L4 Quantum Transport with Charging and Fields

Sunday, September 5, 2021 12:10 AM

Lecture notes by Ethan Minot, visiting from Oregon State University
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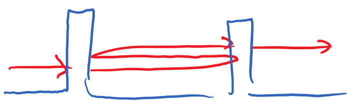
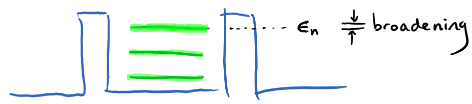
Nanoelectronics Class at Aalto University, Autumn, 2021.

QUANTUM DOTS, (continued from last week)



First picture:
Summation of different paths

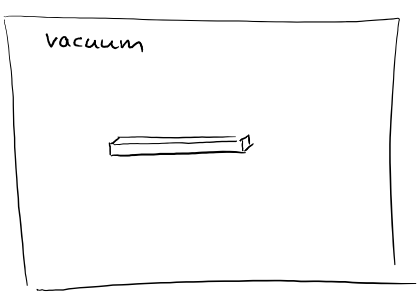


New Picture:
Discrete electron orbitals that have finite broadening
Probability of tunneling will depend on energy detuning between incoming electron and the nearest ϵ_n

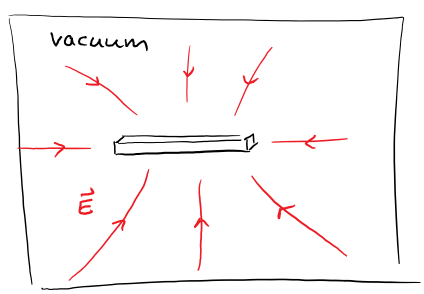
This new picture is like atomic energy levels.
"artificial atoms"

We will add onto this new picture an additional phenomena:

Electrostatic charging energy.



Charge neutral

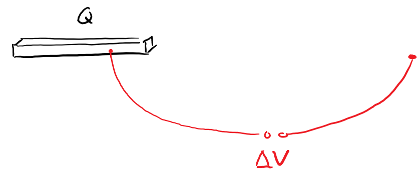


One excess electron

Energy stored in this field, E_c

$$\frac{\epsilon_0}{2}\int |E|^2 \, d^3\vec{r} \quad \text{or} \quad \frac{e^2}{2C}$$

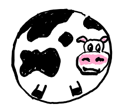
where C relates Q & ΔV



$$Q = C\Delta V$$

A quick approximation for C is

$$C \approx 4\pi\epsilon_0 r_0$$ "Spherical cow approximation"

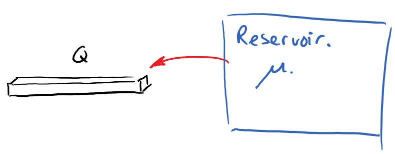


$k_B = 1.4 \times 10^{-23}$ J/K
$\epsilon_0 = 8.85 \times 10^{-11}$ F/m

Exercise: Approximately how small is an object in which $E_c > k_B T$ at $T = 1K$?

For n excess electrons $E^{(el)} = \frac{e^2}{2C} n^2 = E_c n^2$

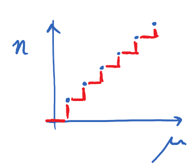
(In some contexts, you may find charging energy defined as $E_c = \frac{e^2}{C}$)



Raise μ by E_c, 1st electron enters, system energy up E_c
Raise μ by $3E_c$, 2nd electron enters, system energy up $4E_c$ $(1E_c + 3E_c)$
Raise μ by $5E_c$, 3rd electron enters, system energy up $9E_c$ $(1E_c + 3E_c + 5E_c)$

Important to remember $\mu = U(n) - U(n-1)$

Charging energy is quadratic in n
But n has linear dependence on μ.



Coulomb blockade.

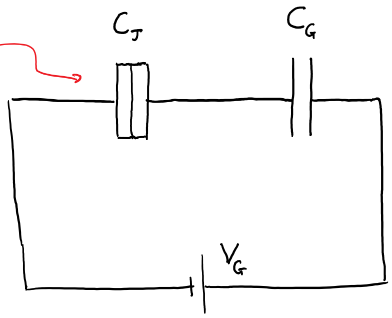
This staircase of charge number is only a significant effect in small devices with opaque contacts (small C, large E_c)

The simplest device that combines tunneling and Coulomb blockade
SINGLE ELECTRON BOX (§7.1.1 in text book)

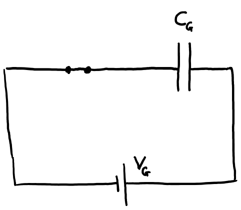
The simplest device that combines tunneling and Coulomb blockade

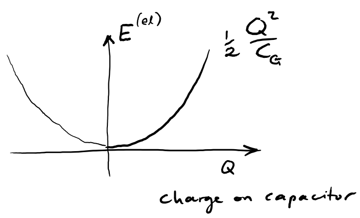
SINGLE ELECTRON BOX (§7.1.1 in text book)

capacitor that allows charge to tunnel through.

