31E11100 - Microeconomics: Pricing

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Part 3: Topics on dynamic pricing
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Pricing over Time

- Many interactions between buyers and sellers take place over time
  - Durable goods:
    - ★ One unit purchased lasts for several periods.
    - ★ Buyers may decide when to buy.
    - ★ How should the monopolist set the price over time? Increasing or decreasing prices?
    - ★ Implications of declining price path for buyers' purchasing time?
  - Experience goods:
    - ★ Consumers learn their taste by consuming.
    - ★ Repeat purchases over time: consumers' belief of the product depends on their consumption history
    - ★ Introductory offers to induce effective learning, or skimming the market through high and declining prices?

- Or, the product is perishable (e.g. service, consumed at a given time), but demand varies dynamically or is uncertain
- Leads to revenue management problems such as:
  - Peak-load pricing:
    - Demand varies systematically across seasons
    - Service provider must purchase durable capital
    - Capital cannot be liquidated or rented to others easily
  - Advance booking
    - Fixed capacity and perishable services as before
    - Consumers arrive over time before the consumption date
    - Buyers differ in terms of their willingness to pay

Examples

- Durables:
  - Books
  - New generations of electronics such as iPhones
- Experience goods:
  - Restaurants
  - Pharmaceuticals
- Peak load pricing:
  - Electricity
  - Transportation
- Advance booking:
  - Flights
  - Hotels

Durable Goods

- Consider first the case of identical buyers:
- A monopolist sells a durable good that yields services to the buyers over two consecutive periods.
- Let the buyers' use values for the good be denoted by v.
- Hence a buyer with valuation v for the good gets utility $2v$ if she purchases the good in period 1 and she gets utility v if she purchases in period 2.

- If all buyers are alike, the optimal pricing strategy of the monopolist is easy:
  - Set prices $p_1 = p_2 = 2v$.
  - All buyers buy in period 1 at these prices and therefore the second period prices are also optimal for the monopolist.

- Consider then buyers with different valuations:
- Suppose the buyers have valuations v drawn from the uniform distribution on $[0, 1]$
- Suppose that the marginal cost of providing the good is 0.
- If the monopolist sells only in period 1 (i.e. no transactions are possible in period 2), then $p_1 = 1$ and the monopolist sells to all buyer with $v \geq \frac{1}{2}$ and makes a profit of $\frac{1}{2}$.

- Consider now the case where the monopolist can sell also in period 2. Will this increase her profit?
- If the first period prices and purchases are unchanged, then the monopolist will set $p_2 = \frac{1}{4}$ and make a profit of $\frac{1}{16}$ in the second period on top of the first period profit.

- Unfortunately (for the monopolist), given (p_1, p_2) it is no longer optimal for all buyers with $v \geq \frac{1}{2}$ to purchase in period 1: If

$$v - \frac{1}{4} \geq 2v - 1 \text{ or } v \leq \frac{3}{4},$$

  then it is optimal for the buyer with valuation v to shift her purchases to period 2.
- Hence first period sales are only $\frac{1}{4}$ and total profit is only $\frac{3}{8}$.
- Notice that for these first period purchases, $p_2 = \frac{1}{4}$ is no longer optimal in period 2.

- We look for an equilibrium solution that is time consistent in the sense that every player chooses optimally in every period (technically, this is closely related to sub-game perfect equilibrium, see additional material on game theory)
- To get an equilibrium solution to the problem we need to determine four variables:
    - Prices p_1 and p_2.
    - The quantities sold at those prices. They are determined by the lowest type v_1 that purchases in period 1 and the lowest type v_2 that purchases in period 2.

- Since p_2 must be optimal given buyers' decisions in period 1, we have $p_2 = \frac{v_1}{2}$. But then $v_2 = \frac{v_1}{2}$.
- The buyer at v_1 must be indifferent between the purchases in periods 1 and 2:

$$2v_1 - p_1 = v_1 - p_2.$$

  - From this we get:

$$v_1 = \frac{2}{3}p_1.$$

- The firm's profit is

$$\pi = p_1 (1 - v_1) + p_2 (v_1 - v_2) .$$

- Substituting in from above, we have:

$$\pi = p_1 \left(1 - \frac{2}{3} p_1\right) + \frac{1}{9} p_1^2.$$

  The first order condition gives:

$$p_1 = \frac{9}{10}.$$

  Using the formulas above, we get $v_1 = \frac{3}{5}, v_2 = p_2 = \frac{3}{10}$.
- Equilibrium profit is then $\frac{45}{100} < \frac{1}{2}$.

Discussion of the results

- The result is quite general
  - If we increase the number of periods, the problem gets worse for the seller (better for the buyers)
  - If number of periods is large and discounting between periods small, then even the first offer p_1 converges to marginal cost 0.
  - Hence the monopolist gets no profit in this limit.

