1. (**A little warm-up**) (4p)

   Calculate the first three velocity moments of the distribution function $f_s(\mathbf{x}, \mathbf{v}; t)$, i.e., the 0th, 1st and 2nd (energy density) moments. The velocity space dependence is given by the Maxwellian distribution $f_{Ms}(\mathbf{v})$ (normalized to unity).

2. (**More warm-up**) (4p)

   We obtained the ideal MHD equations by assuming that our interest is in phenomena where the time scale is fast compared to the collision rate. Recalling that during the first lecture we found that the fastest time scale on which a plasma responds to a perturbation is given by the plasma frequency, find out if ideal MHD could be applicable (for at least some phenomena) in

   (a) Fusion plasmas: $n = 10^{19}\ \mathrm{m}^{-3}$, $T = 10$ keV

   (b) Earth's nightside magnetotail: $n = 10^3\ \mathrm{m}^{-3}$, $T = 1$ keV.

   The expression for the collision frequency was given at the end of the third lecture. Consider the electrons in the plasma, since they are the fastest to respond.

3. (**Straight-forward and challenging)** (6p)

   Derive the continuity equation
   $$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \boldsymbol{V}_s) = 0$$
   starting from the Vlasov equation
   $$\frac{\partial f_s}{\partial t} + \dot{\boldsymbol{r}} \cdot \nabla f_s + \dot{\boldsymbol{v}} \cdot \nabla_v f_s = 0.$$
   Hints: Integrate the Vlasov equation over velocity, i.e., take the zeroth velocity moment. For the first term, you can interchange the derivation and integration (why?). For the second term, use the vector identity
   $$\nabla \cdot (f_s \boldsymbol{A}) = f_s \nabla \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \nabla f_s$$
   to simplify the integrand. For the force term, start as you did for the second term, then show that for the Lorentz force
   $$\nabla_v \cdot \boldsymbol{a} = \nabla_v \cdot \left(\frac{q}{m} (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \right) = 0.$$
   The rest is vector algebra (use NRL when needed).

4. (**Food for thought: 'rocket science ...'**)

   Find out what *magnetohydrodynamic drive* is all about and what applications it has. Return your short write-up in MyCourses before the next lecture.