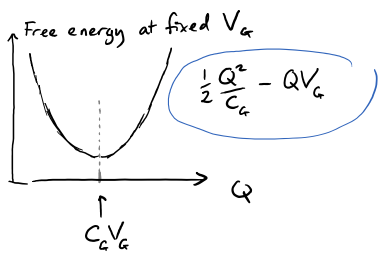


Familar circuit #1

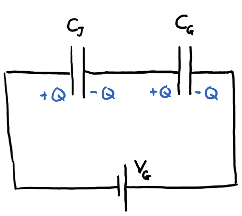




Charge on the capacitor that minimizes free energy is $C_G V_G$



Familar circuit #2

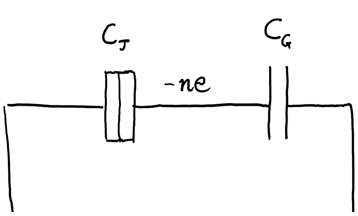


Total charge on the island is fixed at zero $-Q+Q=0$

$$V_G = \frac{Q}{C_J} + \frac{Q}{C_G} = Q\left(\frac{1}{C_J} + \frac{1}{C_G}\right) = Q\left(\frac{C_J + C_G}{C_J C_G}\right)$$

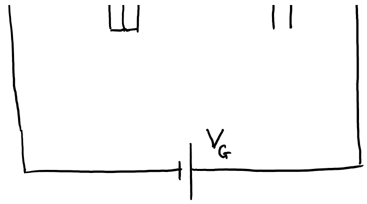
effective capacitance $\frac{C_J C_G}{C_J + C_G} < C_J, C_G$

A hybrid of these two familiar circuits

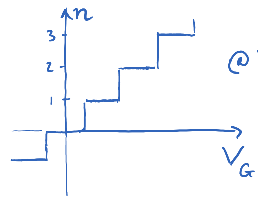


n is integer.

n only changes when it is energetically



n only changes when it is energetically favorable.



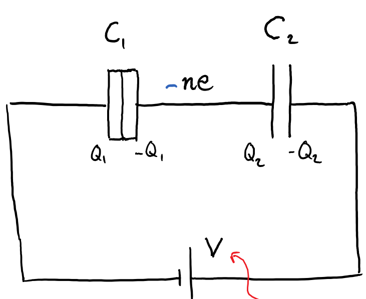
@ $T = 0$

we already can guess the slope of this staircase.

$ne = C_G V_G$ if n could change continuously.

Our goal is to calculate free energy of the system for a given V_g and various possible integer values of n.

(will later use these free energies in a Hamiltonian, or use to find $\langle n \rangle$ at finite T.)



$$V = \frac{Q_2}{C_2} + \frac{Q_1}{C_1} \qquad \text{— ①}$$

$$Q_2 - Q_1 = -ne \qquad \text{— ②}$$

I dropped the subscript because it's clear what voltage I mean.

Using ① & ② we can write

Q_1 in terms of V, C_1, C_2, ne

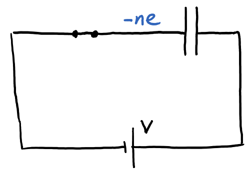
Q_2 in terms of V, C_1, C_2, ne

$$Q_1 = \frac{C_1 C_2 V + neC_1}{C_1 + C_2} \qquad Q_2 = \frac{C_1 C_2 V - neC_2}{C_1 + C_2}$$

$$\underbrace{E^{(el)} = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}}_{\text{For a given value of } V \text{, this is minimized by } -ne = C_2 V}$$

C_2

lowest energy state,
let all the charges pass thru C_1
voltage drop only happens at C_2



$$E^{(el)} = \frac{Q_2^2}{2C} = \frac{n^2e^2}{2C_2}$$

Confirming
$Q_1 = 0$
$Q_2 = C_2V$
for the special case
$-ne = C_2V$

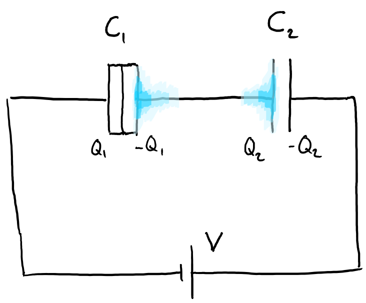
$$Q_1 = \frac{C_1C_2V + \overset{-C_2V}{\cancel{ne}}C_1}{C_1+C_2} = 0$$

$$Q_2 = \frac{C_1C_2V - \overset{-C_2V}{\cancel{ne}}C_2}{C_1+C_2} = \frac{(C_1C_2 + C_2^2)V}{C_1+C_2} = C_2V$$

BUT
n is not a continuous variable, there must be times when $n \neq \frac{C_2V}{-e}$

When n = integer and $V \neq \frac{-ne}{C_2}$

see situations like $n = 1$, $Q_1 = \frac{-e}{2}$, $Q_2 = \frac{e}{2}$ Fractional charge?



Total charge on the island is e. $Q_2 - Q_1 = e$.

The charge has distributed itself between the two ends of the island.

Try different integer values

Treat as fixed

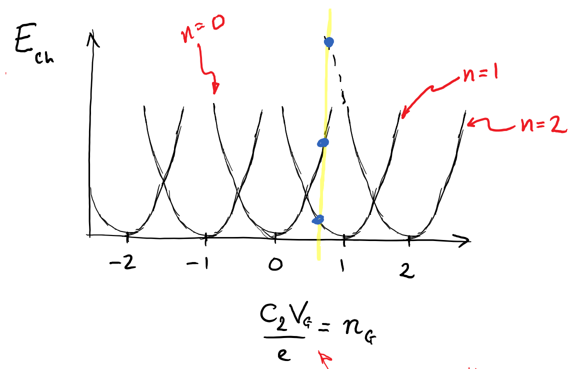
Be careful of sign here. It depends on which capacitor plate has the positive Q_2.

$$\text{Free Energy} = E_{ch}(n;V) = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \underbrace{VQ_2}$$

If I change n, I expect charge to move accross the battery.
If charge moves across the battery, Q_2 will change.
Thus VQ_2 keeps track of the relative free energy when n changes.

