Ice Mechanics 2021; MEC-E4003

Deformation and Failure Mechanisms in Ice and Their Mathematical Representations

Welcome!

Dr Kari Santaoja, room K1 139
Phone +358 50 432 6623
E-mail Kari.Santaoja@aalto.fi

Contents of the lecture

SSDC - Harrison Bay - Alaska

1 Introduction

Why we are interested in ice mechanics?

Navigation in ice-covered waters is important for many countries.
In Finland during several months all the harbours are only reachable through ice.

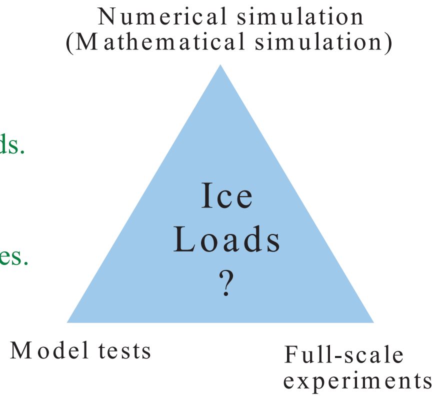
Good knowledge on ice loads is needed because of
- Everyday transportation of goods
- Exploring and exploiting of natural resources under ice-covered waters

Knowledge on ice loads helps to reduce expenses of operation in ice-covered waters.

Traditionally ice loads exerted on structures have been determined:
- By full-scale field tests
- By model tests in ice tanks.

Mathematical models were very simple
Thus, they gave very rough estimate on ice loads.

Corners show above-mentioned three approaches.



Full-scale exper. work is vital as it uses real structures under real ice conditions.

On the other hand, field experiments with real structures have problems:
- In Arctic field work is usually very expensive.
- In Arctic field work is harder than expected.
- Results are seldom generally valid.

Advantages of model tests and numerical simulation are:

(a) Structure and ice condition can be modified.
Thus, design is easier.

(b) Different ice conditions can be modelled.

Model tests are less expensive than full-scale work.

However, they have certain draw-backs.
- Ability of model ice to simulate natural ice is limited.
- Complicated ice conditions are difficult to duplicate in model tests.
  - (e.g. rubble field or contact with the ground)

Mathematical simulation can be either analytical or numerical.

Analytical solutions for ice loads exerted on structures cannot be derived, since
- Material response is nonlinear.
- Ice-structure interaction processes are complex.

Thus, mathematical simulation means numerical simulation by a computer.

Reliable mathematical models for description of ice response plays important role.
Preparation of material models for ice is the task for the present lecture.

Advantage of numerical simulation over full-scale work and model tests lies on its potential.

Full-scale experiments and model tests have long traditions.
Thus, no drastic advancement can be expected.

Numerical simulation is rather young concept in evaluation of ice loads.
It has much potential as papers by Kolari et al. (2009) and Ranta et al. (2010) show.

Determination of ice loads on structures

Numerical simulation
(Mathematical simulation)

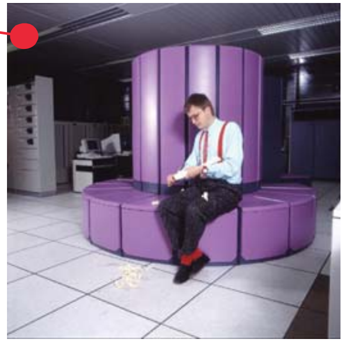


Photo: Anne Kinnunen

Ice
Loads
?

Model tests

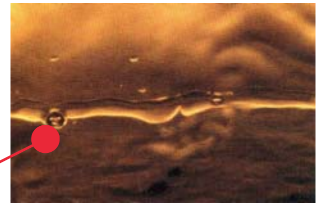
Full-scale
experiments

Photo: Aker Arctic

Material modelling

Dennis Blanchet

Experimental work



Micromechanical investigation

Preparation of macroscopic material model

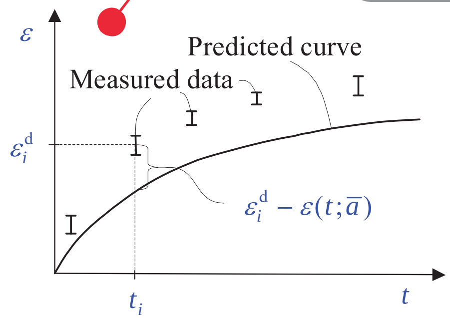
$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon}^{e}$

Thermodynamical validation

Clausius-Duhem inequality

$\boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^{i} + \boldsymbol{\beta} : \dot{\boldsymbol{\alpha}} \geq 0$

Determination of values for material parameters



Numerical verification

Numerical simulation

Different types of mechanical response of material

(a) Static response
Does not evolve with time t .

(b) Quasi-static response
Evolves with time t but inertia forces are negligible. $\vec{F} \approx \vec{0}$.

(c) Dynamic response
Evolves with time t and inertia forces are important $\vec{F} = m\,\vec{a}$.

Response on material also depends on loading

Standard AISI 316 steel is studied.

(a) Stress σ is below yield point R_e and temperature T is close to room temp.
Deformation is elastic.

(b) Stress σ is above yield point R_e and temperature T is close to room temp.
Deformation is elastic-plastic.

(c) Temperature T is above 450 °C.
Also creep is remarkable deformation.

On the other hand:
Response of ceramics are “always” elastic-brittle.

Material is not elastic or plastic.
Response of material is elastic, plastic, ...

Dennis Blanchet

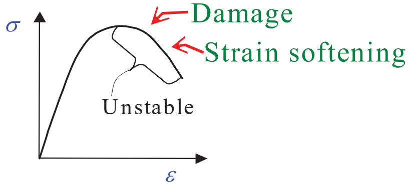
Different types of material models

(I) Instantaneous response

There is no time-scale in the material model.

Can be used if both of the following conditions hold:
- Deformation mechanisms are faster than the rate of loading.
- Constant loading does not cause response.

Loading can be mechanical, thermal etc.



Can lead to a simple material model.
- Theory of elasticity.

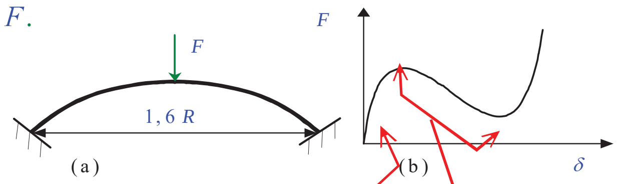
Can lead to an unstable material behaviour.
- Computational problems.

Stress waves may travel back and forth across the structure.
- Computational problems.

When the stiffness matrix K is not positive definitive?

The arch is loaded by the force F.

Displacement in the centre of the arch is δ



The force-deflection curve is highly nonlinear.

Between these points the stiffness matrix K is not positive definitive.

Different types Cont...

