# 31 E11100 - Microeconomics: Pricing 

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Topics on dynamic pricing: time-inconsistent buyers (extra material)

## Introduction

- So far we have assumed that buyers are rational and know what is best to them
- The broad literature on behavioral economics seeks to understand situations, where decision makers are boundedly rational, make mistakes, or have otherwise non-standard preferences.
- For example, decision makers may exhibit:
- self-control problems
- loss aversion
- inattention
- overconfidence
- For pricing, a relevant question is:
- Can firms exploit buyers' bounded rationality, and how?
- Of course, one could also analyze biases by firms' managers, but this appears to be less first-order in this context


## Pricing with time-inconsistent buyers

- In this lecture we take up one interesting example: buyers with time-inconsistent preferences
- Preferences change over time
- Hyperbolic discounting or $\beta-\delta$ preferences as the simplest example
- Such preferences lead to self-control problems, preference for commitment, procrastination
- How should seller take into account such preferences in pricing?
- To make the question interesting, we think about products, where benefits and costs occur at different times:
- Investment goods: current costs and future benefits (health clubs, healthy food, ...)
- Leisure goods: current benefits and future costs (unhelthy food, credit card borrowing, ...)


## Standard (geometric) Discounting

- Decision making over time.
- Denote time periods by $t=0,1,2, \ldots$
- Instantaneous utility in period $t$ from action $a_{t}$ is $u\left(a_{t}\right)$.
- Geometric discounting: for all $s$, and $t>s$, the discounted utility in period $s$ from action $a_{t}$ in period $t$ is $\delta^{t-s} u\left(a_{t}\right)$ for some $0 \leq \delta \leq 1$.
- No special importance for current period.


## Hyperbolic Discounting

- An example of hyperbolic discounting: $\beta-\delta$ preferences.
- Utility in period $s$ from action $a_{t}$ in period $t>s$ is $\beta \delta^{t-s} u\left(a_{t}\right)$ for some $\beta<1$.
- Notice that current period has now a special meaning.
- Waiting from now until tomorrow is discounted by $\beta \delta$.
- Waiting from tomorrow until the day after tomorrow is discounted by $\delta$ from today's point of view.
- Notice that tomorrow, the discount rate between tomorrow and the day after tomorrow is $\beta \delta$ and not $\delta$.
- Hence the preferences between periods change as time goes on.
- Hence we talk about time inconsistent preferences.


## Example

- You have money for a single movie ticket.
- Different movies are shown during the next four weeks.
- The instantaneous willingness to pay for these movies is as follows:
- $v_{0}=2, v_{1}=3, v_{2}=5, v_{3}=9$.
- If you have geometric discounting, you should choose the $t$ that maximizes

$$
\delta^{t} v_{t}
$$

- Furthermore, if you plan in period $t=0$ to choose $t=3$, then waiting until $t=3$ will be optimal also at $t=1$ and $t=2$.
- Consider now $\beta-\delta$ preferences. Let $\beta=\frac{1}{2}$ and $\delta=1$.
- How will you choose in this case?
- Suppose you are sophisticated and you understand what your preferences look like.
- By backward induction, you realize that in $t=2$, you choose between $v_{2}=5$ and $\beta \delta v_{3}=\frac{9}{2}<5$.
- Hence you know that if you reach $t=2$, you will choose $v_{2}=5$.
- In $t=1$, you therefore compare $v_{1}=3$ and $\beta \delta v_{2}=\frac{5}{2}<3$ and you choose $v_{1}$.
- So in $t=0$, you choose $v_{0}=2>\beta \delta v_{1}=\frac{3}{2}$.
- Assume now that you are naive:
- You think that your preferences for all future periods coincide with your current preferences.
- Starting in $t=0$, you compare $v_{0}=2$ to $\max _{t} \beta \delta^{t} v_{t}=\max \left\{\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right\}=\frac{9}{2}$.
- Hence you decide to wait expecting to wait until $t=3$.
- In $t=1$, you compare $v_{1}=3$ to $\max \left\{\frac{5}{2}, \frac{9}{2}\right\}=\frac{9}{2}$ and decide to wait.
- In $t=2$, you compare 5 to $\frac{9}{2}$ and therefore you choose $v_{2}$.
- Notice that you make a false prediction about your future behavior at $t=0$.
- Still you gain relative to the sophisticated agent: From first period perspective, your payoff is $\frac{5}{2}>2$.