$\vdots$

$$= \frac{e^2}{2(C_1+C_2)}(n - n_G)^2 - \frac{e^2}{2C_2}n_G^2 \qquad \text{where } n_G = \frac{C_2V}{e}$$

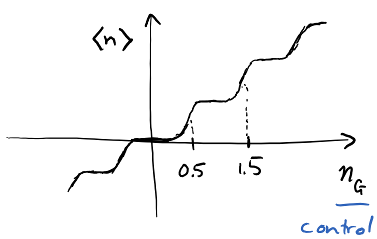


I added the subscript back V_G is same as V.

Note:
The $\frac{e^2}{2C_\Sigma} n_G^2$ term is not included in the plot.

How to use the plot:
Pick a value of n_G, then compare relative values of E_{ch} for various n.

Using E_{ch}, I can calculate $\langle n \rangle$ at finite T



control this with voltage on battery.

$$\langle n \rangle = \sum_n n\, p(n)$$

$$p(n) = \frac{\exp\left(-\frac{E_{ch}(n)}{k_B T}\right)}{\sum_n \exp\left(-\frac{E_{ch}(n)}{k_B T}\right)}$$

You may have seen the Hamiltonian for a Cooper-pair box (the charge qubit)

$$H = \sum_n E_C (n - n_G)^2 |n\rangle\langle n| - \frac{1}{2} E_J \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right)$$

$$E_C = \frac{(2e)^2}{2(C_J + C_G)} \qquad n_G = \frac{C_G V_G}{(2e)}$$

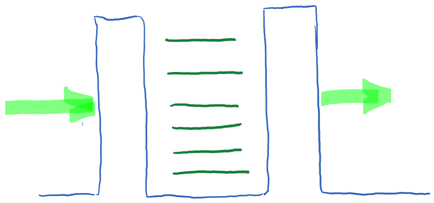
Josephsen energy

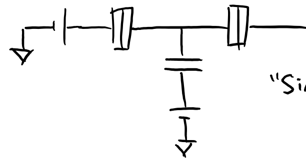
Now you know the details of the first term in this Hamiltonian.

Now, back to where we started. Studying the conductance of a quantum dot (two tunnel barriers, closed system).

Small variation: single electron box $\Rightarrow$ single-electron transistor (SET)

$\xrightarrow{I}$

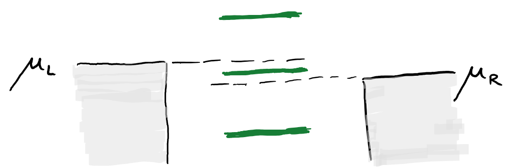


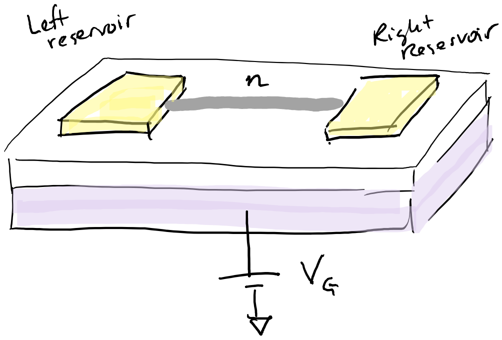


"Single electron transistor"
"SET"

Connect the system to 2 reservoirs.

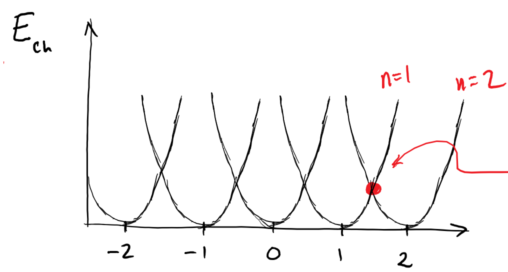
Establish a transport window





Include a gate voltage that can adjust number of electrons, n.

For current to flow, n must fluctuate.



$$\frac{C_G V_G}{e} = n_G$$

pick a gate voltage where $E_{ch}(n) = E_{ch}(n+1)$

Express this idea in language of chemical potential

$$\mu = U(n) - U(n-1)$$

$$\mu_{dot}(n) = \epsilon_n + \underbrace{\frac{e^2}{C_\Sigma}\left(n - \tfrac{1}{2}\right)} - \underbrace{e\,\frac{C_g}{C_\Sigma}\,V_G}$$

quantum ("chemical") energy of the highest occupied orbital.

Term derived from $E^{(el)} = \frac{e^2}{2C} n^2$

$E^{(el)}(\ ) - E^{(el)}(\ )$

V_G is the parameter controlled by the experimenter.