Instantaneous response

Elastic deformation | Elastic-plastic deformation | Damage ?

(II) Time-dependent response

More realistic material models can be formulated.

Easier to avoid stability problems.

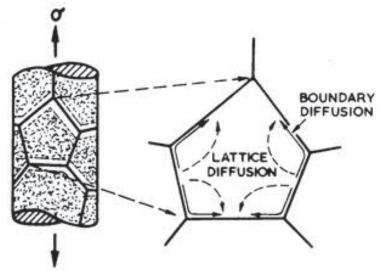
Time-dependent response

Creep | Damage

Creep mechanisms in crystalline materials

Creep remarkable when temp. T above 30 - 40% of melting point in Kelvin scale.

Main mechanisms.



(a) Diffusional creep:
- Important when T is close to melting point.
- Stress σ is low.

(b) Dislocation creep (power law creep):
- Temperature T is lower compared to diffusional creep.
- Stress σ is higher compared to diffusional creep.

(c) Grain boundary sliding:
- Temperature T is very close to the melting point.
- Stress σ is low.

Strain terms due to different deformation mechanisms are additive

If the strain $\varepsilon < 0.1$, deformation is infinitesimal (small).
If deformation mechanisms do not interact,
strain terms due to different deformation mechanisms are additive, i.e.

$$\varepsilon = \varepsilon^{e} + \varepsilon^{d} + \varepsilon^{Th} + \varepsilon^{p} + \varepsilon^{v} + \ldots \quad (1)$$

- ε^{e} Hookean strain.
- ε^{d} Damage strain.
- ε^{Th} Thermal strain.
- ε^{p} Plastic strain.
- ε^{v} Viscous strain.

Other strain terms are possible to introduce.
For example, fatigue strain ε^{f}.

In this presentation the damage-elastic strain is defined to be

$$\varepsilon^{de} = \varepsilon^{e} + \varepsilon^{d} . \quad (2)$$

Although uniaxial quantities are used, Expression (1) is valid for 3D case.
In 3D case Equation (1) reads

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{e} + \boldsymbol{\varepsilon}^{d} + \boldsymbol{\varepsilon}^{Th} + \boldsymbol{\varepsilon}^{p} + \boldsymbol{\varepsilon}^{v} + \ldots \quad (3)$$

There are cases where small deformation assumption is not valid.
They are not for this course.

Glacier of Svartisen - Norway

2 Background for material modelling of ice

Structure of ice

Brief discussion is carried out here.

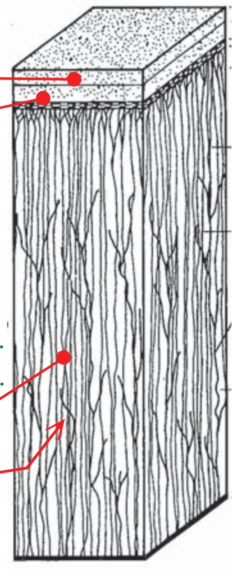
Level ice is normally built up in three layers.

(1) Snow.

(2) Snow ice. (e.g. in the Baltic sea)
Isotropic.
Grain size normally less than five millimetres.

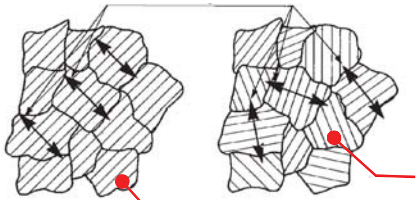
(3) Columnar-grained ice frozen from sea water at bottom.
Horizontal diameter of crystals can be few centimetres.

Brine drainage channels



Vertical view

Direction of optic axis



Horizontal plane of S2 ice is isotropic.

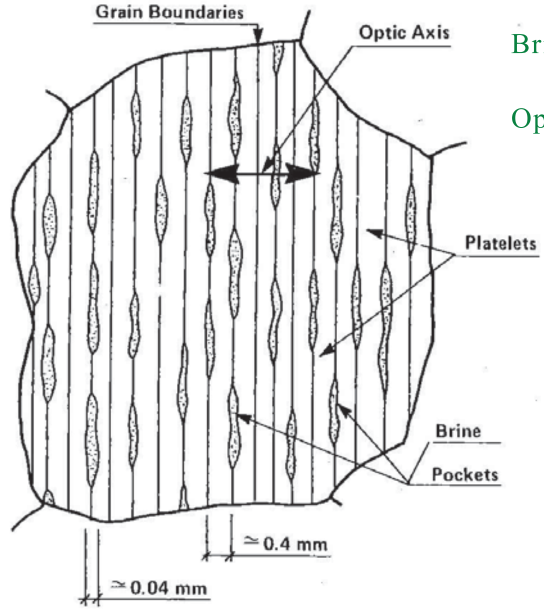
Horizontal plane of S3 ice is anisotropic.
Currents are the reason for anisotropy.

Horizontal view of columnar ice

Columnar-grained S2 ice is transversely isotropic.

Structure of ice Cont...

Horizontal view of one crystal of columnar ice

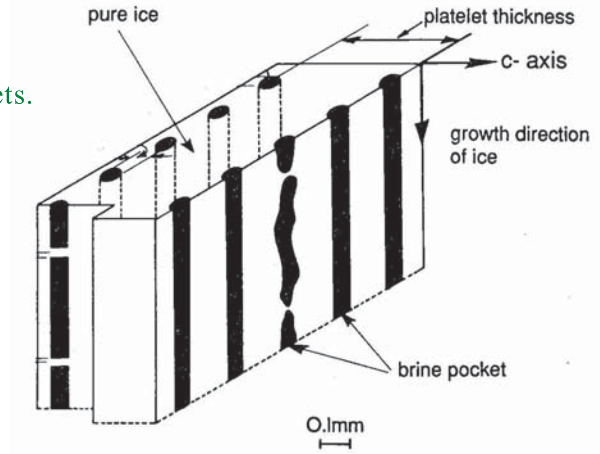


Brine pockets are vertical tubular cells.

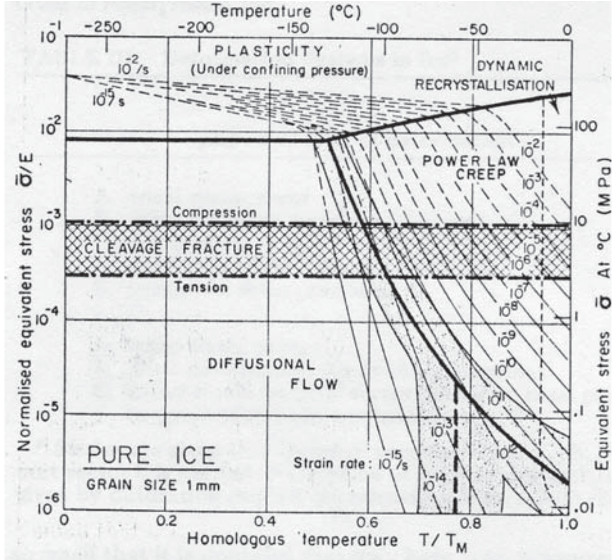
Optic axis is called c-axis.