## Hyperbolic Discounting

- This models became very popular in the end of 1990's.
- Laibson: too little pension savings because of $\beta-\delta$ preferences.
- It can explain why people sometimes commit (at a cost) to options that restrict future choices.
- Naive agents: Explains procrastination.
- Yields new insights but at a cost:
- What about normative economics here? If preferences change, whose preferences should matter?
- As you can see in the above simple example, the analysis is really the analysis of a game, not a single decision problem.
- How to select between various equilibria in games where an agent plays against her future selves?


## Pricing with Time-Inconsistent Buyers

- Model by DellaVigna and Malmendier, QJE 2004.
- Three periods $t=0,1,2$.
- A monopolist offers a service to be consumed in $t=2$.
- Setup cost of $K$ per customer, service cost $\alpha$ for providing the service.
- Buyers with $\beta-\delta$ preferences.
- Consumption has immediate cost $c$ and delayed benefits $b$ that occur in $t=2$.
- Health club is the leading example in the paper.
- Also calling plans, credit cards etc.


## Pricing with Time-Inconsistent Buyers

- Costs are unknown in $t=0$ and drawn from the uniform distribution in $t=1$.
- Buyer and firm are both risk-neutral.
- The benefits are known in advance and $0<b<1$.
- Seller proposes a contract of the form $(L, p)$.
- $L$ is an (unconditional) up-front payment that is paid regardless of whether the service is eventually used.
- Contract is accepted or rejected in $t=0$.
- $p$ is the fee for using the service after learning $c$ in period 2 .


## Pricing with Time-Inconsistent Buyers

- Summarizing the timing:
- $t=0$
$\star$ Firm offers contract, buyer accepts and pays $L$ in $t=1$ or rejects and just gets reservation utility $\bar{u}$ in period $t=1$.
$\star$ If agent accepts, firm pays setup cost $K$ in $t=1$.
- $t=1$
$\star$ Buyer learns her cost $c$.
$\star$ Decides whether to consume the service or not.
$\star$ If yes, pays $p$ and incurs cost $c$.
- $t=2$
« If buyer consumer, she experiences utility $b$ otherwise 0 .



## Time-Consistent Buyer

- Start with the benchmark case with $\beta=1$, i.e. time-consistent buyer.
- Buy if and only if

$$
\delta b-c-p \geq 0 \text { or } c \leq \delta b-p
$$

- Hence expected payoff at $t=0$ from accepting the contract is

$$
U^{T C}=\delta\left[-L+\int_{0}^{\delta b-p}(\delta b-p-c) d c\right]
$$

- By not consuming, she gets $\delta \bar{u}$.
- The firm makes expected profit $\pi(L, p)$ if the agent accepts:

$$
\pi(L, p)=\delta\left[L-K+\int_{0}^{\delta b-p}(p-\alpha) d c\right]
$$

- Hence firm chooses $(L, p)$ to maximize $\pi(L, p)$ subject to $U^{T C}=\delta \bar{u}$.


## Time-Consistent Buyer

- Hence we can write the problem as:

$$
\max _{p} \delta\left[\int_{0}^{\delta b-p}(\delta b-\alpha-c) d c-K-\bar{u}\right]
$$

- Notice that this is the problem of maximizing the sum of profit and consumer surplus.
- Hence the optimal choice here is:

$$
p^{T C}=\alpha
$$

- Since this is a completely standard model, price $=$ MC and lump sum extracting all surplus is optimal.


## Sophisticated Time-Inconsistent Buyer

- Now $\beta<1$.
- Buy iff

$$
c \leq \beta \delta b-p
$$

- Ex ante payoff to a sophisticated buyer:

$$
U^{S}=\beta \delta\left[-L+\int_{0}^{\beta \delta b-p}(\delta b-p-c) d c\right]
$$

- By rejecting, reservation utility is $\beta \delta \bar{u}$.


