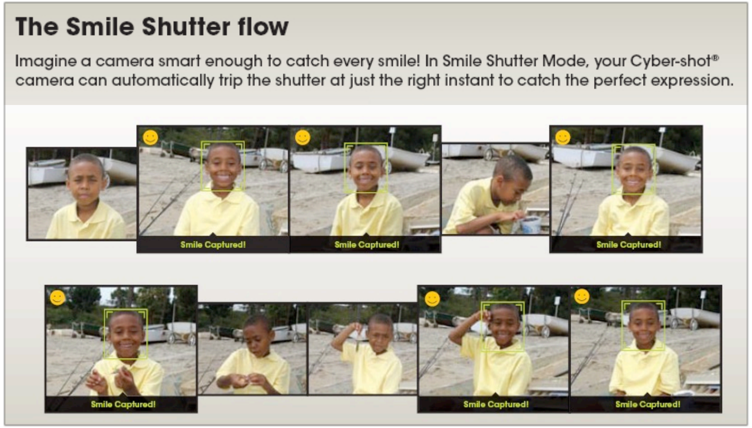
# Motivation - Success stories

# Recognizing “simple” patterns

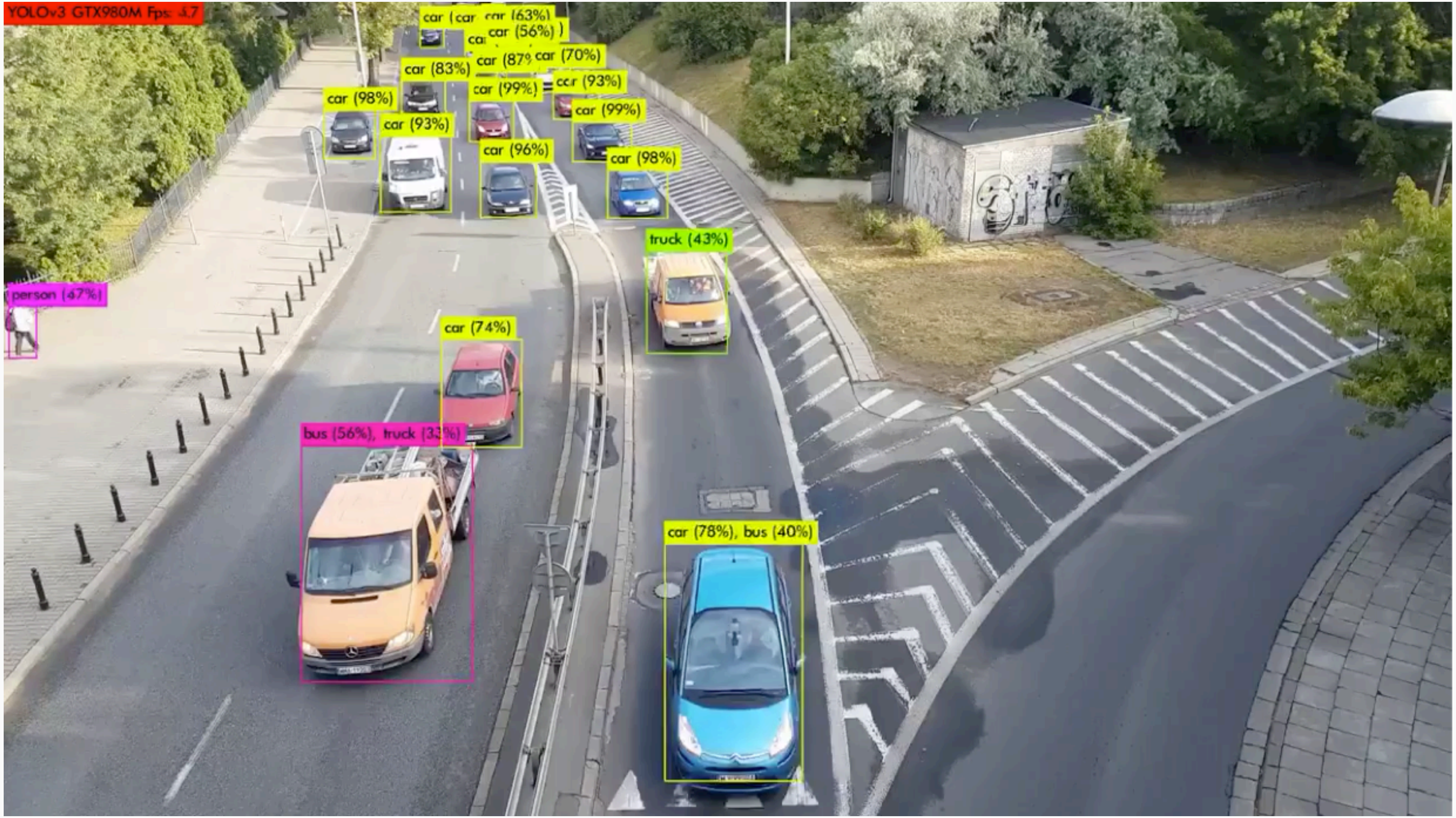



# Face recognition



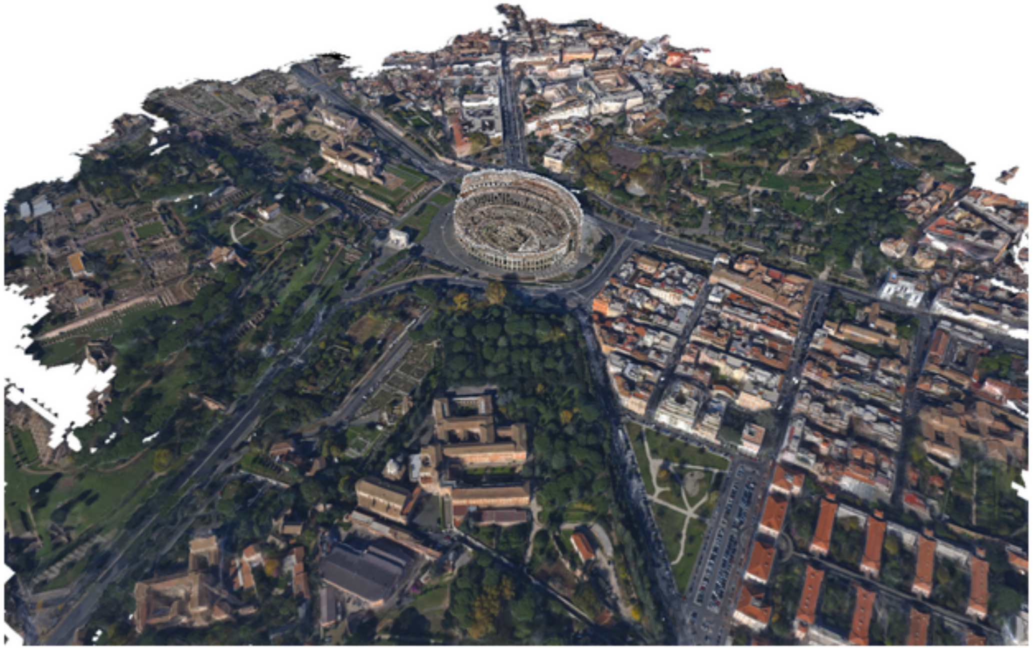




# Object detection and recognition



# Reconstruction: 3D from photo collections

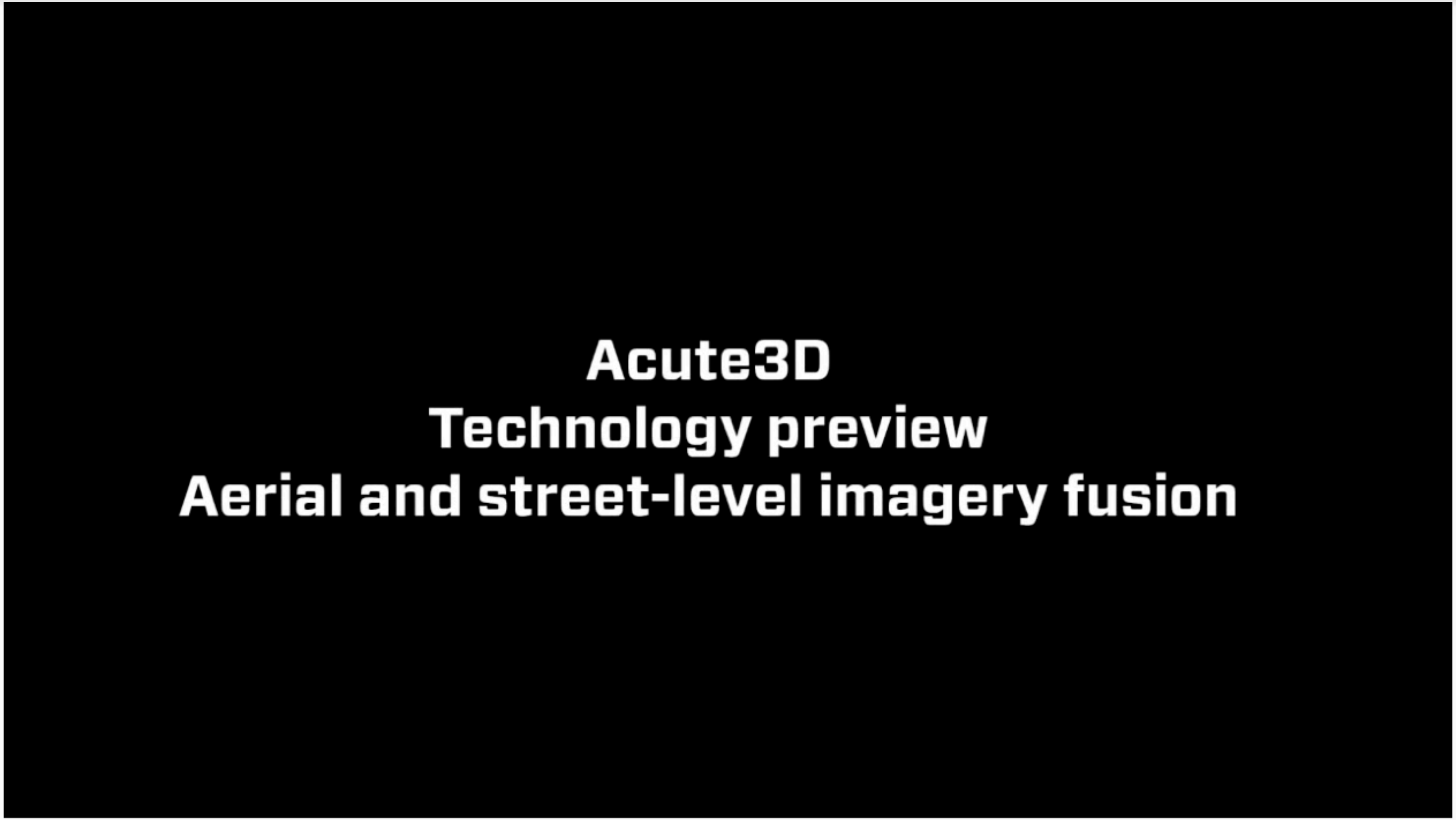
Colosseum, Rome, Italy



San Marco Square, Venice, Italy

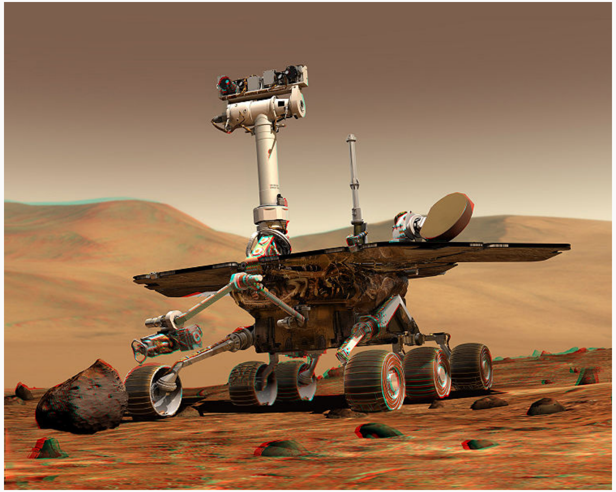
The Visual Turing test for Scene Reconstruction,
Shan, Adams, Curless, Furukawa, Seitz, in 3DV 2013. [YouTube video].