Sea water in the Gulf of Bothnia has low salinity (≈ 1o/oo).
Thus, the sea ice in the Gulf of Bothnia is almost equivalent to freshwater S2 ice.

Idealised picture of brine pockets.



Deformation mechanisms map of ice



For fine-grained ice (pity).

Variable $\bar{\boldsymbol{\sigma}}$ of the vertical axis stands for

$$[(\sigma^{\mathrm{vM}})^2 =] \qquad \bar{\boldsymbol{\sigma}}^2 := \frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \frac{3}{2} s_{ij} s_{ij} \cdot (1)$$

In Equation (1) the quantities σ_1, σ_2 and σ_3 are the principal stresses.

Variable $\bar{\boldsymbol{\sigma}}$ is the von Mises stress σ^{vM}.

In Eq. (1) notation s_{ij} stands for scalar component of deviatoric stress tensor $\mathbf{s}$.

Map shows that plasticity does not occur since ice fractures before plastic yield.

Important strain rate values for ice-structure interaction cases are $\dot{\varepsilon} > 10^{-5}\ 1/\mathrm{s}$.

Excluding structures in level ice under ice temperature rise.

There is discussion on the role of power law creep.

Diffusional flow is too slow process for ice-structure interaction.

Dynamic recrystallisation is too slow process for ice-structure interaction.
(it is not dynamic)

Deformation mechanisms of ice

1) Elastic distortion of crystal lattices. Modelled by Hooke's law.
2) Grain boundary sliding. Also called delayed elasticity.
3) Microcrack formation.
4) Effect of microcracks on the (elastic) properties of ice.

} Important for us

5) Creep by dislocation movement. (Power law creep).
6) Recrystallization, diffusional flow etc.
7) Pressure melting.

von Mises stress σ^{vM}: (next to power 2)

$$[\sigma^{\mathrm{vM}}]^2 = \frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \frac{3}{2} s_{ij} s_{ij}, \qquad (1)$$

where the quantities σ_1, σ_2 and σ_3 are the principal stresses.

or in the $(\sigma_x, \sigma_y, \sigma_z)$ coordinate system

$$[\sigma^{\mathrm{vM}}]^2 := \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2\right] + 3\left[\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right]. \qquad (2)$$

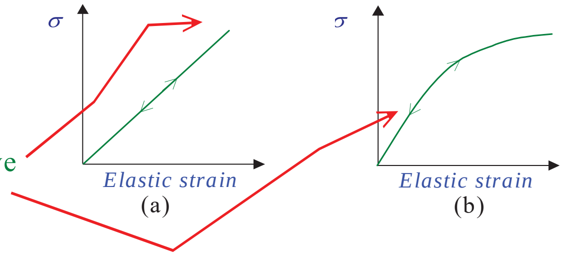
Elastic deformation

Response of the material is so fast that it can be modelled as time-independent.

Elastic deformation:
Processes during unloading happen in reverse order compared to loading.

Elastic deformation can be:
- (a) Linear
- (b) Nonlinear

Stress-strain curve

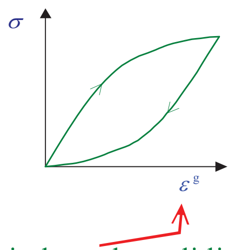


In elastic response (elastic strain ε^{e}):
During unloading stress-strain curve takes same path back as it took during loading.

Viscoelastic deformation

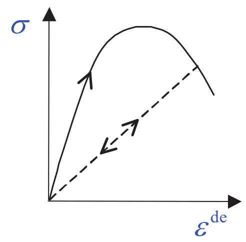
It describes recoverable time-dependent processes (viscoelastic strain ε^{ve})

It can be related to:
1. (1) Dislocation movement.
2. (2) Grain boundary sliding.
3. (3) Diffusion of atoms



Grain boundary sliding (for example)

Damage-elastic deformation



Response of Hookean material with damage.

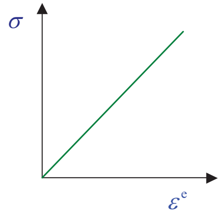
Descending part of the curve is due to damage.

In ice also other deformation mechanisms exist.

Thus, horizontal axis should be e.g. $\varepsilon^{de} + \varepsilon^{v}$.

Above figure is prepared for studying how damage-elastic deformation is modelled.

3 Hookean deformation



When deformation is reversible and linear,
It is said to be Hookean deformation.

Notation ε^{e} stands for Hookean strain.

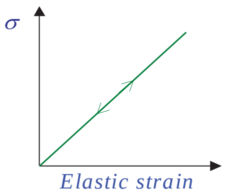
Micromechanical processes associated with Hookean deformation are fast.
Usually they are faster than the applied load.
Thus, time-dependency of the load determines the rate of deformation.
Therefore, the material model can be without a time-scale.

Thus, Hookean strain ε^{e} is modelled by time-independent, instantaneous process.
This leads to a simple material model.
It is called Hooke's law.

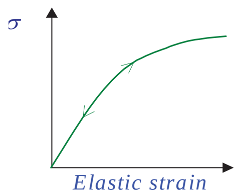
Hooke's law is widely used, since
Almost all materials follow Hooke's law.

Response of a material can consist of several deformation mechanisms.
(Almost) in every case one of them is reversible and with good accuracy Hooke.

Linear elastic deformation



(a)



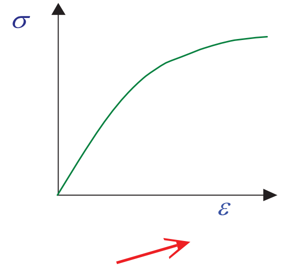
(b)

Non-linear elastic deformation

Thus, elastic deformation can be nonlinear.
It would be clearer to use for Hooke's law something else than ε^{e}. (Sorry)

Misinterpretations

Some researchers have introduced e.g. following terms:



(1) Effective modulus.
(2) Strain modulus.
(3) Elastic modulus. (maybe equal to Young's modulus)

The background for the above terms is shown in figure.
Thus, researchers have measured something else than ε^{e}.

instead of ε^{e}

The horizontal axis may be

$$\varepsilon = \varepsilon^{e} + \varepsilon^{v} . \qquad (4)$$

From above nonlinear curve ice researchers have defined the other moduli.
The error is:
The horizontal axis is incorrect.

Sometimes also (4) "dynamic modulus" is introduced.
The definition of dynamic modulus equals that of Young's modulus.