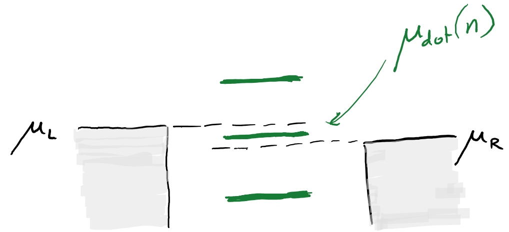
("chemical") energy of the highest occupied orbital.

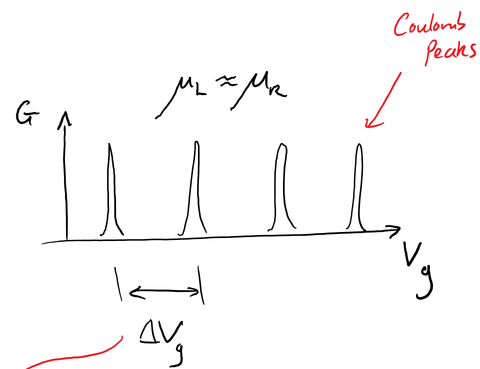
$E^{(el)} = \frac{e^2}{2C} n^2$

$E^{(el)}(n) - E^{(el)}(n-1)$

controlled by the experim...

Sometimes $\frac{e^2}{C_\Sigma}$ is called "charging energy"





$$e \frac{C_g}{C_\Sigma} \Delta V_g = \frac{e^2}{C_\Sigma} + \Delta \epsilon_n$$

$$\Delta V_g = \frac{e}{C_g} + \frac{C_\Sigma}{C_g} \frac{\Delta E_n}{e}$$

The spacing between Coulomb peaks, ΔV_g,
is a useful way to determine $\Delta \epsilon_n$ in small structures.

Applications of SETs

1. Low power transistor? "Carbon nanotube single-electron transistor at room temperature" Science 293 76 (2001)

At low temperatures,
the transistor can be turned on & off by a tiny gate voltage (< 1 mV)
Unfortunately, the SET suffers from "Boltzmann's tyranny" just like conventional transistors... ($60 \frac{mV}{decade}$ when $k_B T = 25\,meV$.) So, it will still take hundreds of millivolts to make it switch. No dramatic savings in power consumption.

2. Thermometry

Coulomb Blockade Thermometers (CBT) cover a temperature range from 0.01 - 60K

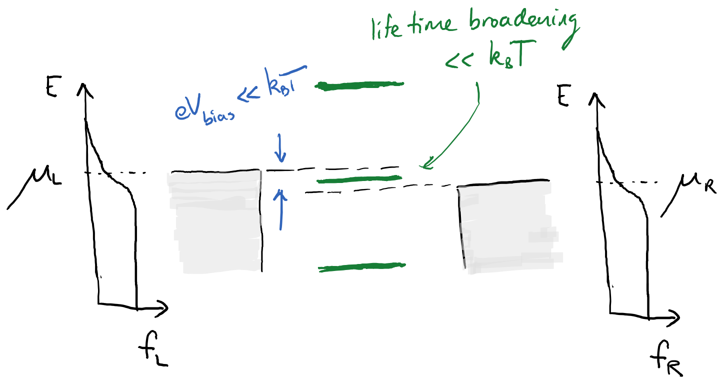
Coulomb Blockade Thermometers (CBT) cover a temperature range from 0.01 – 60K

They are "primary" thermometers, (no calibration is needed).

They work equally well in magnetic fields.

Accuracy is ~1%.

There are various schemes, see §7.6.1. Here is one simple scheme.



Probability of transmission proportional to $f_L(1-f_R)$

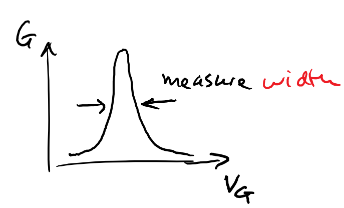
$$\propto \frac{1}{e^{(\mu_{dot}-\mu_L)/k_BT}+1}\left(1-\frac{1}{e^{(\mu_{dot}-\mu_R)/k_BT}+1}\right) \qquad \mu_{dot} = \text{const} - \frac{eC_G}{C_\Sigma}V_G$$

Use V_G to move the QD level up and down relative to μ_L & μ_R.

Note that $\left(\frac{1}{e^x+1}\right)\left(1-\frac{1}{e^x+1}\right) = \frac{1}{4}\frac{1}{\cosh^2\left(\frac{x}{2}\right)}$

Full result

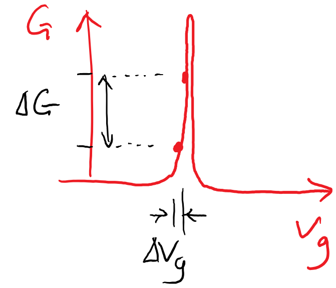
$$G(V_G) = \frac{e^2}{k_BT}\left(\frac{1}{\Gamma^S}+\frac{1}{\Gamma^D}\right)^{-1}\frac{1}{4}\frac{1}{\cosh^2\left(\frac{\frac{eC_G}{C_\Sigma}\left(V_G^{max}-V_G\right)}{2k_BT}\right)}$$

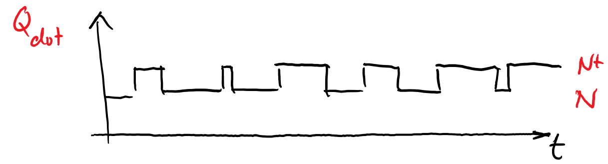


See Beenakker PRB 44 1646 (1991)

③ Ultrasensitive Electrometers (charge detectors)

A small change in the electrostatic environment shifts the effective gate voltage and changes the conductance.