- What can the monopolist do to limit damages?
  - Do not sell, rent.
  - If rental contracts last for a single period, then the monopolist can rent at price $p = \frac{1}{2}$ in each period. This gives the same profit as selling in period 1 alone.
  - Similarly, with an inflow of new buyers, the problem becomes smaller.

Experience Goods

- Monopolist sells to buyers who do not know their true valuation v of the good.
- Valuations v are distributed uniformly on $[a, b]$, where $0 < a < b$.
- Willingness to pay for a single consumption opportunity is $\mathbb{E}v = \frac{a+b}{2}$.
- In this market, the monopolist can sell to all buyers once at price $p = \frac{a+b}{2}$.
- After a single purchase, the buyers learn their v.
- In subsequent periods, they buy if and only if their v exceeds the going price p.

Experience Goods

- How to determine willingness to pay with possible repeat purchases?
- Depends on tomorrows price p'.
- If $p' > b$, then the buyer will never purchase tomorrow and willingness to pay is simply $\frac{a+b}{2}$.
- If $p' < a$, then the buyer always buys tomorrow and current purchase gives no valuable information. Again willingness to pay is $\frac{a+b}{2}$.
- If $a < p' < b$, then the buyer will purchase tomorrow if and only if $v > p'$. Her willingness to pay is now above $\frac{a+b}{2}$ since learning the type now gives information that leads to better future decisions.

Experience Goods

- Option value for future purchases.
- As an excercise, solve this simple model with some different values a,b: start by solving the optimal price in the second period assuming all buyers know their valuation.
- Proceed to compute their option value
- How should the monopolist price in the first period?
- The problem becomes hard in a more general framework (many periods, less than perfect learning, etc.)
- Since current willingness determines optimal current prices and since current willingness to pay depends on future prices, all prices must be determined in equilibrium.

Experience Goods

- Bergemann and Välimäki "Optimal Pricing of New Experience Goods", Journal of Political Economy, 2006, present a very nice and elegant fully dynamic analysis.
- Two types of price paths possible:
  - Initially low prices leading to a buildup of knowledgeable customers followed by higher extracting prices.
  - High and declining prices skimming rent from the high value buyers over time.
- The first likely in a niche market where the monopoly price of the fully informed market exceeds the expected valuation. The latter happens in a mass market, where expected valuation exceeds the monopoly price in a fully informed market.

Discussion

- We saw two examples of how dynamics enrich the basic model.
- Durable goods model illustrates commitment problem.
- Experience goods model illustrates option values from consumption for the buyers.

Revenue Management

- We next take a brief look at Peak-load Pricing and Advance booking: two models that are sometimes considered as a part of larger literature on revenue management or yield management
- Peak-load Pricing of Services
  - Perishable good (service) with dynamically varying demand
  - Examples
    - Vacations: Airlines, restaurants, hotels,...
    - Utilities: Electricity, broadband,...
  - Question: How much capacity to build and how to set prices to maximize profit (or alternatively to achieve socially optimal utilization of resources).
    - Relates to the question of correct allocation of fixed costs within firms.
    - Should another nuclear plant be built?

Revenue Management

- Advance Booking
  - Perishable good with uncertain demand that resolves dynamically in advance of consumption
  - Examples
    - Airlines, other transportation, hotels.
    - Related practices: Overbooking, refund policies.
  - New element here: Booking classes
    - Basically just the same product offered at a different prices (sometimes comes with some versioning).
    - Booking classes and prices are decided.
    - Capacity allocated to booking classes.

Peak-load Pricing

- For this model, we should distinguish between two types of costs:
  - Cost of capacity.
  - Operating cost.
- Cost of capacity is to be differentiated from the operating cost because demands in different seasons are different.
  - Cost of capacity: building another plant.
  - Operating cost: fuel cost to produce electricity.
- Key question: how to decide the optimal capacity?

Peak-load Pricing

- A monopolist sells a service over a time horizon consisting of two different seasons, S, W.
- The inverse demand for services q^i in season i is given by

$$p^i = \alpha^i - q^i.$$

- The monopolist incurs a cost $f > 0$ per unit of capacity k (measured in the units of services per period) that she installs and a cost $c > 0$ per unit of services q that she produces. Capacity constraint says: $q \leq k$.
- Each unit of capacity is available for production in both seasons.

- For a fixed level of capacity k, the problem of the monopolist is season i is simple:

$$\max_{q_i \le k} p^i q^i - cq^i$$
$$\text{subject to} \quad : \quad p^i = \alpha^i - q^i.$$

- Substituting for the price in the objective function from the constraint, we see that the problem is concave in q^i and the first-order condition for optimality is $MR^i\left(q^i\right) = MC\left(q^i\right)$ or:

$$\alpha^i - 2q^i \ge c,$$

where equality holds if $q < k$.