Arno Keinonen

4 Effect of microcracks on the elastic properties of ice

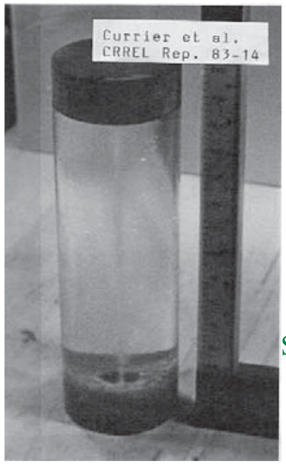
Damage of material can be:

- Carbide coarsening. (high-temperature power-plant materials)
- Grain boundary cavitation. (high-temperature power-plant materials)
- Breakage of fibres. (paper, polymer-matrix composites)
- Microcracks in matrix material. (polymer-matrix composites)
- Microcracks. (ice)

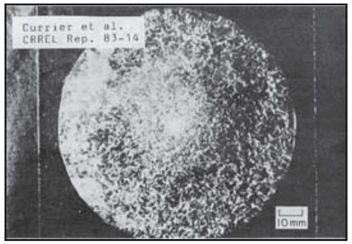
Damage can be accommodated to:

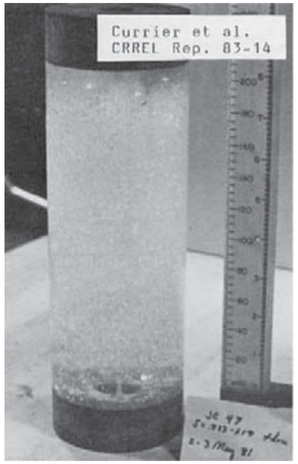
- Inelastic deformation. (Carbide coarsening)
- Elastic deformation. (Voids and microcracks)

Damage of ice = Microcracks.



Specimen after loading.



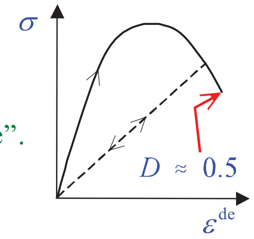


Specimen before test.

Traditional description of damage (Hooke + damage)

The stress strain relation reads

$$\sigma = (1 - D)\, E\, \varepsilon^{de} . \qquad (1)$$



In Equation (1): D scalar-valued variable “damage”.
virgin material $D = 0$.
fully damaged material $D = 1$.

Damage evolution equation (Kachanov 1958)

$$\dot{D} = \left[\frac{\sigma}{A_0 (1 - D)} \right]^r \qquad (2)$$

Eqs (1) and (2) lead to the curve shown in the figure, but they are too simple.

The effective stress $\tilde{\sigma}$ is defined by (Rabotnov 1968)

$$\tilde{\sigma} := \frac{\sigma}{1 - D} . \qquad (3)$$

Definition (3) is acceptable, but it is too simple.

Note! Mathematical descriptions for micromechanisms are needed.

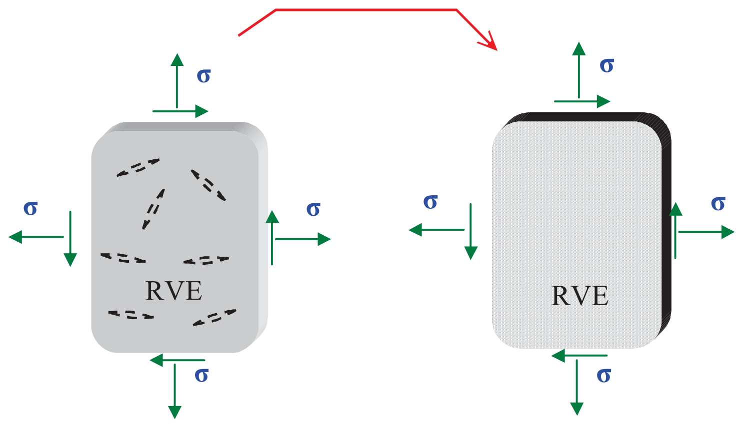
Homogenisation and the representative volume element (RVE)

Representative volume elements is:

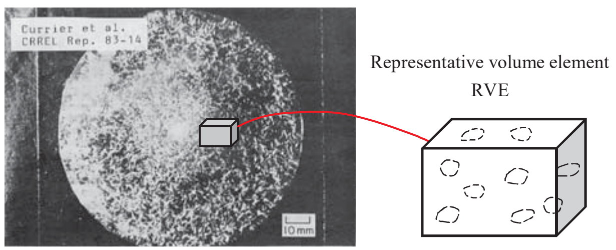
(a) Large enough to be statistically representative of modelled material properties.

(b) Small enough to be treated as a material point in the structural analysis.

Concept of RVE is to smooth out local discontinuities $\Rightarrow$ Continuum mechanics.

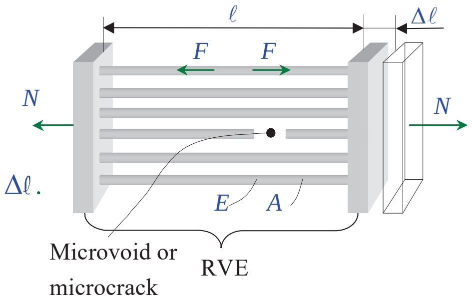


Representative volume element (RVE) for ice.



Simple bar model for description of stress measures σ and $\tilde{\sigma}$

Model is as follows:



Cross-sectional area of a single bar is A.
Length of a bar is ℓ.
Elongation of single bar due to loading is $\Delta\ell$.
Force over 1 unbroken bar is F.

Figure gives

$$\sigma^* = E\,\varepsilon^{\mathrm{e}}, \quad \text{where} \quad \sigma^* = \frac{F}{A} \quad \& \quad \varepsilon^{\mathrm{e}} = \frac{\Delta\ell}{\ell} \qquad \text{For unbroken bar.} \quad ,(4)$$

where σ^* is the stress in unbroken bars. (The phrase “unbroken” is left out.)