# A recent commercial 3D reconstruction system

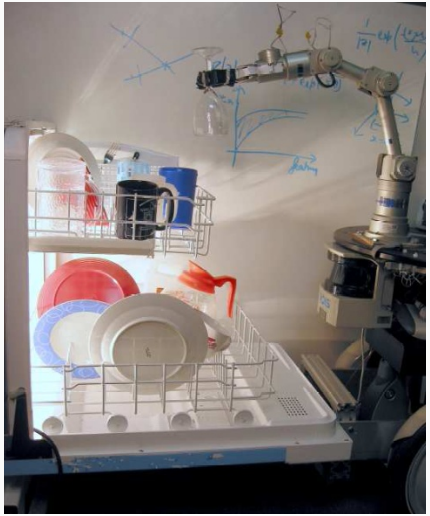



[YouTube]

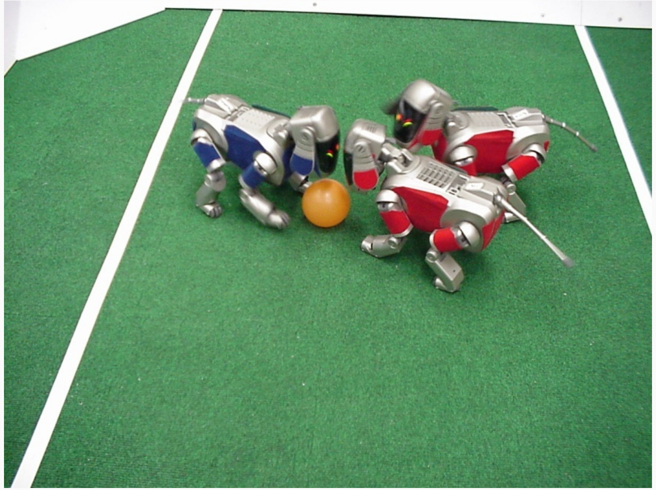
# Robotics



NASA's Mars Rover
See "Computer Vision on Mars"