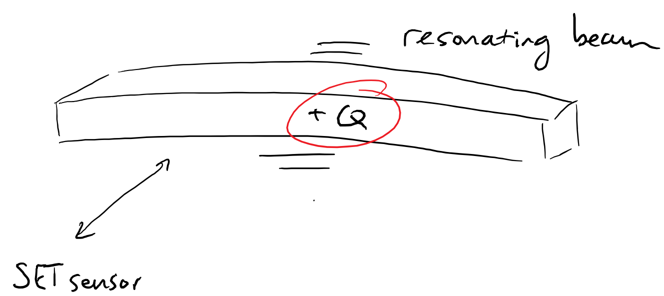
Lu et al. "Real-time detection of electron tunnelling in a quantum dot" Nature **423** 422 (2003)

Petta et al. "Coherent manipulation of couple electron spins in semiconductor quantum dots" Science **309** 2180 (2005)

As current flows through one SET, a second SET can measure the fluctuating number of charges on the first SET. Similar detection schemes were needed for reading out a quantum dot qubit.

Knobel and Cleland "Nanometer-scale displacement sensing using a single electron transistor" Nature **424** 291 (2003)

Lahaye et al. "Approacing the Quantum Limit of a Nanomechanical Resonator" Science **304** 74 (2004)



Motion of beam changes effective gate voltage
Detect change in G
(can be done with rf reflection §7.6.2)

On-going area of research...

How much measurement back-action comes from this kind of electrometer? Are there better ways?

④ Spin qubits in semiconductor QDs... these are essentially SET devices.

Petta et al. "Coherent manipulation of couple electron spins in semiconductor quantum dots" Science **309** 2180 (2005)

Out of favor due to short coherence times.

⑤ Cooper pair box ... part of the transmon qubit.

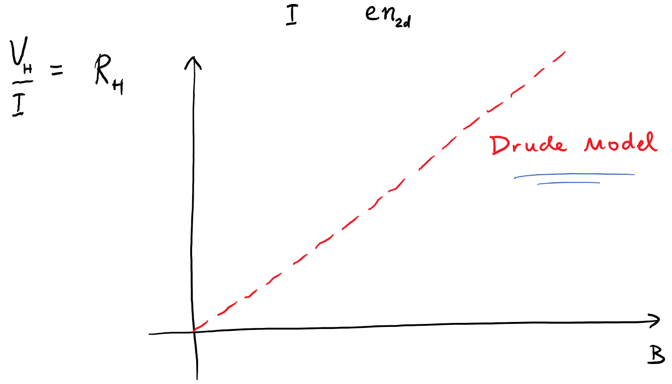
QUANTUM HALL EFFECT

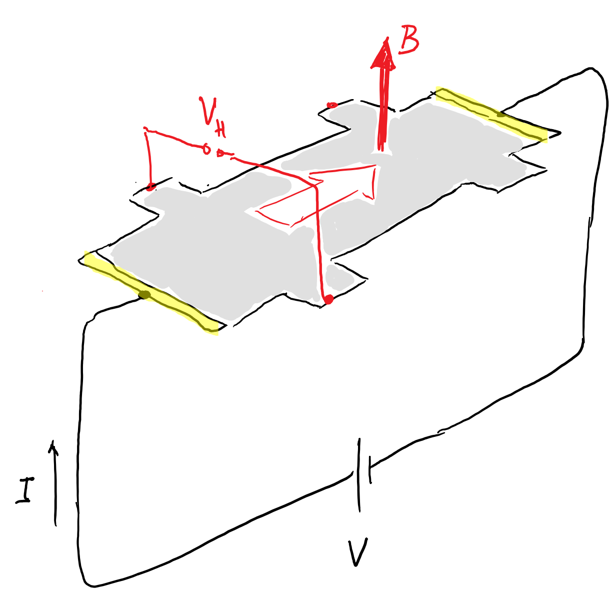
The first half of the lecture focused on using gate voltage to control the a nanoelectronic device.

Now I'll talk about devices that are controlled by magnetic field.
A basic knowledge of this QH phenomena is useful for understanding many nanoelectronic experiments. We'll also find a satisfying connection to the quantum of conductance $\frac{e^2}{h}$.

Hall bar geometry

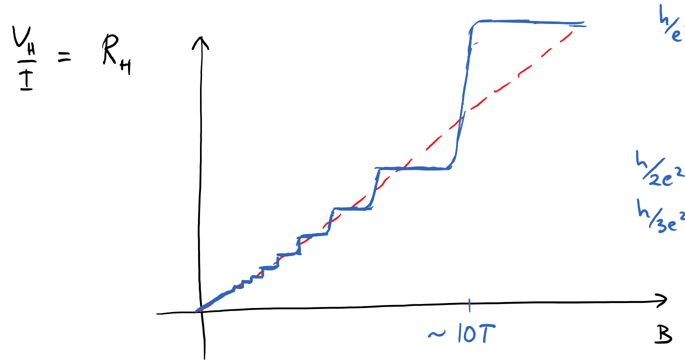
$$\frac{V_H}{I} = \frac{B}{e n_{2d}}$$





Quantum Hall Effect.