Equation $(4)_1$ and $(4)_3$ give $\sigma^* = E\,\dfrac{\Delta\ell}{\ell}$ For bar. (5)

Effective stress $\tilde{\sigma}$ is defined to be

$$\tilde{\sigma} := E\,\varepsilon^{\mathrm{de}}. \qquad (6)$$

Figure gives

$$\varepsilon^{\mathrm{de}} = \frac{\Delta\ell}{\ell} \qquad \text{For bar \& for RVE} \quad (7)$$

Equation (7) is valid for: unbroken bar, since $\varepsilon^{\mathrm{de}} = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{d}}$, where $\varepsilon^{\mathrm{d}} = 0$.
RVE

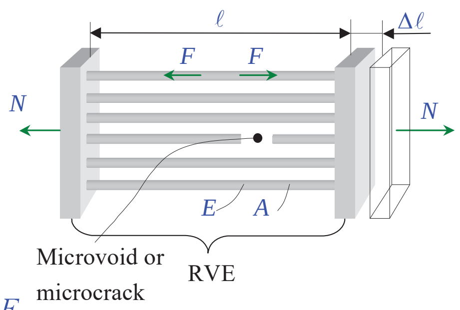
Damage-elastic strain $\varepsilon^{\mathrm{de}}$ is substituted from Eq. (7) into Def. (6)

$$\tilde{\sigma} = E\,\frac{\Delta\ell}{\ell}. \qquad (8)$$

Comparison of Eqs (5) and (7) gives $\sigma^* = \tilde{\sigma}$ For bar. (9)

Eq. (9) shows that effective stress $\tilde{\boldsymbol{\sigma}}$ is microscopic stress in matrix material.
Since stress $\tilde{\boldsymbol{\sigma}}$ takes same value over all unbroken bars, it is homogeneous stress.
This means that the effective stress $\tilde{\boldsymbol{\sigma}}$ is an averaged microscopic stress.

Number of equal tensile bar is n.
Cross-sectional area of a single bar is A.
Number of broken bar is m.



Total tensile force over bar system is N.
Force over 1 unbroken bar is F.

Equations $(4)_2$ and (9) are

$$\sigma^* = \frac{F}{A} \quad \text{and} \quad \sigma^* = \tilde{\sigma} \quad \Longrightarrow \quad \tilde{\sigma} = \frac{F}{A} \qquad \text{For bar.} \quad (10)$$

(Total) force over the RVE is N and it is sum of forces along unbroken bars, i.e.

$$N = (n - m)\, F \qquad \text{RVE} = \text{RVE} \times \text{bar.} \quad (11)$$

The relation between an RVE quantity and a microscopic quantity is in Eq. (11).

Equation (11) is divided by nA, which gives

$$\frac{N}{nA} = \frac{(n - m)}{n}\frac{F}{A} \qquad \text{RVE} = \text{RVE} \times \text{bar.} \quad (12)$$

The RVE (macroscopic) stress σ is defined to be

$$\sigma := \frac{N}{nA} \qquad \text{For RVE.} \quad (13)$$

Denominator in Def. (13) is sum of areas of all bars, although some are broken.
Thus, the stress σ is a macroscopic averaged stress in the material of the RVE.
Substitution of Eq. $(10)_3$ and Def. (13) into Eq. (12) yields

$$\sigma = \frac{(n - m)}{n}\tilde{\sigma} \qquad \text{RVE} = \text{RVE} \times \text{bar.} \quad (12)$$

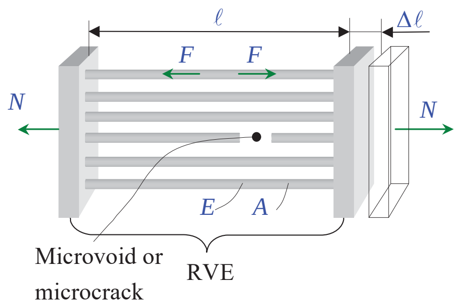
Damage D is the number of broken bars divided by the number of all bars.

$$D := \frac{m}{n} \quad \Longrightarrow \quad 1 - D = \frac{n - m}{n} \qquad \text{For RVE.} \quad (13)$$

Substitution of Eq. $(13)_2$ into Eq. (12) gives $\tilde{\sigma} = \frac{1}{1 - D}\sigma$ bar = RVE (14)

Compare Eq. (3) by Rabotnov.

Number of equal tensile bars is n.
Length of a bar is ℓ.
Elongation of single bar due to load is $\Delta\ell$.
Number of broken bar is m.



Effective Young's modulus $\tilde{E}$ is defined

$$\tilde{E} := \frac{(n - m)}{n} E \qquad \text{RVE} = \text{RVE} \times \text{bar.} \quad (15)$$

Eq. (12) and Eq. (8) are

$$\sigma = \frac{(n - m)}{n}\tilde{\sigma} \quad \text{RVE} = \text{RVE} \times \text{bar} \qquad \text{and} \qquad \tilde{\sigma} = E\,\frac{\Delta\ell}{\ell}. \quad (16)$$

Substitution of Eq. $(16)_2$ into Eq. $(16)_1$ gives

$$\sigma = \frac{(n - m)}{n} E \frac{\Delta\ell}{\ell} \quad \Longrightarrow \quad \sigma = \tilde{E}\frac{\Delta\ell}{\ell} \qquad \text{RVE} = \text{RVE} \times \text{bar.} \quad (17)$$

Equation (7) is

$$\varepsilon^{\text{de}} = \frac{\Delta\ell}{\ell} \qquad \text{For bar \& for RVE} \quad (18)$$

Substitution of Eq. (18) into Eq. $(17)_2$ gives

$$\sigma = \tilde{E}\,\varepsilon^{\text{de}} \quad \Longrightarrow \quad \varepsilon^{\text{de}} = 1/\tilde{E}\,\sigma = \tilde{S}\,\sigma \qquad \text{For RVE,} \quad (19)$$

In Expr. $(19)_2$ effective compliance $\tilde{S}$ is defined $\tilde{S} := 1/\tilde{E}$ For RVE. (20)

The effective stress tensor $\tilde{\sigma}$ is defined in Def. (6), viz.

$$\tilde{\sigma} := E\,\varepsilon^{\text{de}}. \quad (21)$$

Substitution of Eq. $(19)_2$ into Def. (21) gives

$$\tilde{\sigma} = E\,\tilde{S}\,\sigma \qquad \text{bar} = \text{bar} \times \text{RVE.} \quad (22)$$

Micromechanics gives "exact" relation

$$\tilde{\boldsymbol{\sigma}} = \mathbf{E} : \tilde{\mathbf{S}} : \boldsymbol{\sigma}. \quad (23)$$

Use this instead of this $\tilde{\sigma} = \frac{1}{1 - D}\sigma$

Conclusions:

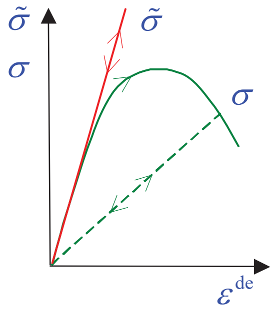
Effective stress tensor $\tilde{\boldsymbol{\sigma}}$ describes the "averaged" stress between the microcracks.
Thus, eff. stress tens. $\tilde{\boldsymbol{\sigma}}$ is driving force for creep, microcracking, GB sliding etc.

Stress tensor $\boldsymbol{\sigma}$ is the macroscopic averaged stress in the material of the RVE.