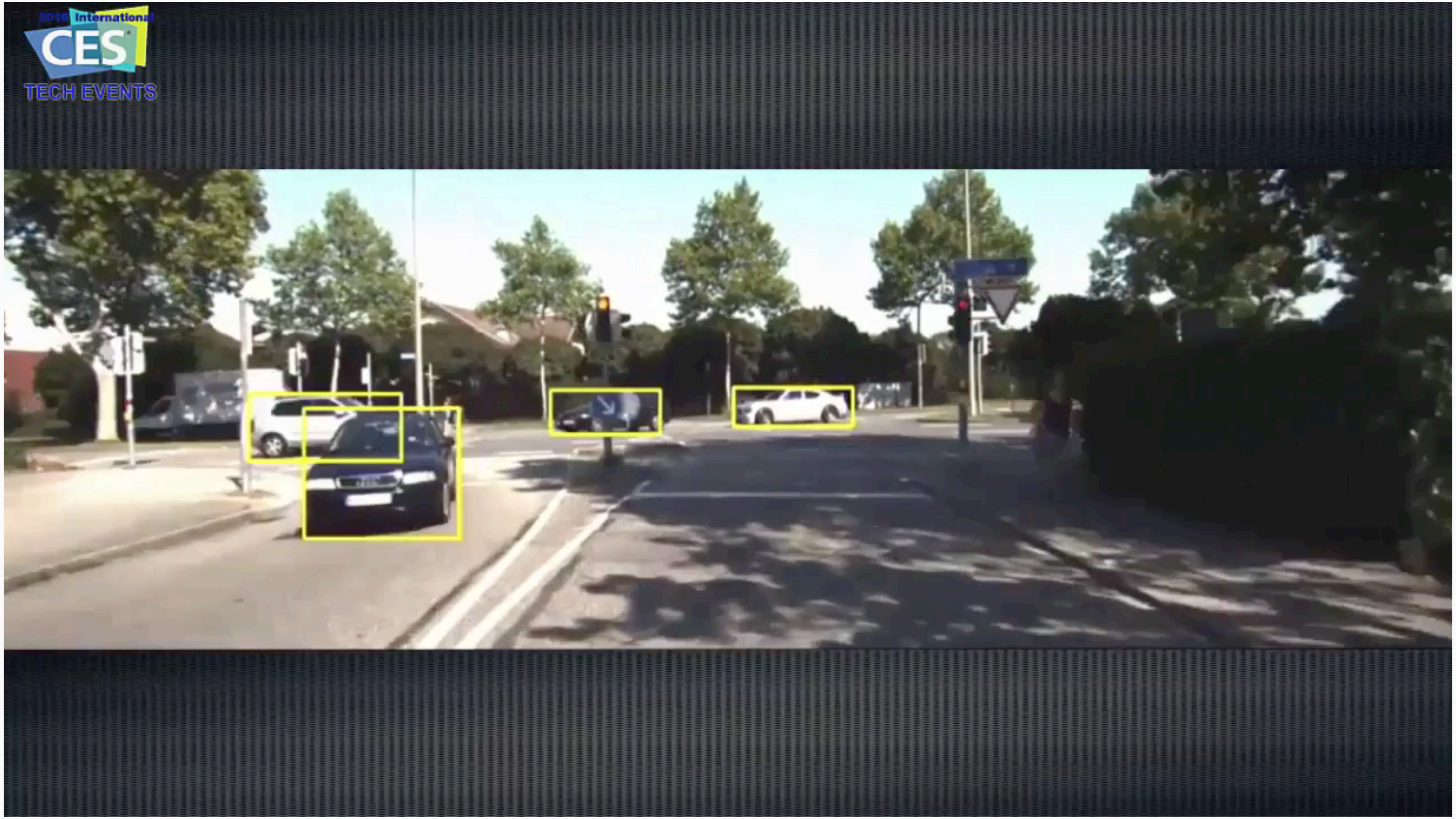


STAIRS at Stanford
Saxena et al. 2008

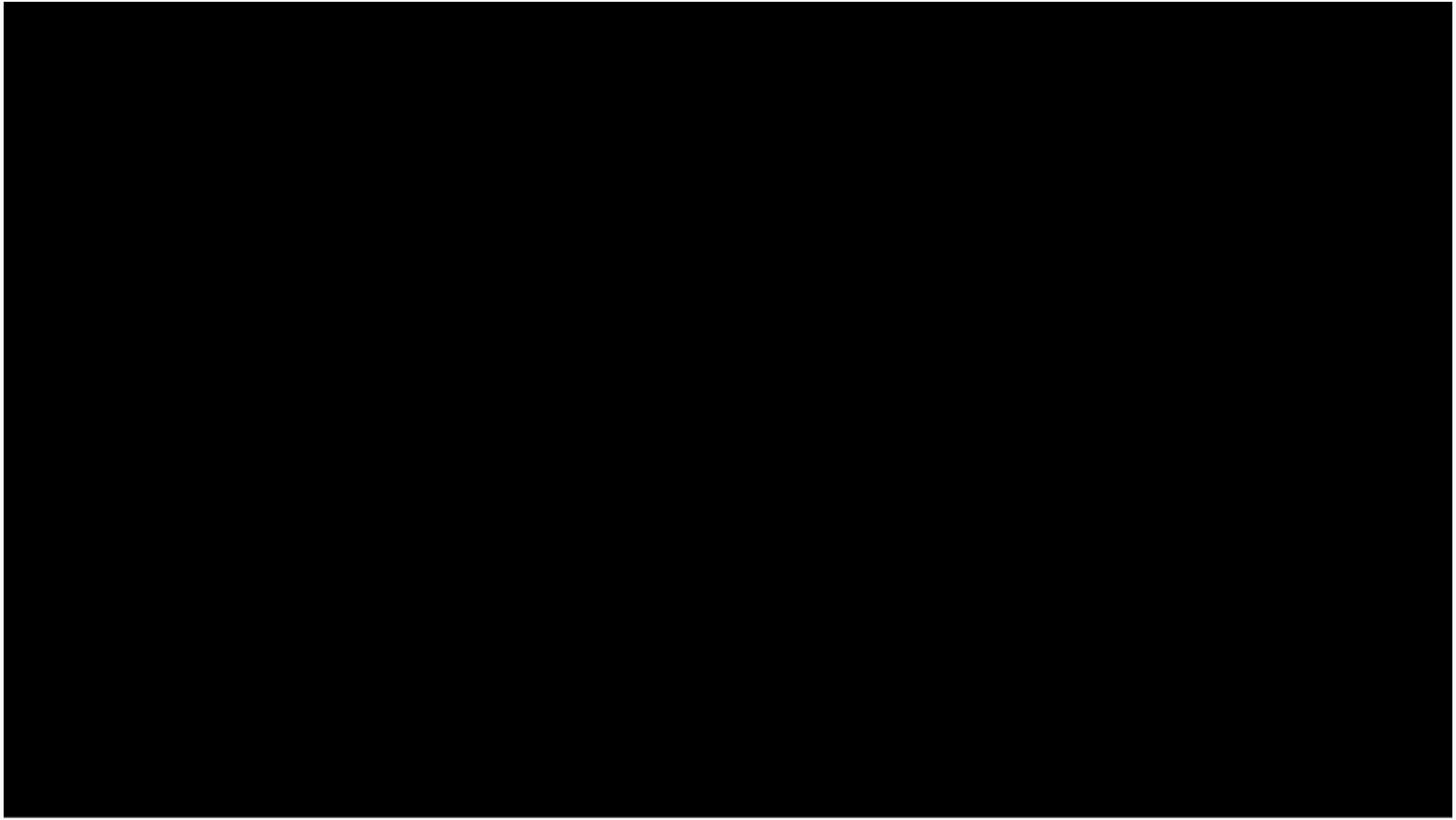


Robocup
See www.robocup.org

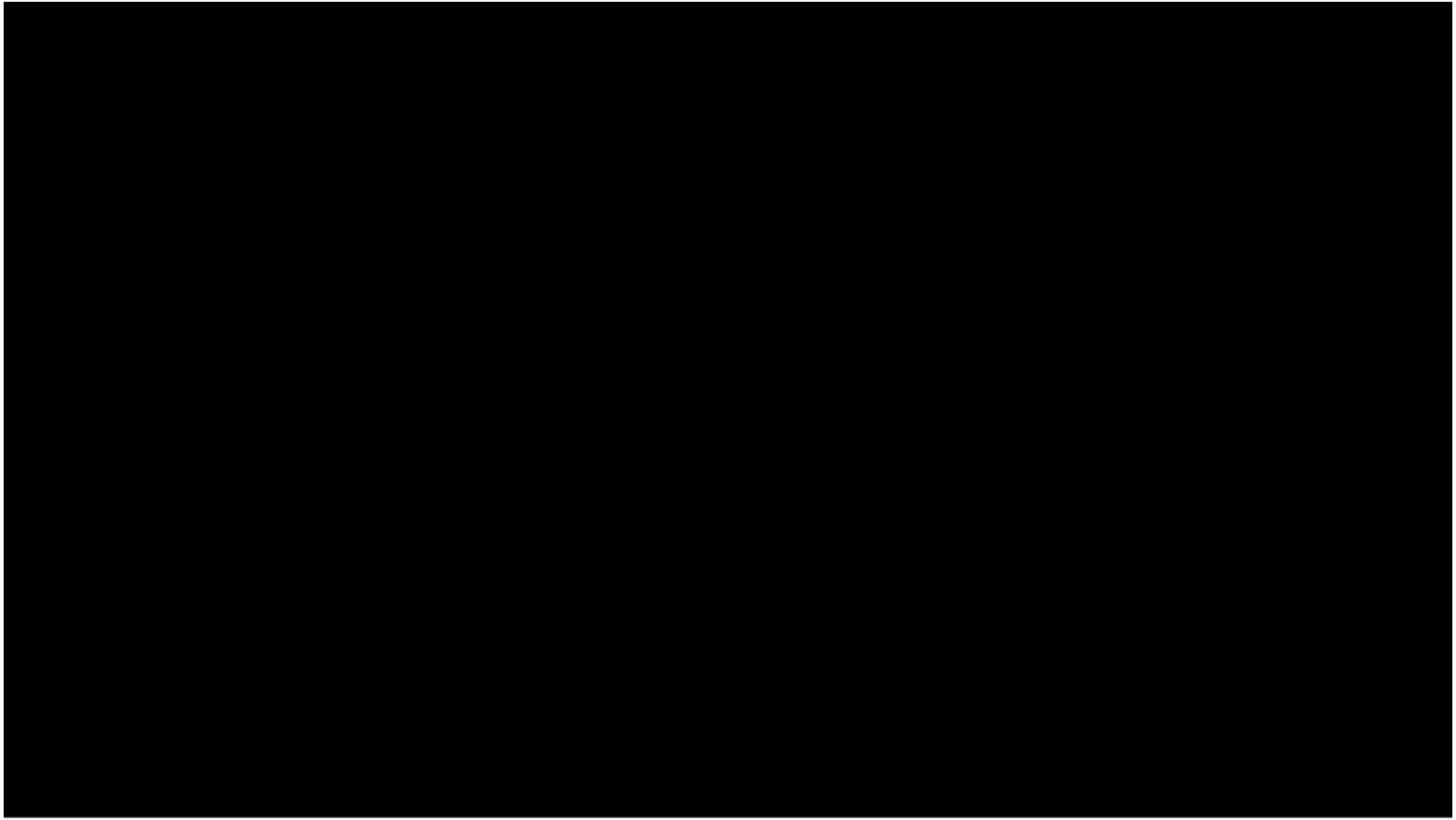
# Self-driving cars (Nvidia @ CES 2016)



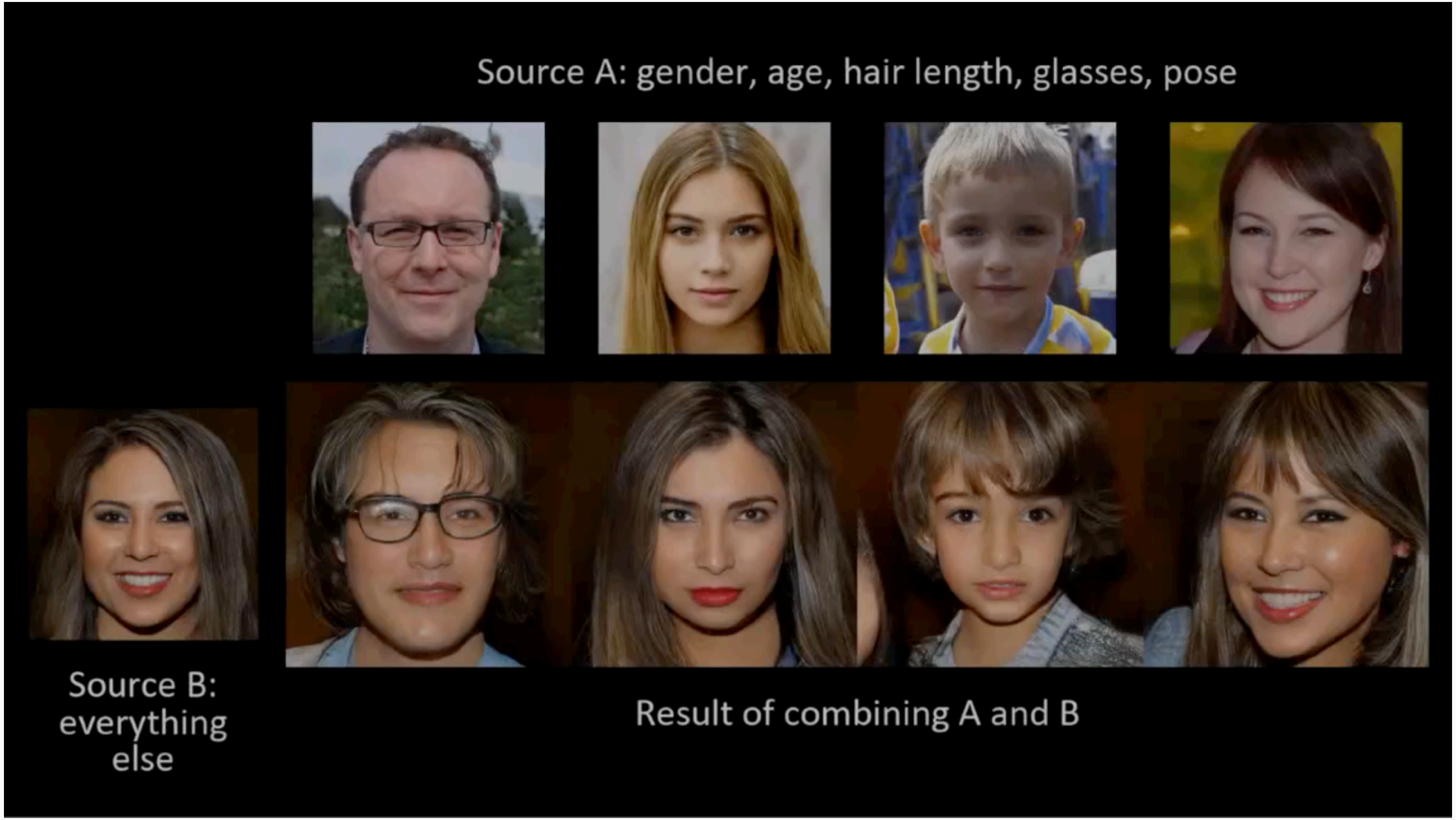
# Visual odometry and SLAM



# Augmented Reality (AR) and Virtual Reality (VR)



# Image generation

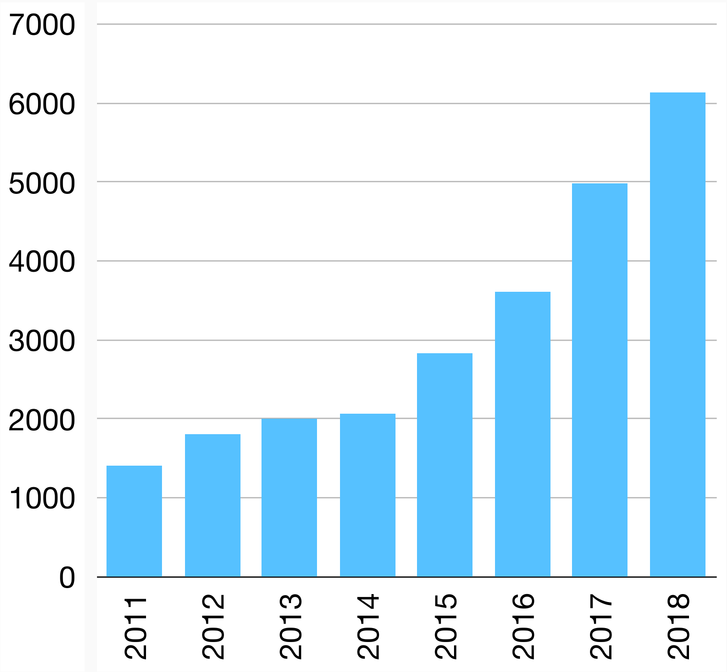


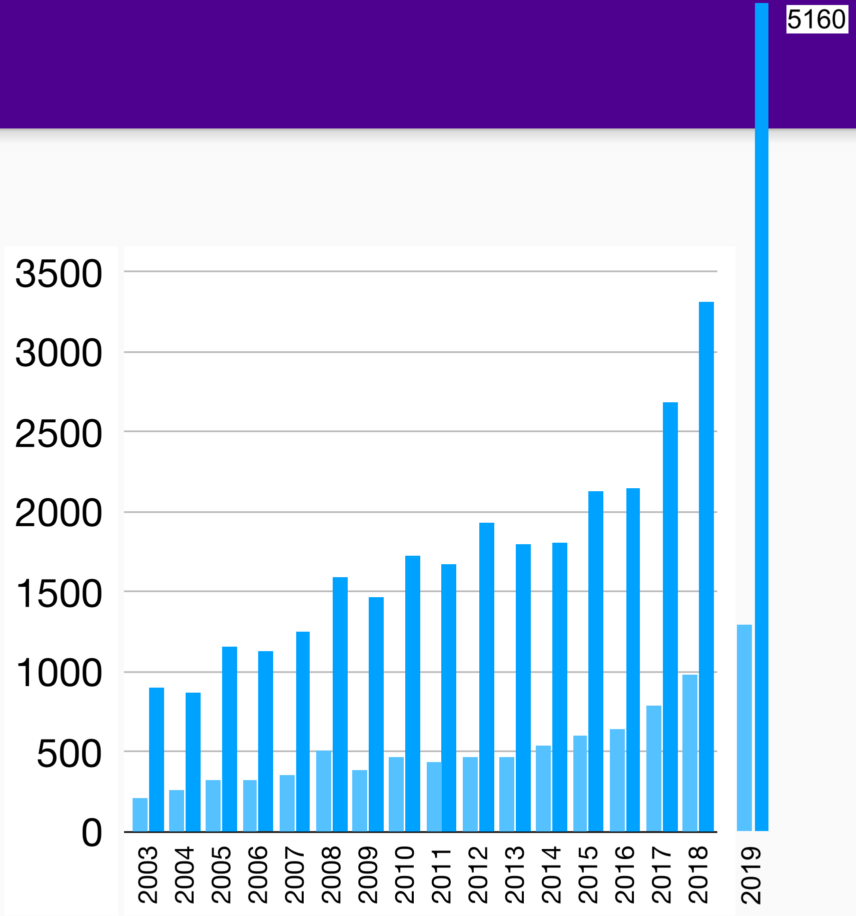

A style-based generator architecture for generative adversarial networks. Karras, Laine, Aila. CVPR 2019.

# Current state of the affairs

- Many of the previous examples are less than 5 years old!
- Many new applications to appear in the next 5 years
- Strong open source culture
  - Many recent state-of-the-art methods are freely available
  - See papers from top conferences like CVPR, ECCV, ICCV, and NeurIPS

# Rapidly growing area




Attendees and submissions to IEEE Conference on Computer Vision and Pattern Recognition (CVPR)

# Rapidly growing area

| | Publication | h5-index | h5-median |
|---|---|---|---|
| 1. | Nature | 362 | 542 |
| 2. | The New England Journal of Medicine | 358 | 602 |
| 3. | Science | 345 | 497 |
| 4. | The Lancet | 278 | 417 |
| 5. | Chemical Society reviews | 256 | 366 |
| 6. | Cell | 244 | 366 |
| 7. | Nature Communications | 240 | 318 |
| 8. | Chemical Reviews | 239 | 373 |
| 9. | Journal of the American Chemical Society | 236 | 309 |
| 10. | Advanced Materials | 235 | 336 |
| 11. | Proceedings of the National Academy of Sciences | 226 | 291 |
| 12. | Angewandte Chemie International Edition | 213 | 295 |
| 13. | JAMA | 209 | 309 |
| 14. | Nucleic Acids Research | 208 | 392 |
| 15. | ACS Nano | 199 | 279 |
| 16. | Physical Review Letters | 197 | 286 |
| 17. | Energy and Environmental Science | 196 | 330 |
| 18. | Journal of Clinical Oncology | 196 | 279 |
| 19. | Nano Letters | 194 | 281 |
| 20. | IEEE Conference on Computer Vision and Pattern Recognition, CVPR | 188 | 302 |

Ref. Google Scholar top publications.