Notes based on David Tong, arxiv: 1606.06687v2



$h/e^2 = 25.8\ k\Omega$ similarities with 1 d wires.

What does B do to the wavefn of an electron?

$$\frac{\hat{p}^2}{2m}\psi(x,y) = E\,\psi(x,y)$$

no B-field

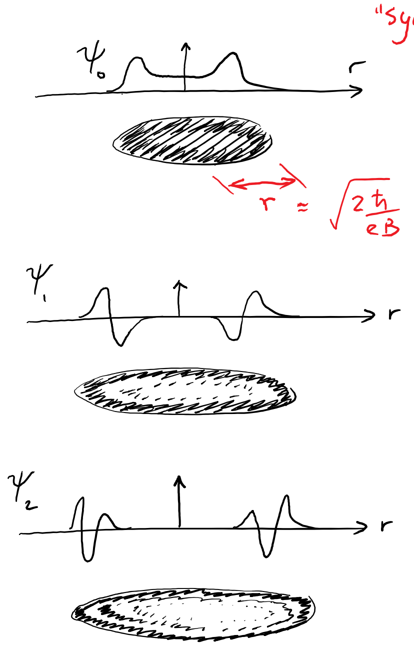
$$-\frac{\hbar^2}{2m_{eff}}\vec{\nabla}^2\psi = E\psi$$

$$\psi = e^{i\vec{k}\cdot\vec{r}}$$

with B-field

$$\frac{1}{2m_{eff}}\Big(\underbrace{-i\hbar\vec{\nabla} + e\vec{A}}_{\hat{p}}\Big)^2\psi = E\psi$$

Can be solved in various gauges. Here I show solutions in the "symmetric gauge".



A good way to remember this length scale is the flux quanta h/e.
The area of an orbit is such that $(B)(\pi r^2) \approx h/e$

Quantum cyclotron orbits.

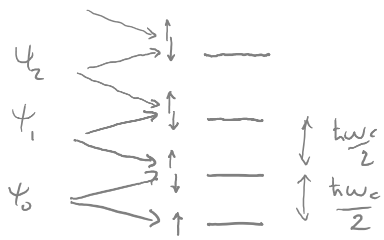
$$\omega_c = \frac{eB}{m} \qquad E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega_c \qquad n = 0, 1, 2, 3 \ldots$$

cyclotron freq.

I've been ignoring spin, but I should note that

$\hbar\omega_c = \frac{\hbar eB}{m}$ is similar to Zeeman splitting $\mu_B g B$

$$\approx \frac{\hbar e}{2m}\,\frac{1}{2}\,B \approx \frac{\hbar\omega_c}{4}$$



Using the ground state energy we can estimate the space occupied by a Landau orbital,

$$\Delta x \Delta p = \frac{\hbar}{2\pi} \qquad \frac{\Delta p^2}{\cancel{2m}} = \frac{\hbar\omega_c}{2} = \frac{\hbar eB}{\cancel{2m}}$$

$$\Delta p = \sqrt{\hbar eB}$$

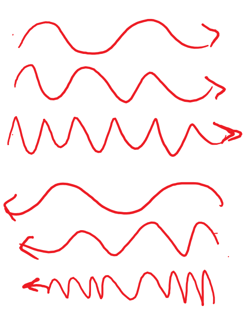
$$\Delta x = \frac{\hbar}{2\pi}\frac{1}{\sqrt{\hbar eB}}$$

$$\approx \sqrt{\frac{\hbar}{eB}}$$

B = 0

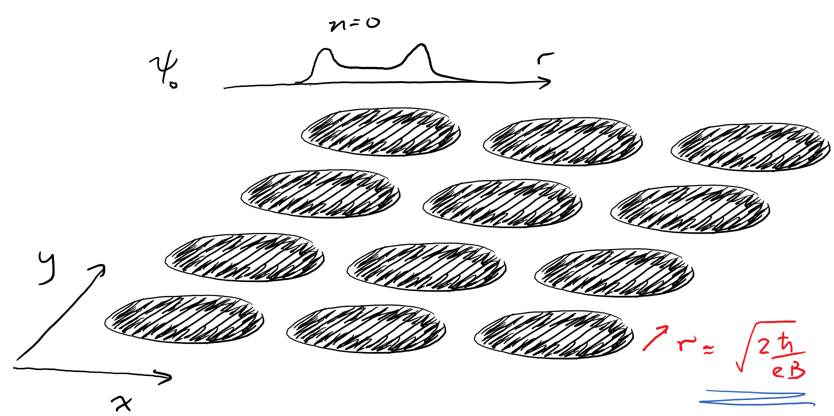
Plane wave basis

$\psi = e^{i\vec{k}\cdot\vec{r}}$



$\longrightarrow$

B > 0



$$r \approx \sqrt{\frac{2\hbar}{eB}}$$

The 2D material filled with circular electron orbits

Pauli exclusion principle: Identical wavefns should not overlap each other.

