1. (**Warm up**) (5p)

   Calculate the electron plasma frequency, Debye length, plasma parameter Λ, and Coulomb logarithm $\ln\Lambda$ for the following common plasmas.

   (a) Fusion experiment: $T_e = 10\,\mathrm{keV}, n_e = 10^{20}\,\mathrm{m}^{-3}$
   (b) High-latitude ionosphere at 150 km altitude: $T_e = 0.1\,\mathrm{eV}, n_e = 10^5\,\mathrm{cm}^{-3}$
   (c) Solar wind at 1 AU: $T_e = 10\,\mathrm{eV}, n_e = 10\,\mathrm{cm}^{-3}$
   (d) Core of the Sun: $T_e = 1\,\mathrm{keV}, n_e = 10^{26}\,\mathrm{cm}^{-3}$
   (e) Neutron star: $T_e = 100\,\mathrm{keV}, n_e = 10^{38}\,\mathrm{cm}^{-3}$

2. (**Simple algebraic epiphany**) (2p)

   Show that the plasma parameter Λ, introduced in the first lecture, is actually the number of particles in a Debye sphere (up to a small constant).

3. (**A bit harder**) (6p)

   (a) Solve the expression for the perturbation potential due to a point charge $q\delta(\boldsymbol{r})$ in an electron-proton plasma that is in a thermal equilibrium. Since the ions are much more massive than the electrons, they can be considered stationary ($n_i = n_0 = \text{constant}$). The electron density, in the presence of an arbitrary potential Φ, is

   $$n_e(\boldsymbol{r}) = n_0 e^{-q_e \Phi(\boldsymbol{r})/T}$$

   First, write down the Poisson equation, then perturb the system with an additional charge, $q\delta(\boldsymbol{r})$, and use Taylor expansion for the electron density around the equilibrium. Solve the differential equation using spherical coordinates to get the result

   $$\delta\Phi = \frac{q}{4\pi\epsilon_0 r} e^{-r/\lambda_D}$$

   (b) Sketch the perturbation potential for a hydrogen ion in the core of the Sun, a neutron star, and in the solar wind at 1 AU. How well do the other particles see the perturbation in these different environments? Remember that the densities are different as well. You can use the average distance between the particles for the comparisons.

4. (**Cool down)** (4p)

   The degree of ionization is described by the Saha equation

   $$\frac{n_i}{n_n} = 3 \times 10^{27} T^{3/2} n_i^{-1} e^{-U/T}$$

   where n_i is the number density of ions and n_n of neutral atoms, T the temperature in eV, and U the ionization energy. Assume that the dominating ion species in the ionosphere is O^+ and their density $10^{11}\,\mathrm{m}^{-3}$ and temperature 0.3 eV. The ionization energy of oxygen is 13.62 eV. What is the ionization degree of this plasma? Using the neutral oxygen density found here, calculate the ionization degree also for temperatures of 0.1 eV, 0.2 eV, and 0.5 eV.

5. (**Food for thought: lightning**)

   Consider the natural phenomenon called lightning. Research the phenomenon and explain in writing what it has to do with plasma physics. Return your short write-up in MyCourses before the next lecture.