# Rapidly growing area - substantial commercial interest

Platinum Sponsors

Gold Sponsors

Silver Sponsors

CVPR 2018 sponsors

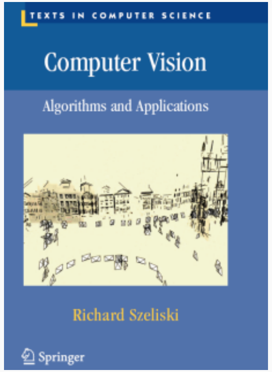
# Plenty of job opportunities

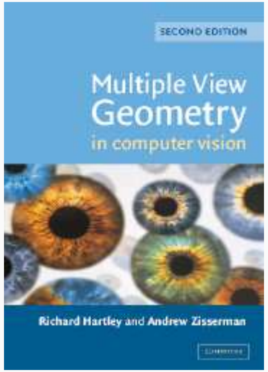
- Companies are looking for computer vision and deep learning experts.
- Big Internet players are investing heavily (Apple, Google, Facebook, Microsoft, Baidu, Tencent, …) as well as car industry (Tesla, BMW,…)
- Strong imaging ecosystem also in Finland

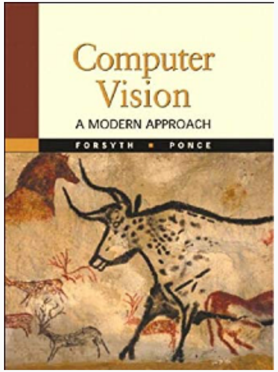
# Specifics of this course

# Course textbooks

- Szeliski: Computer Vision
  - Full-copy freely available
- Hartley & Zisserman: Multiple View Geometry in Computer Vision
  - Available as an e-book via library
- Forsyth & Ponce: Computer Vision
  - Full-copy freely available



# What will you learn on this course?

- Course content (numbers refer to chapters in Szeliski's book, 1st edition):
    - Image formation and processing (2, 3)
    - Feature detection and matching (4)
    - Feature based alignment and image stitching (6,9)
    - Optical flow and tracking (8)
    - Basics of image classification and convolutional neural networks
    - Object recognition and detection (14)
    - Structure from motion, stereo and 3D reconstruction (7, 11, 12)

# What will you NOT learn on this course?

- Software packages
    - PyTorch, TensorFlow, Keras, Caffe, etc.
    - We have simple exercises with Python/Matlab though
- In-depth deep learning
    - Tweaking architectures, loss functions, etc.
    - Note that there exists a separate deep learning course (CS-E4890)
- All the bells and whistles in the state-of-the-art systems
    - We concentrate on the basic concepts (get them right and the rest is easier for you)

# Organization

- Lectures on Mondays at 10-12 (12 lectures)
- Exercises on Fridays at 12-14 (12 sessions)
  - The solutions of weekly homework assignments should be returned before the session
  - The solutions are presented in the session
- Guidance available if needed
  - Slack and teacher's receptions (see MyCourses)
- Virtual presence is not rewarded, only returned homework counts

# Requirements

- Get more than 0 points from at least 8 exercise rounds (i.e. solve at least 1 task from 8 different weekly rounds)
- Pass the exam

# Hints

- Doing homework takes time but is often a good way to learn in depth
- Try to do more than the minimum - homework points are taken into account in the grading (i.e. weighted exercise points are added to exam points)
- Note that the amount of work and bonus points varies a bit between weeks - exercises are published early so that you can do them in advance if needed

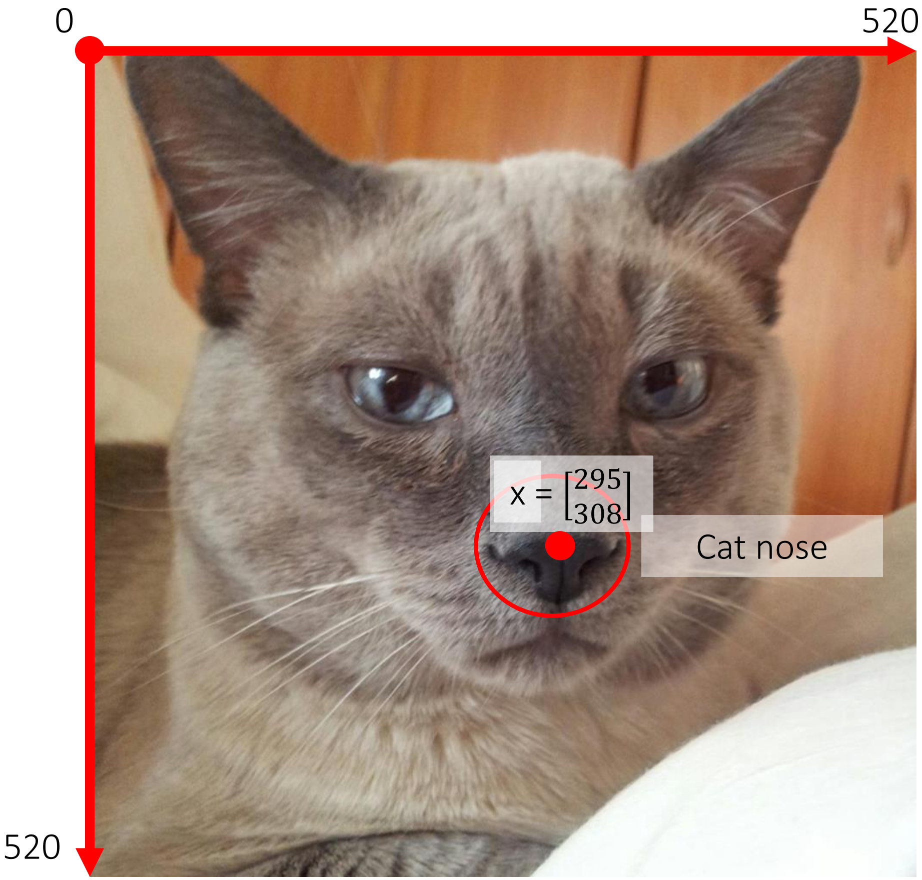
Questions at this point?

# Lecture 1: Camera model

# Relevant reading

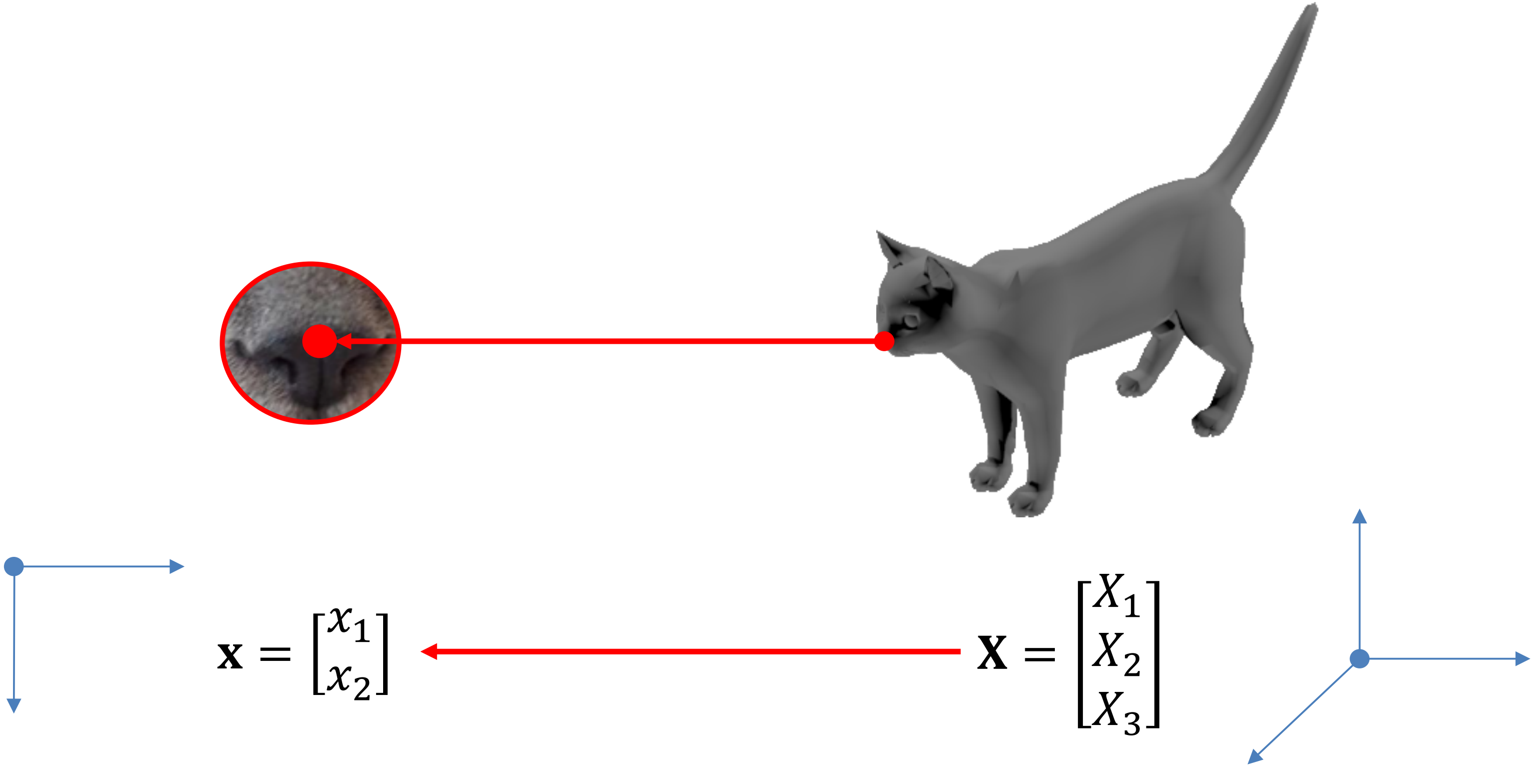
- Chapters 2, 3, and 6 in [Hartley & Zisserman]
  - Comprehensive presentation of the core content
- Chapter 2 in [Szeliski]
  - Broader overview of the image formation

# This is (a picture of) a cat




Credits: Victor Prisacariau

# Cat lives in a 3D world

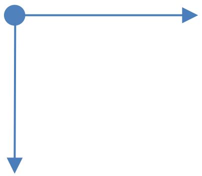


$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longleftarrow \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

The point **X** in world space projects to the point **x** in image space.