The number of ψ_0 states per unit area in the sample is

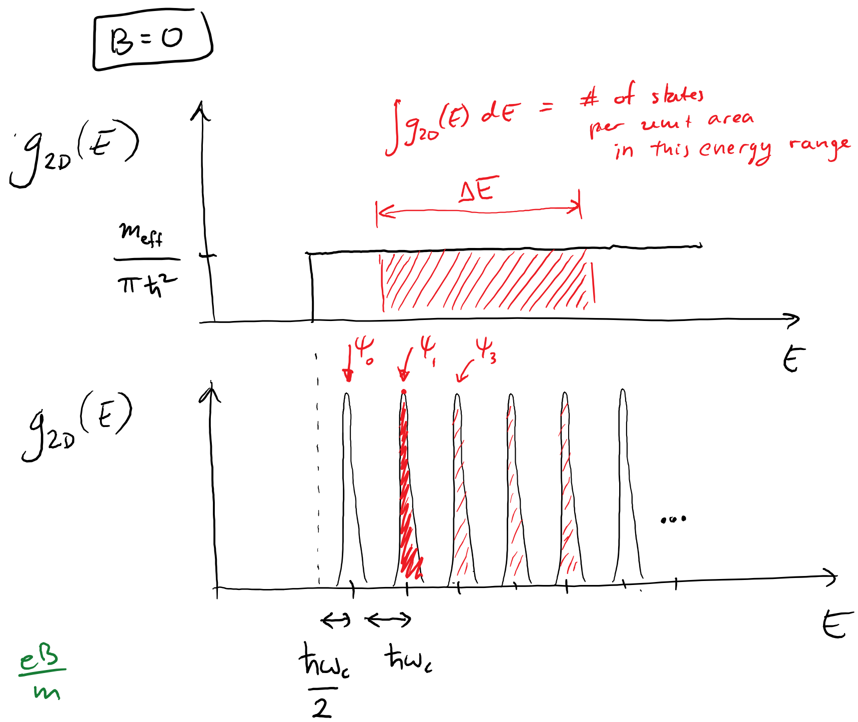
$$2 \cdot \frac{1}{\pi r^2} \qquad \text{where } r = \sqrt{\frac{2\hbar}{eB}}$$

spin degeneracy

Similarly, The number of ψ_1 states per unit area in the sample is also

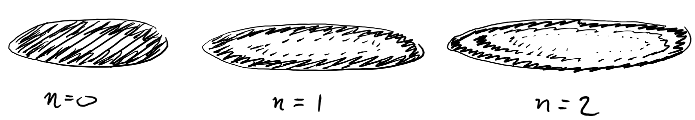
$$2 \cdot \frac{1}{\pi r^2} \qquad \text{where } r = \sqrt{\frac{2\hbar}{eB}}$$

density of states evolves as B field is increased



Each set of states with a unique energy is a Landau level.

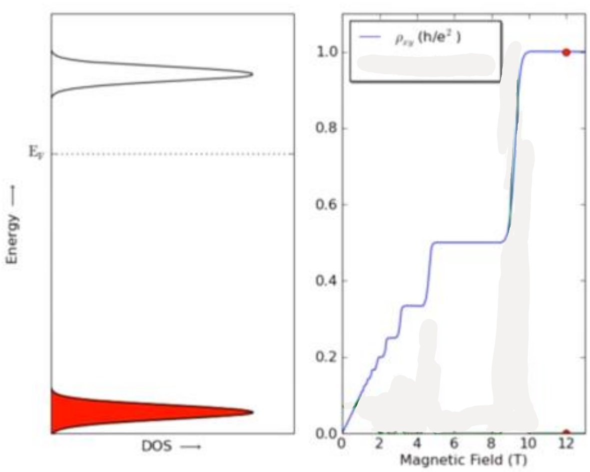
$$\omega_c = \frac{eB}{m}$$



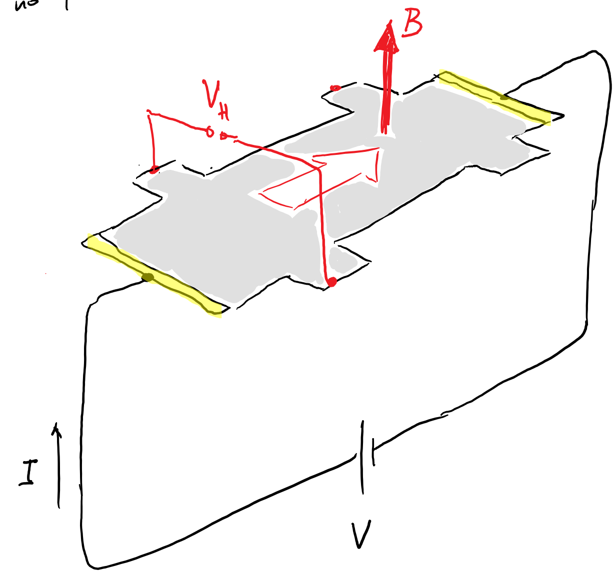
Exercise: If I integrated $g_{2D}(E)$ over a range of energy, ΔE, that includes many peaks, show that the total number that includes many peaks, show that the total number of states per unit area is still $\frac{m_{eff}}{\pi\hbar^2}\Delta E$

Watch animation on wikipedia

The ratio of voltage to current for a 1d channel with no spin degeneracy.