## Sophisticated Time-Inconsistent Buyer

- So the maximization problem for the firm is now:

$$
\max _{p} \delta\left[\int_{0}^{\beta \delta b-p}(\delta b-\alpha-c) d c-K-\bar{u}\right] .
$$

- Again, the firm wants to sell whenever it is first-best from the point of view of the firm and the buyer in period $t=0$.
- But now this implies setting

$$
p=\alpha-\delta b(1-\beta)
$$

## Sophisticated Time-Inconsistent Buyer

- Pricing below marginal cost.
- This is done to help the agent.
- Since the agent in period $t=0$ understands her preferences at $t=1$, she realizes that she would consume too little at $p=\alpha$.
- Commitment to a lower price than marginal cost makes corrects the buyers decisions in $t=1$ from the point of view of the buyer in $t=0$.
- In effect, the firm is selling a commitment device for period $t=1$.
- Pricing is efficient in the sense that total surplus of the firm and the period 0 consumer is maximized.


## Naive Time-Inconsistent Buyer

- Finally we consider the case where $\beta<1$ but in $t=0$, the buyer believes that $\beta=1$ i.e. that she is time-consistent.
- We assume that the firm knows this naive belief.
- This assumption makes sense from a descriptive point of view.
- Firms can learn this feature in the market when dealing with many buyers.
- At the same time, a major assumption in terms of modeling.
- Firms know something about the buyers that contradicts their own view. Goes against the usual assumption of consistent priors.


## Naive Time-Inconsistent Buyer

- In $t=1$, the buyer consumes if and only if $c \leq \beta \delta b-p$.
- In $t=0$, she expects to consume if $c \leq \delta b-p$.
- Hence in $t=0$, the ex-ante payoff to naive buyer is

$$
U^{N}=\beta \delta\left[-L+\int_{0}^{\delta b-p}(\delta b-p-c) d c\right]
$$

- Firm's expected profit (knowing that the buyer is naive):

$$
\pi(L, p)=\delta\left[L-K+\int_{0}^{\beta \delta b-p}(p-\alpha) d c\right]
$$

## Naive Time-Inconsistent Buyer

- Again, $L$ must be such that $U^{N}=\beta \delta \bar{u}$.
- Hence we can write the profit maximization problem as:

$$
\max _{p} \delta\left[\int_{0}^{\beta \delta b-p}(\delta b-c-\alpha) d c+\int_{\beta \delta b-p}^{\delta b-p}(\delta b-c-p) d c-K-\bar{u}\right]
$$

- The first integral is the true surplus.
- The second integral is a fictitious surplus that the naive agent believes that she will get.
- It arises only because she believes that she will accept trades that she will not.
- Also here: price below marginal cost


## Discussion

- Both versions of the model feature price below marginal cost, but for different reasons
- With sophisticated consumer, low price helps the buyer to commit to more use
- With naive consumer, seller exploits buyer's mistake: buyer is willing to pay a higher up-front payment $L$ since she is overly confident of her probability of consuming
- The same model would apply with some relabeling of variables to the case of leisure goods, but with opposite conclusion:
- High price of use, low up-front payment
- Think about credit cards: low fixed fee, high interest rate


## Discussion

- Need to distinguish between time-consistency as such and naivete
- Sophisticated (time-incosistent) consumers benefit from the extra commitment that the pricing scheme gives
- But if buyers are also naive, sellers exploit them by setting a too high up-front payment
- What about differentially sophisticated buyers?
- Second-degree price discrimination possible.
- For example Eliaz and Spiegler, REStud 2006 "Contracting with Diversely Naive Agents" takes first steps in this direction.


## Further readings

- The model analyzed here is based on DellaVigna and Malmendier (2004): "Contract design and self-control: theory and evidence", Quarterly Journal of Economics. Related empirical evidence here: DellaVigna and Malmendier (2006): "Paying Not to Go to the Gym", American Economic Review.
- For hyperbolic discounting and its application on saving behavior, see Laibson (1997): "Golden Eggs and Hyperbolic Discounting", Quarterly Journal of Economics.
- There is a lot of literature on behavioral econonimcs applied in IO and pricing. For example, for pricing and loss-averse consumers, see Heidhues, P., \& Koszegi, B. (2008). Competition and price variation when consumers are loss averse. American Economic Review, or Herweg and Mierendorff (2013): "Uncertain demand, consumer loss aversion, and flat-rate tariffs", Journal of the European Economic Association.
- The book Ran Spiegler (2011): "Bounded Rationality and Industrial Organization", Oxford University Press, is a comprehensive introduction to topics in pricing, IO and behavioral economics.
- A shorter survey on different related topics: Grubb (2015): "Behavioral Consumers in Industrial Organization: An Overview", Review of Industrial Organization.