Credits: Victor Prisacariau

# Going from X in 3D to x in 2D



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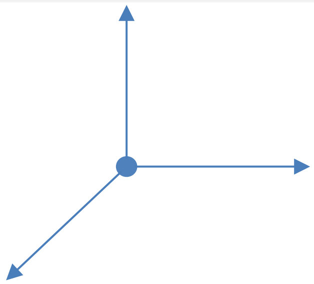
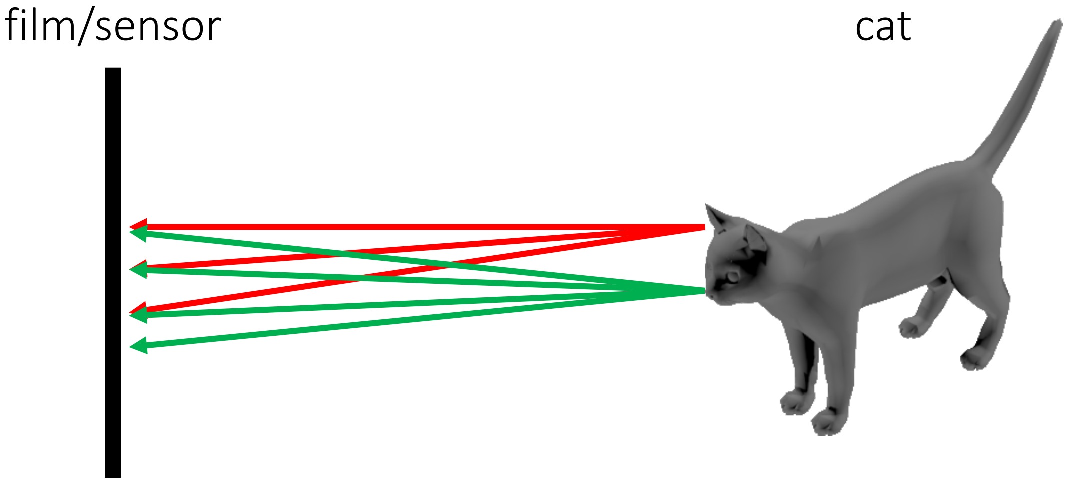
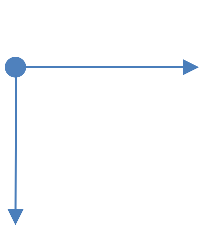





The output would be blurry if film just exposed to the cat.

# Pinhole camera



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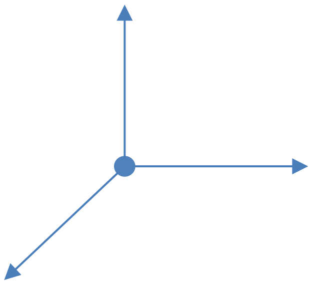
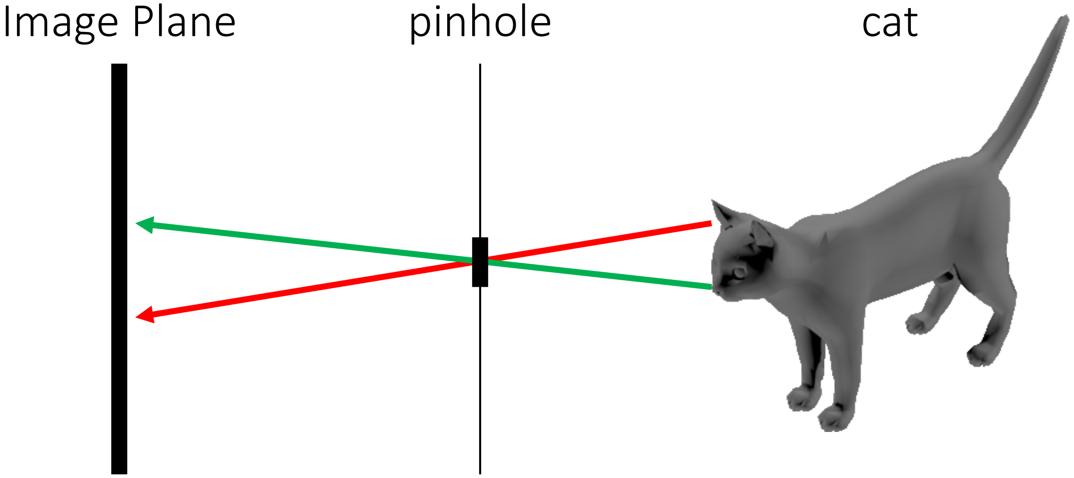
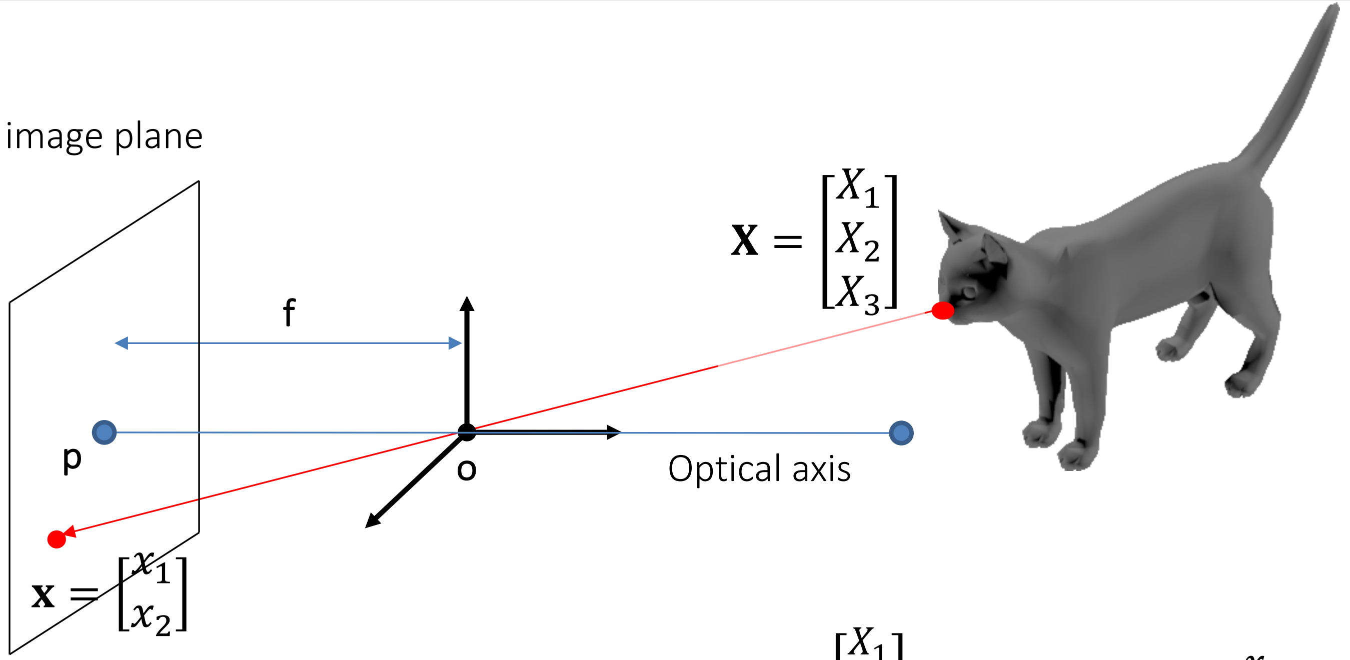





All rays passing through a single point (center of projection)

# Pinhole camera

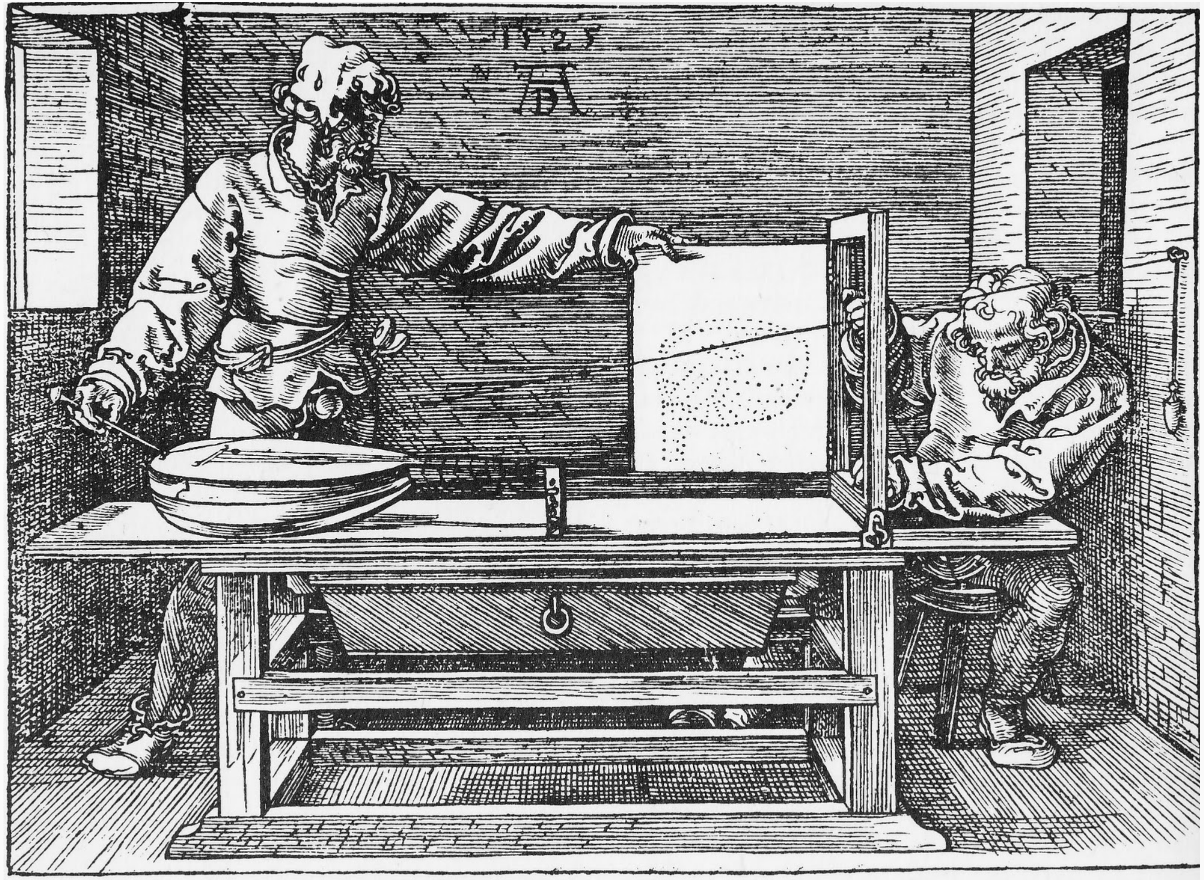



f – focal length
o – camera origin
p – principal point

The 3D point $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ is imaged into $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f\,\frac{X_1}{X_3} \\ f\,\frac{X_2}{X_3} \end{bmatrix}$$

