31C01100 Taloustieteen matemaattiset menetelmät - Mathematics for Economists
Aalto University – Fall 2021
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Problem Set 0
NOTE: This problem set is meant to help you review the material of Chapters 1-5, you don't have to submit your answers

Exercise 1

Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is discontinuous at $x = 1$ and $x = 0$ but continuous at every other point of its domain.

Exercise 2

Calculate the first derivative of each of the following functions:

1. $f(x) = x^a$, with $a > 0$;
2. $f(x) = e^{ax}$, with $a > 0$;
3. $f(x) = (3x + 2)^3$;
4. $f(x) = \frac{3x}{x^2+1}$;
5. $f(x) = 4e^{-3x}$;
6. $f(x) = x \ln x$.

Exercise 3

Consider the function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x - x^3$. Find all the points at which the function attains:

1. a local maximum;
2. a local minimum;

3. a global maximum;

4. a global minimum.

Exercise 4

Let $f : I \to \mathbb{R}$ be a function defined over an interval $I \subseteq \mathbb{R}$. We say that f is **convex** if, for all $x, y \in I$, and all $a \in [0, 1]$, we have

$$f(ax + (1 - a)y) \leq af(x) + (1 - a)f(y).$$

Furthermore, we say that f is **concave** if, for all $x, y \in I$, and all $a \in [0, 1]$, we have

$$f(ax + (1 - a)y) \geq af(x) + (1 - a)f(y).$$

For each of the following functions, determine whether it is convex or concave.

1. $f(x) = 3x^2$

2. $f(x) = e^x$

3. $f(x) = 2 + x$

4. $f(x) = -e^x$

5. $f(x) = \log x$

6. $f(x) = x^3 - 3x$