- Since $f > 0$, the capacity constraint must bind in at least one of the seasons. We see easily that if $\alpha^i > \alpha^j$, then capacity constraint binds in season i.
  - For concreteness, assume from now on that $\alpha^S > \alpha^W$ so that S is the peak season.
- If capacity constraint binds only in season S, then we set $q^S = k$ and maximizing profit over k gives

$$\alpha^S - 2k = f + c.$$

- This formulation assumes that marginal revenue from added capacity comes only in season S. Hence the entire cost of installing the fixed capacity is allocated to season S.

- For the solution to be valid, we must have:

$$q^W = \frac{\alpha^W - c}{2} \leq k = \frac{\alpha^S - f - c}{2},$$

or

$$\alpha^S - \alpha^W \geq f.$$

- Prices are read off the demand curve:

$$p^W = \frac{\alpha^W + c}{2}, p^S = \frac{\alpha^S + f + c}{2},$$

and we have $p^S \geq p^W$.

- If the capacity constraint binds in both seasons, then $q^W = q^S = k$. Hence the revenue from capacity k is given by

$$R(k) = \left(\alpha^S - k\right)k + (\alpha^W - k)k,$$

and marginal revenue is:

$$MR(k) = \alpha^S + \alpha^W - 4k.$$

- The cost of installing capacity and operating at full capacity is $(f + 2c)\,k$.

- Setting $MR(k) = MC(k)$, we get:

$$q^W = q^S = k = \frac{\alpha^S + \alpha^W - f - 2c}{4}.$$

- Since the quantity sold in both seasons is the same, prices are again higher in the peak season S.

Advance Booking

- Capacity must be fixed before the realization of the demand.
- Demand is uncertain.
- How does uncertainty affect optimality.
- We go over three really simple examples to give a flavor of some of the issues related to this rich topic.

Simple example 1

- You have 100 hotel rooms available.
- The buyers have either a valuation 100 or 120 for the rooms.
- There are two equally likely demand conditions. In demand condition 1, there are 50 customers with willingness to pay 100 and none with willingness to pay 120. In the other, there are 100 buyers with willingness to pay of 120. (Congress in town).
- Your task is to maximize expected profit. How do you price the rooms?
- At price 100 or 120 or something else?

Simple example 2

- Advance Booking of a single capacity unit over two periods.
- Buyers have a willingness to pay either $40, 10$, or 0.
- Probabilities of these types are $\frac{1}{10}, \frac{8}{10}$ and $\frac{1}{10}$ respectively.
- There is a single potential customer in each period.
- If you sell in period 1, then you cannot sell again in period 2 since you have used your capacity.
- What are your optimal selling prices for the two periods?

Example: Advance sale or Clearance sale

- This example has no aggregate uncertainty
- Suppose the seller has capacity constraint $\overline{q} < 1$ (e.g. size of concert venue)
- Marginal cost is zero
- Two types of consumer with unit demand:
  - mass $\lambda \in (0, 1)$ of high type and mass $1 - \lambda$ of low type
  - Willingness to pay is 1 for the high type and $v_L < 1$ for the low type
  - Assume that $v_L > \lambda$
- Suppose that there are two periods before the event:
  - Seller sets prices p_1 and p_2, and possibly a quota of tickets to sell at each period
  - If there is overdemand at a particular period, then there is rationing: tickets are allocated randomly

- Consider the following strategies by the seller
  1. Clearance sale
     - ★ Sell at some $p_1 > v_L$ in the first period to all high type buyers
     - ★ Sell the remaining tickets at $p_2 = v_L$ in the second period
     - ★ What is the highest p_1 to attract high type buyers in the first period?
  2. Advance sale
     - ★ Sell ϕ tickets at price $p_1 = v_L$ in the first period (use rationing)
     - ★ Sell at price $p_2 = 1$ for the remaining high type buyers in the second period
     - ★ Who buys in the first period?
- Can you compute the seller's profit with these strategies?
- Which one is better?
- See Belleflamme and Peitz, pgs. 252-254 for details

Further readings

- The basic analysis of a durable good monopoly can be found in Bulow (1982): "Durable goods monopolist", Journal of Political Economy.
- For experience goods, see Shapiro (1983): “Optimal Pricing of Experience Goods”, Bell Journal of Economics, and Bergemann and Välimäki (2006): “Dynamic Pricing of New Experience Goods”, Journal of Political Economy.
- A survey on peak-load pricing is Crew, Fernando, and Kleindorfer (1995): "The theory of peak-load pricing: a survey", Journal of Regulatory Economics.
- For more advanced analysis of advance booking, see e.g. Nocke, Peitz, and Rosar (2011): "Advance-purchase discounts as a price discrimination device", Journal of Economic Theory.