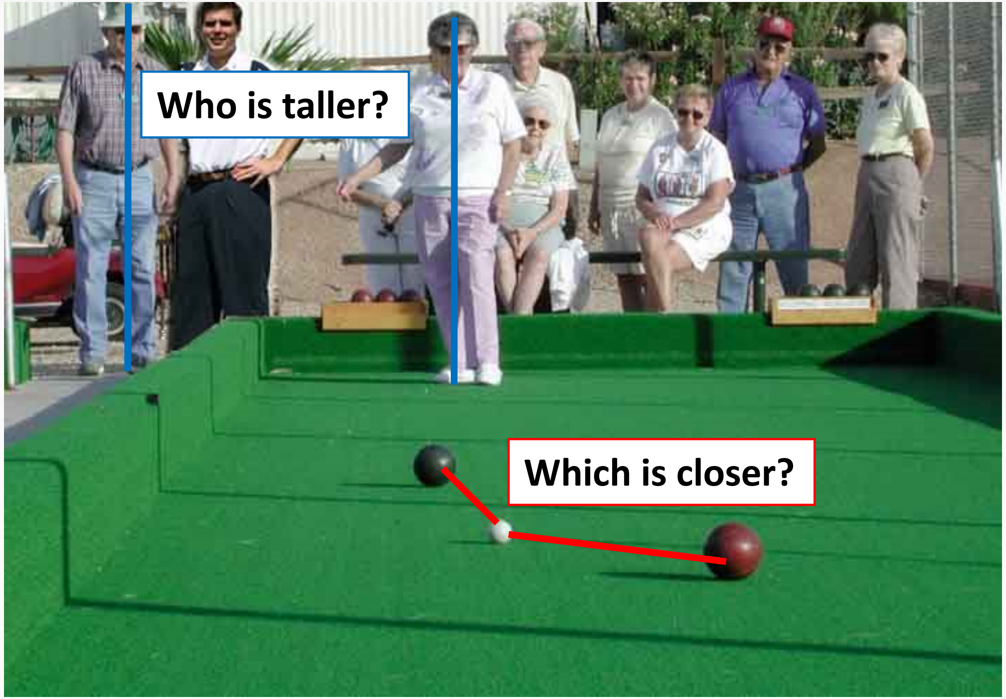
# Pinhole camera



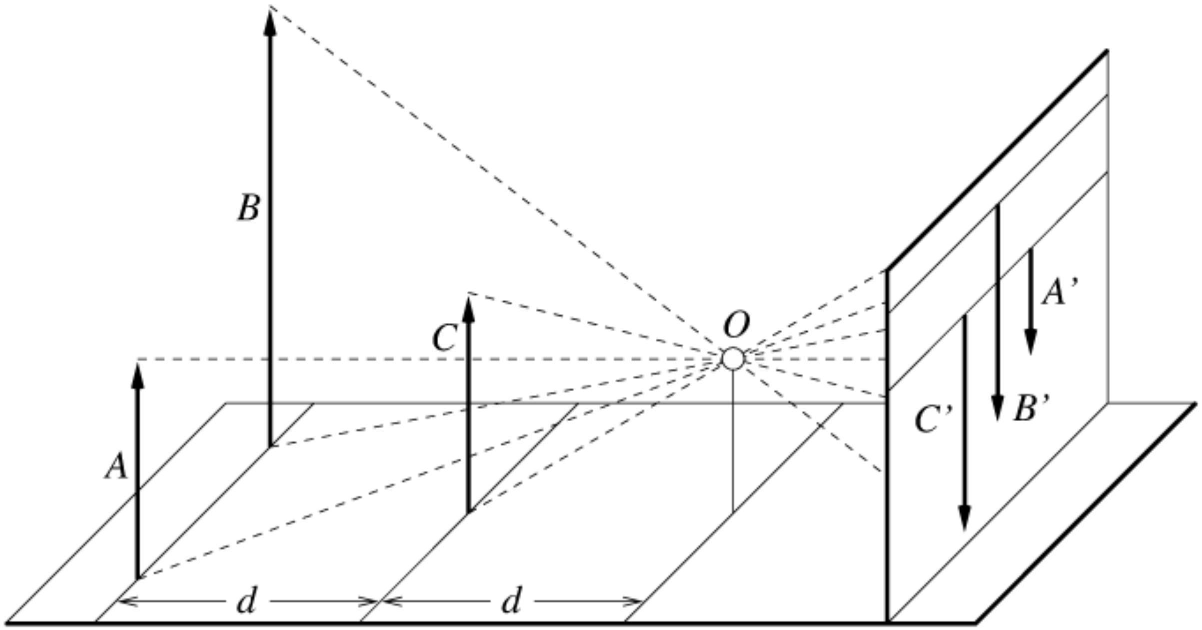
# What happens in the projection?

- Projection from 3D to 2D -> information is lost
- What properties are preserved?
    - Straight lines
    - Incidence
- What properties are not preserved?
    - Angles
    - Lengths

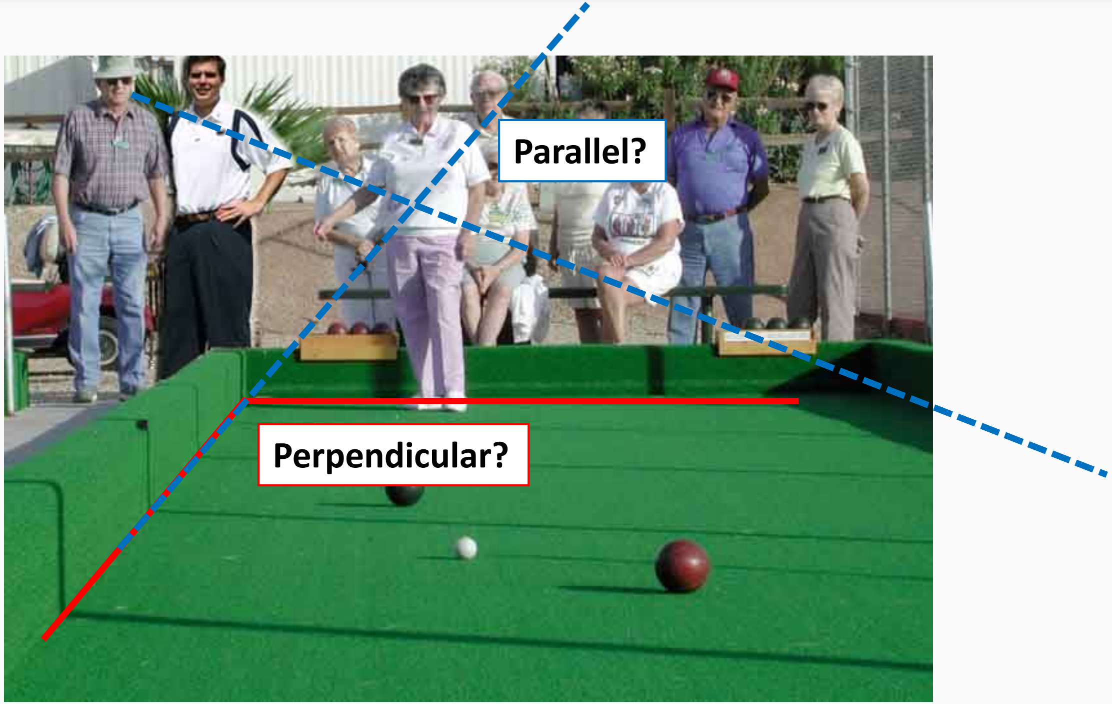
# Projective geometry - what is lost?