The relation between the effective stress tensor $\tilde{\boldsymbol{\sigma}}$ and the stress tensor $\boldsymbol{\sigma}$ is

$$\tilde{\boldsymbol{\sigma}} = \mathbf{E}:\tilde{\mathbf{S}}:\boldsymbol{\sigma}\,. \tag{23}$$

Definition (6) for effective stress $\tilde{\sigma}$ and Expression $(19)_1$ for stress σ read



$$\tilde{\sigma} := E\,\varepsilon^{\mathrm{de}} \quad \text{and} \quad \sigma = \tilde{E}\,\varepsilon^{\mathrm{de}} \qquad (6),\ (19)_1$$

Difference between $\tilde{\sigma} \sim \varepsilon^{\mathrm{de}}$ and $\sigma \sim \varepsilon^{\mathrm{de}}$ curves.

With increasing damage effective stress $\tilde{\sigma}$ increases.

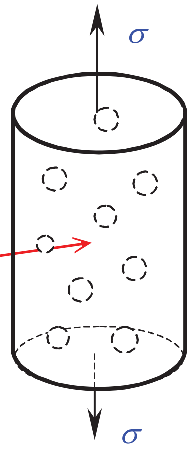
Definition (15) for effective Young's modulus $\tilde{E}$ is

$$\tilde{E} := \frac{(n-m)}{n}\,E \tag{15}$$

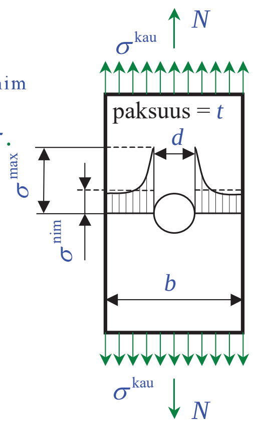
With increasing damage (here m) stress σ decreases.

Interpretation:

Effective stress $\tilde{\sigma}$ is averaged microscopic stress in the matrix



Hole in plate.
Eff. str. $\tilde{\sigma}$ is σ^{nim}
Stress σ^{kau} is σ.



Non-homogeneous microscopic stress.

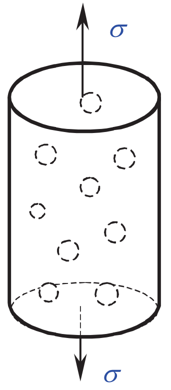
Damage due to voids or microcracks

The notation w^{c} stands for the complementary strain-energy density.

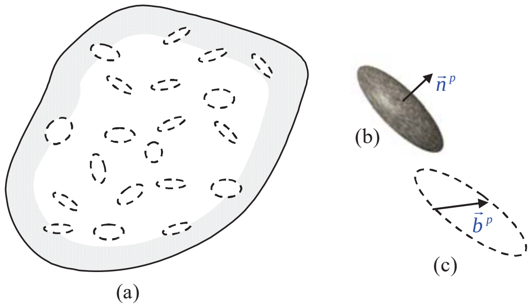
For Hookean matrix deformation with non-interacting voids w^{c} takes appearance

$$w^{c}(\boldsymbol{\sigma}, f) = \frac{1}{2}\left[\frac{1}{3\,(3\,\lambda + 2\,\mu)}\,(1 + A f)\,[\mathbf{1}:\boldsymbol{\sigma}]^{2} + \frac{1}{2\,\mu}\,(1 + B f)\,\mathbf{s}:\mathbf{s}\right]. \quad (24)$$

In Expression (24): λ and μ Lamé elastic constants of the matrix material. (functions of E and ν)



- A and B Express the form of voids.
- $\mathbf{1}$ Second-order unit tensor.
- f Void volume fraction.



For Hookean matrix material with non-interact. penny-shaped microcracks w^{c} is

$$w^{c} = \frac{1}{2}\left[\frac{1}{3\,(3\,\lambda + 2\,\mu)}\,[\mathbf{1}:\boldsymbol{\sigma}]^{2} + \frac{1}{2\,\mu}\,\mathbf{s}:\mathbf{s}\right] + \frac{1}{V^{\mathrm{rve}}}\sum_{p=1}^{k}\frac{1}{2}\int_{A^{p}}\vec{b}^{\,p}\cdot(\boldsymbol{\sigma}\cdot\vec{n}^{p})\,\mathrm{d}A^{p}. \quad (25)$$

In Expression (25):
- V^{rve} Representative volume element.
- k Number of microcracks within V^{rve}.
- A^{p} Area of the p'th microcrack.
- $\vec{n}^{p}$ Unit normal vector to the area A^{p}.
- $\vec{b}^{p}$ Jump between the material points across microcrack.

In both cases the following holds:

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{d}} + \text{Other deformation mechanisms}. \quad (26)$$

Thus, the influence of Hookean deformation and damage are separated.

Damage due to.. Cont...

Equation (26) is

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{d}} + \text{Other deformation mechanisms}. \quad (26)$$

The damage-elastic strain $\varepsilon^{\mathrm{de}}$ is defined to be

$$\varepsilon^{\mathrm{de}} := \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{d}}. \quad (27)$$

Non-interacting voids and microcracks give expressions

$$\varepsilon^{\mathrm{de}} = (S + S^{\mathrm{d}})\,\sigma = \tilde{S}\,\sigma, \qquad \text{where } S := E^{-1}. \quad (28)$$

The quantity S is called compliance.

Conclusions for damage mechanics of ice

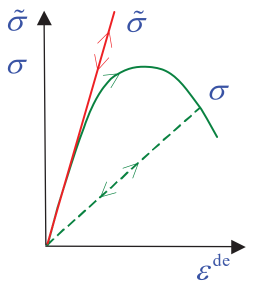
(I) Use micromechanical approach to study the effect of microcracks.
You get the analytical solution for $\tilde{S}$ from Equation (28), viz.

$$\varepsilon^{\mathrm{de}} = (S + S^{\mathrm{d}})\,\sigma = \tilde{S}\,\sigma\,, \qquad \text{where } S := E^{-1}\,. \tag{31}$$

(II) Calculate effective stress $\tilde{\sigma}$ by Equation (22), viz.

$$\tilde{\sigma} = E\,\tilde{S}\,\sigma\,. \tag{32}$$

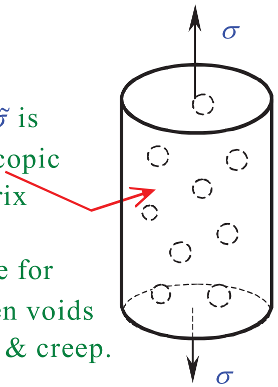
(III) Assume that the effective stress $\tilde{\sigma}$ is the stress in the matrix material.