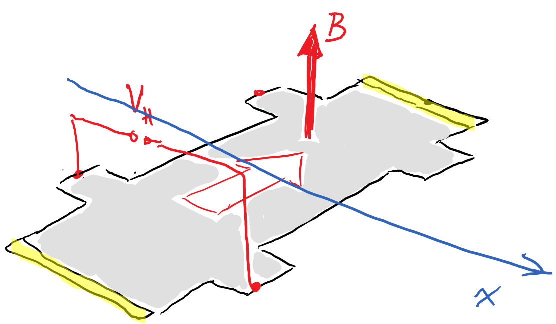
$$R_H = \frac{V_H}{I} = \frac{h}{e^2}$$



This is the density of states of an insulator! (Large gap between filled bands)

Yet a current is still passing from one side of the sample to the other. Explanation requires thinking about the edges.

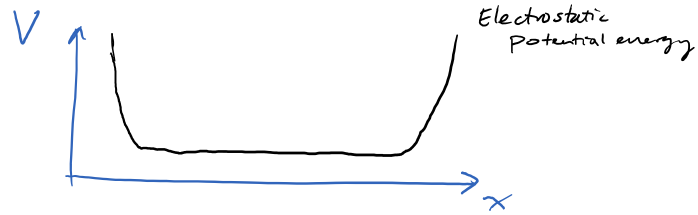
EDGE MODES

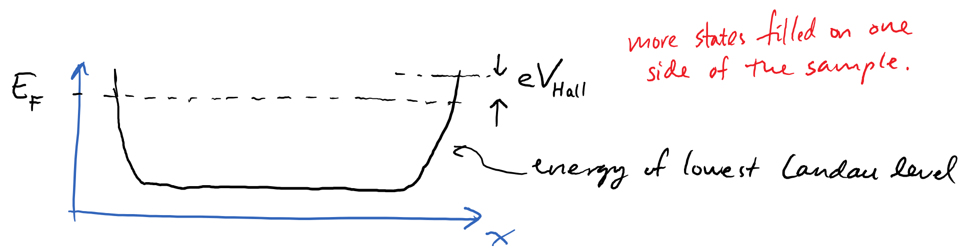


Take cross section in x-direction

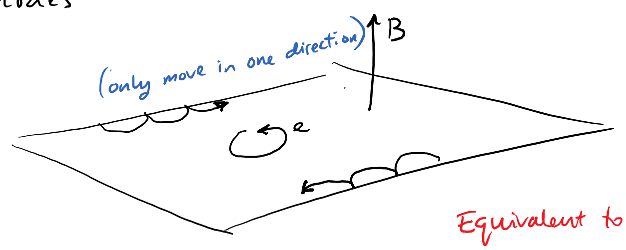
V ↑

Electrostatic potential energy



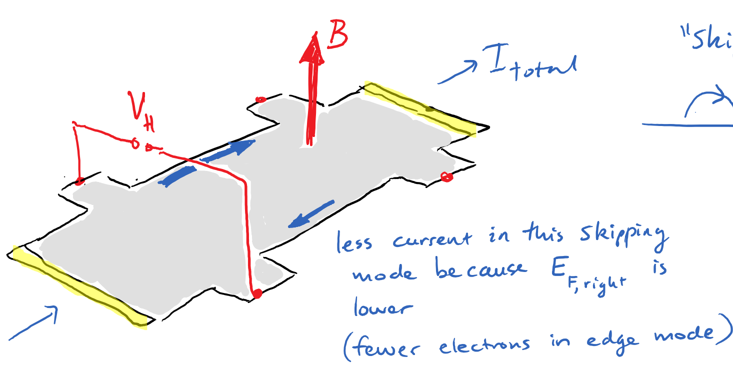


Skipping modes



Equivalent to "$\vec{E}\times\vec{B}$ drifting".

eV_H tell us difference in state filling between left edge and right edge.

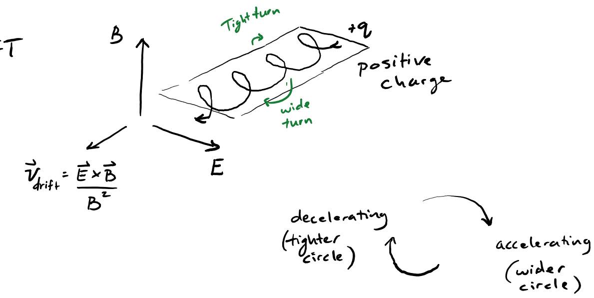


When more left edge is filled, there is a net positive current.

Just like the derivation of current vs. voltage in a 1d conductor.

To calculate the currents carried by these skipping modes we can use $\vec{E}\times\vec{B}$ drift, a uniform density of states, and integration across the sample

$\vec{E}\times\vec{B}$ DRIFT



$$I = e\int_{\text{across sample}} \left(\begin{array}{c}\#\text{ of Landau levels}\\ \text{per unit area}\end{array}\right) \underbrace{\frac{1}{B}\frac{dV}{dx}}_{E\times B\text{ drift velocity}} dx$$

$\frac{dV}{dx}$ is large along the edge of the sample.

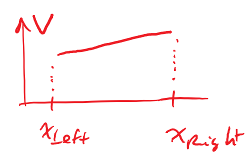
$$= e^2 \int_{\text{Left edge}}^{\text{Right edge}} \frac{eB}{2\pi\hbar} \; \frac{1}{B} \; \frac{dV}{dx} \, dx$$

Because the Landau levels are spin polarized.

$$= \frac{e^2}{2\pi\hbar} V_H$$

$$= \frac{e^2}{h} V_H$$

Note: we'd get the same answer if $V(x)$ was any other shape, for example :



Landau fan.