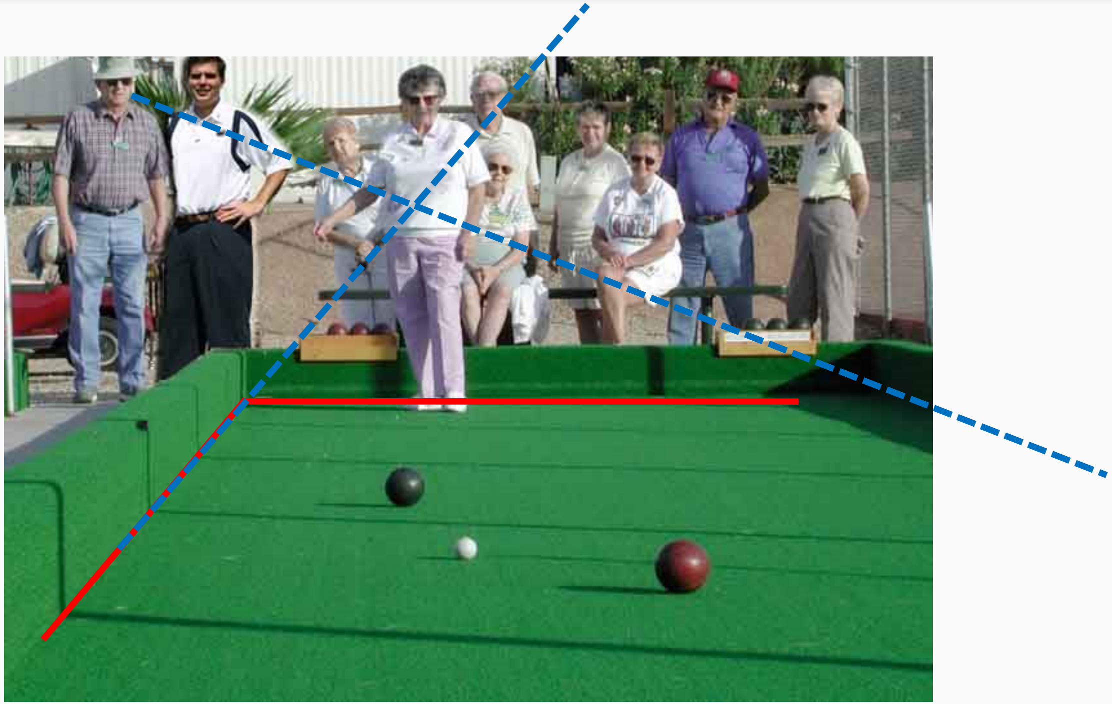
# Length is not preserved



# Angles are not preserved

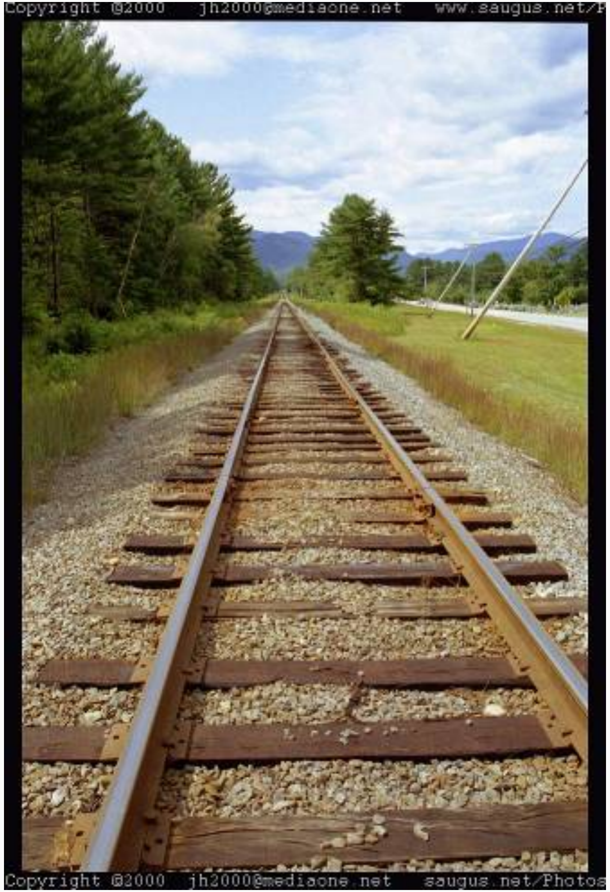


# Straight lines are still straight

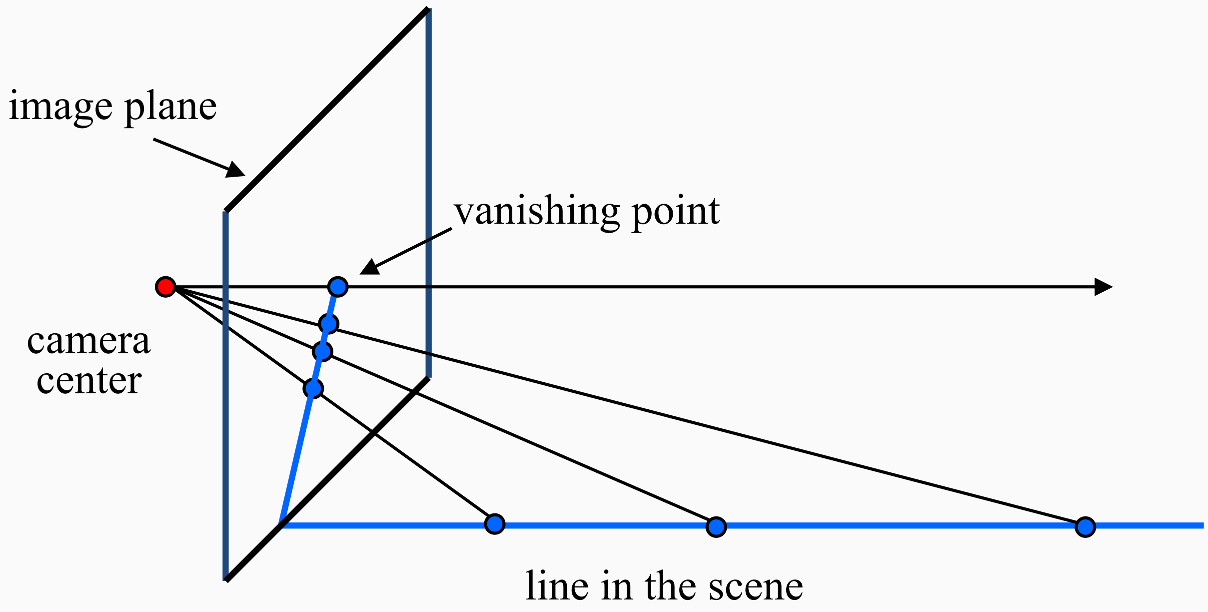


# Vanishing points and lines

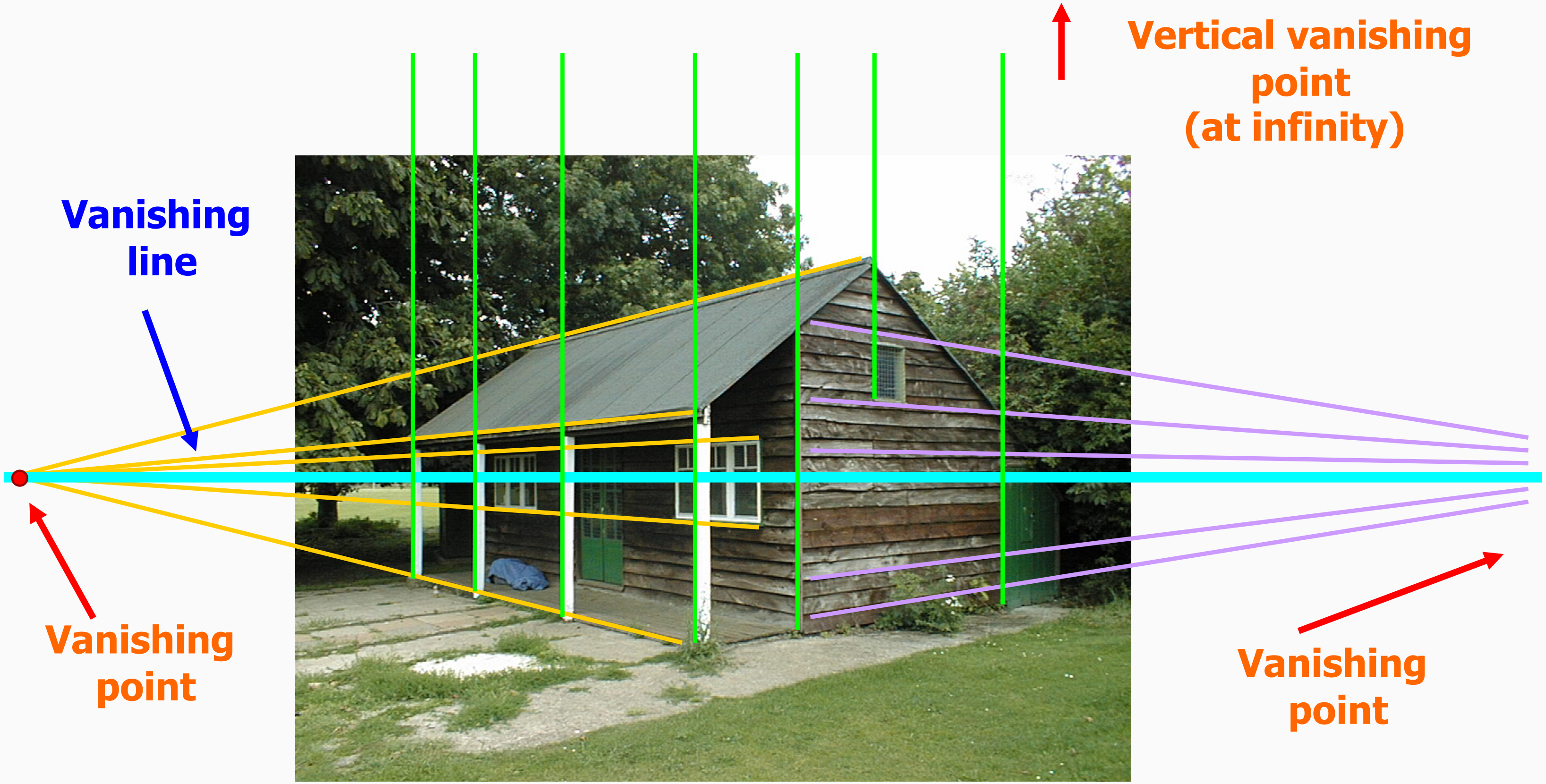
- Parallel lines in the world intersect at a “vanishing point”



# Constructing the vanishing point of a line



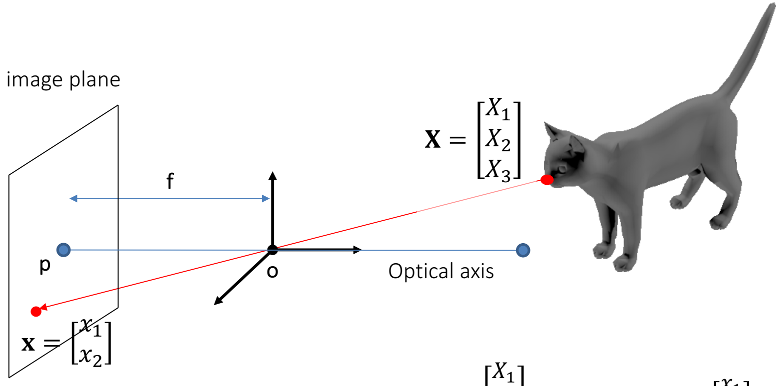
# Vanishing points and lines




All parallel lines will have the same vanishing point.

# Homogenous coordinates

- The projection x = fX/x3 is non linear!
- Can be made linear using homogenous coordinates
- Homogenous coordinates allow for transforms to be concatenated easily

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# Homogenous coordinates

Conversion to homogenous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Conversion from homogenous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

# Invariance to scaling

$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

E.g. [1,2,3] is the same as [3,6,9] and both represent the **same** inhomogeneous point [0.33,0.66].

# Basic geometry in homogenous coordinates

- Line equation: ax+by+c=0

$$line_i = \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix}$$

- A pixel p in homogenous coordinates:

$$p_i = \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix}$$

- Line is given by cross product of two points

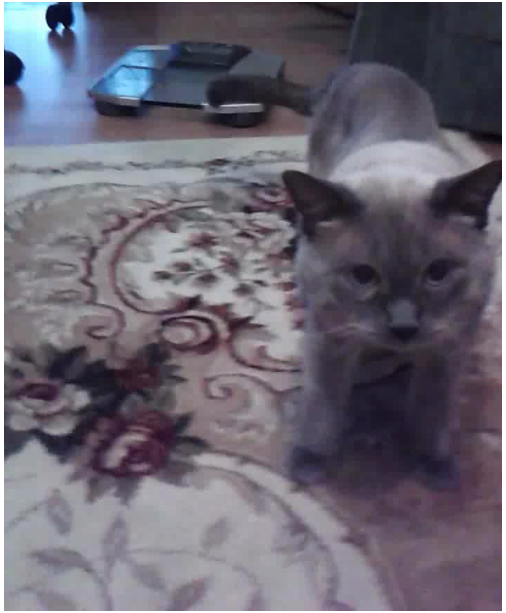
$$line_{ij} = p_i \times p_j$$

- Intersection of two lines is given by cross product of the lines

$$q_{ij} = line_i \times line_j$$

# 3D Euclidean transformation

- Cat moves through 3D space
- The movement of the nose can be described using an Euclidean Transform



$$\mathbf{X}'_{3\times1} = \boldsymbol{R}_{3\times3}\mathbf{X}_{3\times1} + \mathbf{t}_{3\times1}$$

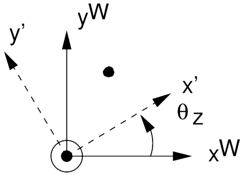
{ rotation } { translation }

$$\begin{pmatrix} \cos\beta\cos\gamma & -\sin\beta & \cos\beta \\ \sin\beta\cos\gamma & \cos\beta & -\sin\beta\sin\gamma \\ \sin\gamma & 0 & \cos\gamma \end{pmatrix}$$

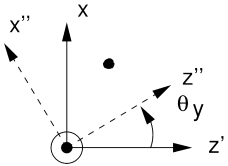
# Building the 3D rotation matrix R

- R can be build from various representations (Euler ang., quaternion)
- Euler angles represent the rotation using three parameters, one for each axis

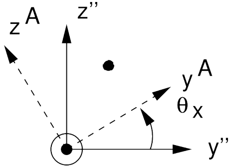
$$X' = R_z X^W = \begin{bmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} X^w$$




$$X'' = R_y X' = \begin{bmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{bmatrix} X'$$




$$X^A = R_x X'' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \pm\sin\theta_x \\ 0 & \mp\sin\theta_x & \cos\theta_x \end{bmatrix} X''$$




$$\boldsymbol{R}^{CW} = \boldsymbol{R}_x \boldsymbol{R}_y \boldsymbol{R}_z$$

Order matters!

# 3D Euclidean transformation

- Concatenation of successive transforms is a mess!

$$\mathbf{X}_1 = \boldsymbol{R}_1\mathbf{X} + \mathbf{t}_1$$

$$\begin{aligned}\mathbf{X}_2 &= \boldsymbol{R}_2\mathbf{X}_1 + \mathbf{t}_2\\ &= \boldsymbol{R}_2(\boldsymbol{R}_1\mathbf{X} + \mathbf{t}_1) + \mathbf{t}_2 = (\boldsymbol{R}_2\boldsymbol{R}_1)\mathbf{X} + (\boldsymbol{R}_2\mathbf{t}_1 + \mathbf{t}_2)\end{aligned}$$

# Homogenous coordinates save the day!

- Replace 3D points $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ with homogenous versions $\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

- The Euclidean transform becomes

$$\begin{bmatrix} \mathbf{X}' \\ 1 \end{bmatrix} = \boldsymbol{E} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

- Transformation can now be concatenated by matrix multiplication

$$\begin{bmatrix} \mathbf{X}_1 \\ 1 \end{bmatrix} = \boldsymbol{E}_{10} \begin{bmatrix} \mathbf{X}_0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} \mathbf{X}_2 \\ 1 \end{bmatrix} = \boldsymbol{E}_{21} \begin{bmatrix} \mathbf{X}_1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{X}_2 \\ 1 \end{bmatrix} = \boldsymbol{E}_{21} \boldsymbol{E}_{10} \begin{bmatrix} \mathbf{X}_0 \\ 1 \end{bmatrix}$$

# More 3D-3D and 2D-2D transformations

Projective (15 dof):

$$\begin{bmatrix} X'_1 \\ X'_2 \\ X'_3 \\ X'_4 \end{bmatrix} = [\boldsymbol{P}_{4\times 4}] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Affine (12 dof):

$$\begin{bmatrix} \mathbf{X}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{3\times 3} & \mathbf{t}_3 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Similarity (7 dof):

$$\begin{bmatrix} \mathbf{X}' \\ 1 \end{bmatrix} = \begin{bmatrix} S\boldsymbol{R}_{3\times 3} & \mathbf{t}_3 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Euclidean (6 dof):

$$\begin{bmatrix} \mathbf{X}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{3\times 3} & \mathbf{t}_3 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$