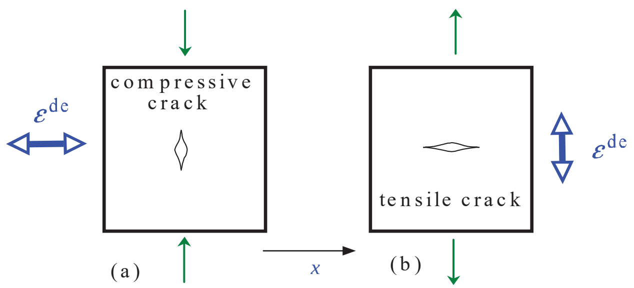
Interpretation:

Effective stress $\tilde{\sigma}$ is averaged microscopic stress in the matrix

$\tilde{\sigma}$ is driving force for processes between voids and cracks. GBS & creep.



(IV) Damage-elastic strain $\varepsilon^{\mathrm{de}}$ is driving force for microcrack formation



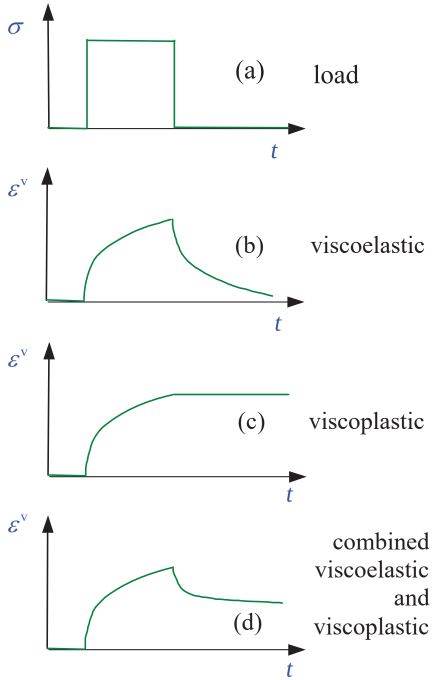
5 Grain boundary sliding and dislocation creep

Viscous strain ε^{v} describes time-dependent deformation mechanisms in materials. Grain boundary sliding; GBS & dislocation creep are time-dependent deformations.

Thus, grain boundary sliding and dislocation creep are parts of viscous strain ε^{v}. This is

$$\varepsilon^{v} = \varepsilon^{g} + \varepsilon^{dc} + \ldots \qquad (1)$$

where ε^{g} grain boundary sliding strain
ε^{dc} dislocation creep strain
… other viscous strains



Viscous deformation may lead to:
- recoverable strain (viscoelastic)
- permanent strain (viscoplastic)
- combination of above 2 strains

In sea ice viscoelastic deformation plays major role.

Viscoelastic strain is:
time-dependent and recoverable.

Thus, in ice mechanics it is called delayed elastic strain. (First for GBS only)

Terms viscous strain and creep strain are synonymous (at least almost).

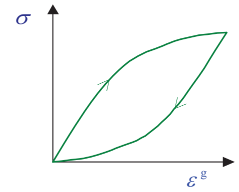
The closer temperature T is to the melting point of material the larger is creep ε^{v}.
Creep is important deformation mechanism since
In engineering applications ice exhibits close to its melting point.

Creep strain of ice can be much larger than Hookean strain.

Model for grain boundary sliding proposed by Sinha

The honoured model is that proposed by Sinha in 1978, viz.

$$\varepsilon^{g} = c_1 \frac{d_1}{d} \left(\frac{\sigma}{E}\right)^{s} \left[1 - e^{-(a_T t)^{b}}\right], \qquad (2)$$



where ε^{g} ≙ delayed elastic strain.
d ≙ average grain size.
a_T ≙ inverse of the relaxation time.
t ≙ time.

GBS is irreversible

Model (2) is for uniaxial tension.
Problems with Expressions (2) are:
Formulation of the model for multiaxial state of stress.
Formulation of the model for varying state of stress (and strain).

Something new is needed.

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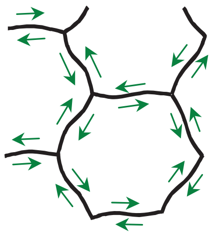
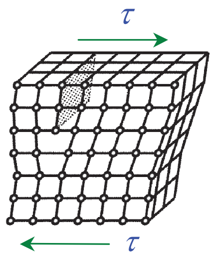
Shear stress is the driving force for GBS and dislocation creep

Grain boundary sliding (GBS)

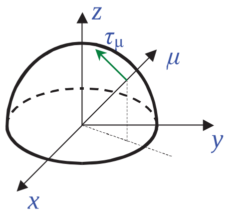


Dislocation creep



Square root of mean square of the shear stresses over all planes of the sphere is

$$I(\tau_{\vec{n}}) := \left[\frac{1}{\Omega} \int_{\Omega} (\tau_{\vec{n}})^2 \, \mathrm{d}\Omega \right]^{1/2} . \qquad (3)$$



The following holds:

$$I(\tau_{\vec{n}}) = \sqrt{\frac{2}{15}} \, J_{\mathrm{vM}}(\boldsymbol{\sigma}) . \qquad (4)$$

Grain boundary sliding and dislocation creep are driven by shear stresses τ .
In granular ice grain boundaries are in all orientations.
Dislocation creep asks 4 independent dislocation gliding planes.

Thus, $J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}})$ is good measure of driving force for GBS and for dislocation creep.

This is

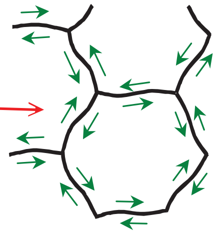
$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} \propto J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}}) \qquad \text{and} \qquad \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} \propto J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}}) . \qquad (5)$$

The effective stress $\tilde{\boldsymbol{\sigma}}$ is the (averaged) stress state between voids and cracks.
Therefore it is the driving force for GBS and dislocation creep.

Thus, $J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}})$ is good measure of driving force for GBS and for dislocation creep.

This is

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} \propto J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}}) \quad \text{and} \quad \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} \propto J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}}) . \quad (5)$$



GBS and dislocation creep are resisted by:
Elastic distortion of crystal lattices.

Elastic distortion makes GBS and dislocation creep recoverable mechanisms.

The resistance for GBS and dislocation creep is modelled by $\tilde{\boldsymbol{\beta}}$.
Thus, Expression (5) yields

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} \propto J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}}) . \quad (6)$$

Model for grain boundary sliding and dislocation creep (no volume changes)

$$(\dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} \propto) \qquad \dot{\boldsymbol{\varepsilon}}^{\mathrm{v}} = \mathring{\varepsilon}_{\mathrm{reg}} \left[\frac{\langle J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}}) - \sigma_{\mathrm{tr}} \rangle}{\sigma_{\mathrm{rc}}} \right]^n \frac{\tilde{\mathbf{s}} - \tilde{\mathbf{b}}}{J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}})} . \quad (7)$$

In Model (7)

$$\dot{\boldsymbol{\beta}} = m_1 \dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} . \quad (8)$$

Increasing grain boundary sliding generates increasing elastic distortion and therefore increasing resistance.

$$\dot{\boldsymbol{\beta}} = m_1 \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} . \quad (9)$$

Same expression for resistance for disloc. creep. Physics is different.