Projective (aka Homography, 8 dof):

$$\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix} = [H_{3\times 3}] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Affine (6 dof):

$$\begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{2\times 2} & \mathbf{t}_2 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

Similarity (4 dof):

$$\begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} = \begin{bmatrix} S\boldsymbol{R}_{2\times 2} & \mathbf{t}_2 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

Euclidean (3 dof):

$$\begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{2\times 2} & \mathbf{t}_2 \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix}$$

# Examples of 2D-2D transforms

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Euclidean
3 DoF

$$\begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Similarity
4 DoF

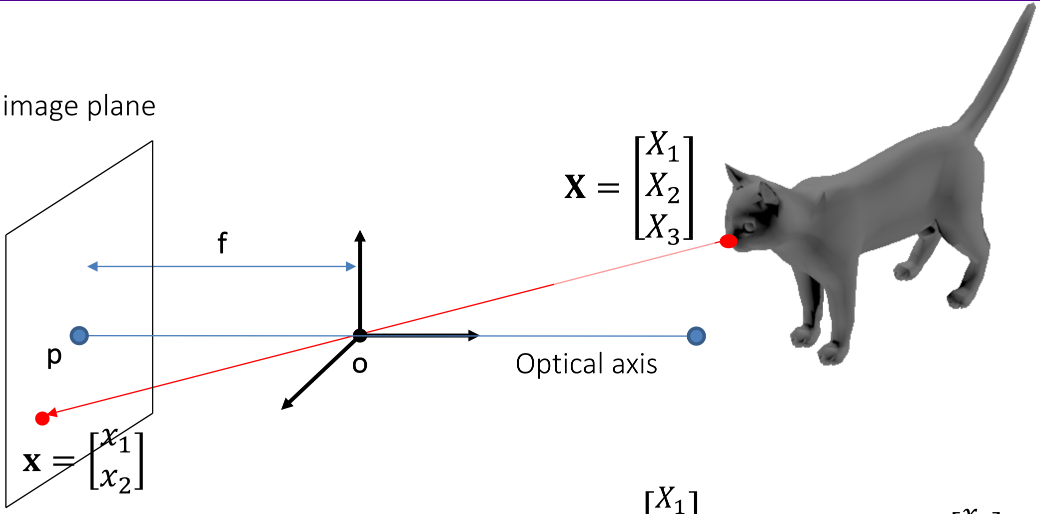
$$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Affine
6 DoF

$$\begin{bmatrix} h_{11} & h_{12} & h_{12} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

Projective
8 DoF

# Perspective transformation (3D-2D)

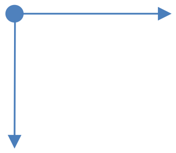



f – focal length
o – camera origin
p – principal point

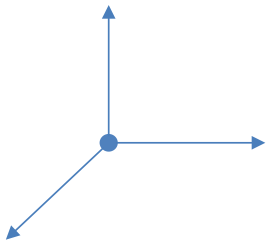
The 3D point $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ is imaged into $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f \frac{X_1}{X_3} \\ f \frac{X_2}{X_3} \end{bmatrix}$$

# Perspective using homogenous coordinates



$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \longleftarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f\,\frac{X_1}{X_3} \\ f\,\frac{X_2}{X_3} \end{bmatrix} \longleftarrow \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$



$$\lambda \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ 1 \end{bmatrix} \rightarrow \begin{matrix} \lambda x_1 = f X_1 \\ \lambda x_2 = f X_2 \\ \lambda = X_3 \end{matrix} \rightarrow \begin{matrix} x_1 = f\,\frac{X_1}{X_3} \\ x_2 = f\,\frac{X_2}{X_3} \end{matrix}$$

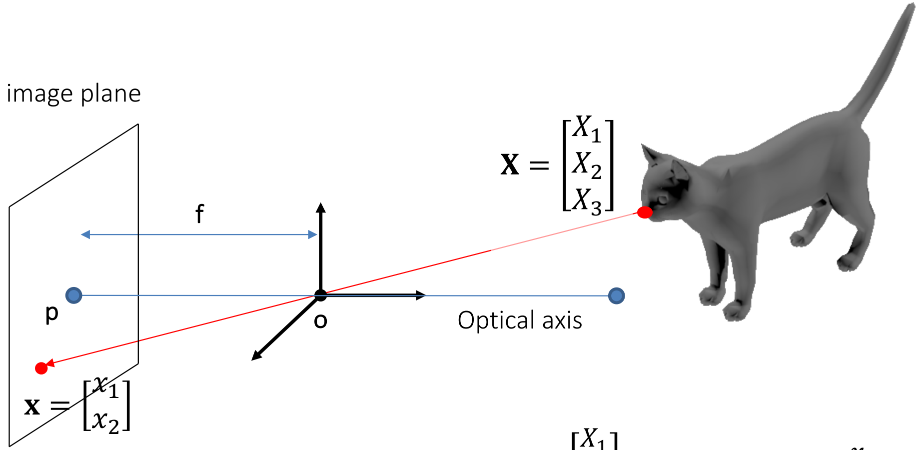
# Perspective using homogenous coordinates

| Image Point | Projection Matrix | World Point |
|---|---|---|

$$\lambda \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ 1 \end{bmatrix}$$

# Wait! Our setup has several assumptions

- Camera at world origin
- Camera aligned with world coordinates
- Ideal pinhole camera




f – focal length
o – camera origin
p – principal point

The 3D point $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ is imaged into $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f\,\frac{X_1}{X_3} \\ f\,\frac{X_2}{X_3} \end{bmatrix}$$

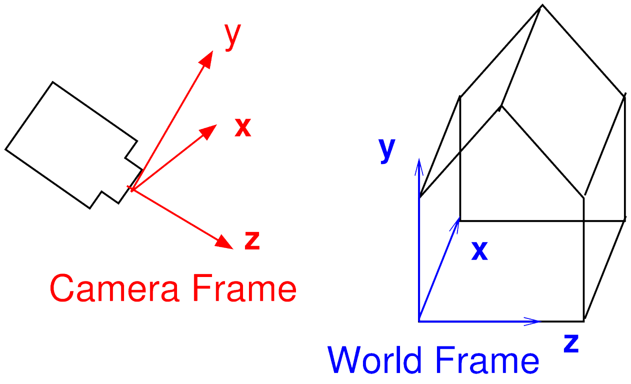
# Removing the initial assumptions

- It is useful to split the overall projection matrix into three parts:
  - A part that depends on the internals of the camera (intrinsic)
  - A vanilla projection matrix
  - An Euclidean transformation between the world and camera frames (extrinsic)
- Assume first that the world is aligned with camera coordinates
  -> the extrinsic camera matrix is an identity

| Image Point | Camera's Intrinsic Calibration | Projection matrix (vanilla) | Camera's Extrinsic Calibration | World Point |
|---|---|---|---|---|
| $\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ | $\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\mathbf{X}$ |

# More realistic setting - camera pose

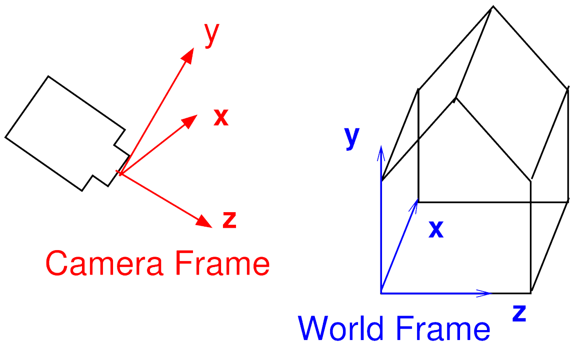
- Assume the camera is translated and rotated with respect to the world

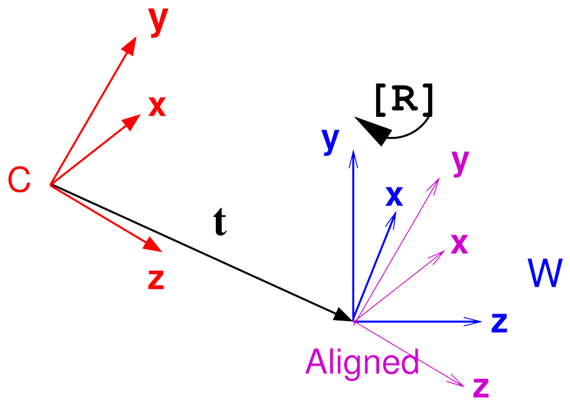


# The camera pose

- The non-ideal camera pose can be taken into account by first rotating and translating points from world frame to the camera frame

$$\begin{bmatrix} \mathbf{X}^c \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}^W \\ 1 \end{bmatrix}$$



# The intrinsic parameters

- Transformation to pixel units from metric units
- Describe the hardware properties of a real camera
  - The image plane might be skewed
  - The pixels might not be square

$$K = \boxed{\begin{bmatrix} f & 0 & 0 \\ 0 & \gamma f & 0 \\ 0 & 0 & 1 \end{bmatrix}} \boxed{\begin{bmatrix} 1 & 0 & \frac{u_0}{f} \\ 0 & 1 & \frac{v_0}{\gamma f} \\ 0 & 0 & 1 \end{bmatrix}} \boxed{\begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} f & sf & u_o \\ 0 & \gamma f & v_o \\ 0 & 0 & 1 \end{bmatrix}$$

different scaling on x and y
$\gamma$ is the aspect ratio.

Origin offset, $(u_o, v_o)$ is the principal point.

s accounts for skew

# Summary of steps from scene to image

- Move the scene point $(X^w,1)^T$ into camera coordinate system by 4x4 (extrinsic) Euclidean transformation:

$$\begin{bmatrix} \mathbf{X}^C \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X}^W \\ 1 \end{bmatrix}$$

- Project into ideal camera via the vanilla perspective transformation

$$\begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix} = [\boldsymbol{I}|\mathbf{0}] \begin{bmatrix} \mathbf{X}^C \\ 1 \end{bmatrix}$$

- Map the ideal image into the real image using intrinsic matrix

$$\begin{bmatrix} \mathbf{x} \\ 1 \end{bmatrix} = \boldsymbol{K} \begin{bmatrix} \mathbf{x}' \\ 1 \end{bmatrix}$$

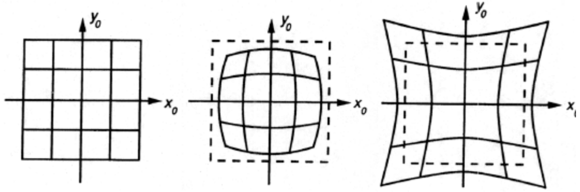
# Camera projection matrix P

| Image Point | Camera's Intrinsic Calibration | Projection matrix (vanilla) | Camera's Extrinsic Calibration | World Point |
|---|---|---|---|---|
| $\lambda\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ | $\begin{bmatrix} f & sf & u_0 \\ 0 & \gamma f & v_o \\ 0 & 0 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | $\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | $\mathbf{X}$ |

$$\mathbf{P}_{3\times 4} = \boldsymbol{K}[\boldsymbol{R}|\mathbf{t}]$$

# Beyond pinholes: Radial distortion

- Common in wide-angle lenses
- Creates non-linear terms in projection
- Usually handled by solving non-linear terms and then correcting the image



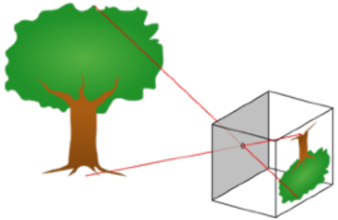

No Distortion Barrel Distortion Pincushion Distortion

Original

Corrected

# Things to remember

- Pinhole camera model



- Homogenous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Camera projection matrix

$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}]\mathbf{X}$$

# The end