Notation $\mathbf{s}$ is the deviatoric stress tensor. It is a “3D shear stress”.
Notation $\mathbf{b}$ is the deviatoric part of $\boldsymbol{\beta}$.
Notation σ_{tr} is the threshold value.

The Macaulay brackets $\langle \cdot \rangle$ have the following property:

$$\langle x \rangle := \begin{cases} 0 & \text{when } x < 0 \\ x & \text{when } x \geq 0 . \end{cases} \quad (10)$$

Equations (7)...(9) are

$$(\dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} \propto) \qquad \dot{\boldsymbol{\varepsilon}}^{\mathrm{v}} = \mathring{\varepsilon}_{\mathrm{reg}} \left[\frac{\langle J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}}) - \sigma_{\mathrm{tr}} \rangle}{\sigma_{\mathrm{rc}}} \right]^n \frac{\tilde{\mathbf{s}} - \tilde{\mathbf{b}}}{J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}})} . \quad (7)$$

and

$$\dot{\boldsymbol{\beta}} = m_1 \dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} \quad \text{and} \quad \dot{\boldsymbol{\beta}} = m_1 \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} . \quad (8) \; \& \; (9)$$

Based on Eqs (7)...(9) GBS and dislocation creep are modelled with same model.
Thus, the following is written

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}} = \mathring{\varepsilon}_{\mathrm{reg}} \left[\frac{\langle J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}}) - \sigma_{\mathrm{tr}} \rangle}{\sigma_{\mathrm{rc}}} \right]^n \frac{\tilde{\mathbf{s}} - \tilde{\mathbf{b}}}{J_{\mathrm{vM}}(\tilde{\boldsymbol{\sigma}} - \tilde{\boldsymbol{\beta}})} . \quad (11)$$

where

$$\dot{\boldsymbol{\beta}} = m_1 [\dot{\boldsymbol{\varepsilon}}^{\mathrm{g}} + \dot{\boldsymbol{\varepsilon}}^{\mathrm{dc}}] . \quad (12)$$

Ice researchers are discussing the roles of GBS and dislocation creep in sea ice.
From measured data of sea ice it is difficult to distinguish GBS and disloc. creep.

Advantage of Model (11) & (12) is that GBS and dislocation creep are modelled in one set of equations.

This may be best solution until better information on creep of sea ice is obtained.

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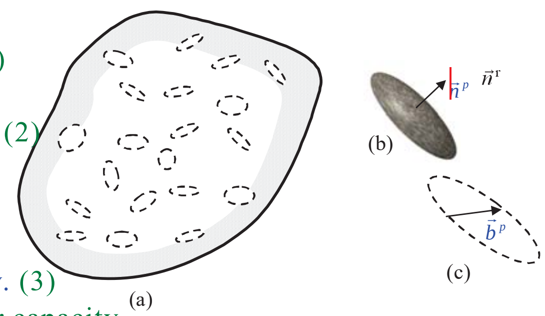
6 Microcrack formation and examples

In ice the following hold:
Size of microcracks ≈ Grain size. (1)

Microcracks are randomly oriented. (2)



Expressions (1) and (2) lead to:
Microcracks have to treat separately. (3)
It requires huge amount of computer capacity.

Santaoja proposes to introduce microcrack groups.

Microcrack group: Collection of microcracks having same orientation & same size
Unit normal of surface of microcrack $\vec{n}^r$ determines orientation of microcracks within a microcrack group.

Two opposite orientations are possible for $\vec{n}^r$, but they lead to same response.

The number of microcrack groups is denoted by M.

According to preliminary computations, acceptable value for engineering $M = 15$.
This implies a huge reduction in computing time.

Thermodyn. variables related to microcrack groups are microcrack densities Q^r.
They are defined by

$$Q^r = \frac{m^r}{\rho_0 V^{\text{rve}}}, \qquad \text{where } r = 1 \ldots M. \tag{4}$$

Microcrack densities Q^r are defined as number of microcracks within a microcrack group m^r per mass occupying representative volume element V^{rve}.

Density of the material is denoted by ρ_0.

Santaoja: Damage-elastic strain ε^{de} is driving force for formation of microcracks.

The following evolution equation is thus introduced:

$$\dot{Q}^r = \frac{\mathring{Q}_{re}}{M} \left\langle [\vec{n}^r \cdot (\boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^d) \cdot \vec{n}^r] \langle \bar{a} \rangle - \varepsilon_{acr} \right\rangle Y^r(\tilde{\boldsymbol{\sigma}}) \qquad \text{where } r = 1 \ldots M. \qquad (5)$$

Maximum principal damage-elastic strain ε_I^{de} gives most damage-elastically strained place and orientation.

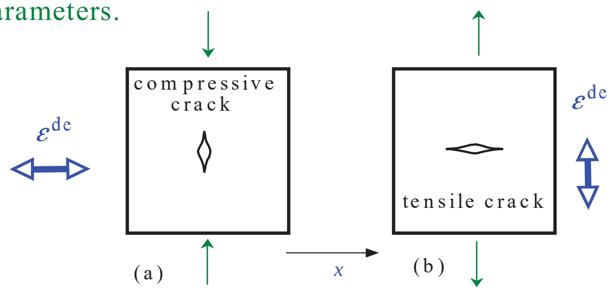
Thus, maximum principal damage-elastic strain ε_I^{de} displays:

Onset of microcrack formation

Orientation of the new microcrack.

This “rule” is not a cause, but it is an effect given by Eq. (5).

$\dot{Q}_{re}$ & ε_{acr} are material parameters.



Term $\vec{n}^r \cdot \boldsymbol{\varepsilon}^{de} \cdot \vec{n}^r$ is $\vec{n}^r$-directed component of damage-elastic strain tensor $\boldsymbol{\varepsilon}^{de}$.
It does not alone ensure microcrack formation; microcrack nuclei are also needed.
The variable $\bar{a}$ is the microcrack nucleus density.

Variables Y^r are conjugate forces for microcrack densities Q^r. (cf. σ & ε)

Therefore, they are part of the driving force for microcracking.

ε_{acr} is microcracking threshold and expresses forces that prevent microcracking.
Therefore, it is called microcracking resistance.

Based on Eq. (5), microcracking occurs when combination of driving force for microcracking $\vec{n}^r \cdot \boldsymbol{\varepsilon}^{de} \cdot \vec{n}^r$ & microcrack nuclei density $\bar{a}$ exceeds threshold ε_{acr}.

The expression for the microcrack nucleus density $\bar{a}$ reads [$\bar{a}$ is a pure number]

$$\bar{a} = \tfrac{2}{3} k^2 \, [J_{vM}(\boldsymbol{\varepsilon}^v)] \,. \qquad (6)$$

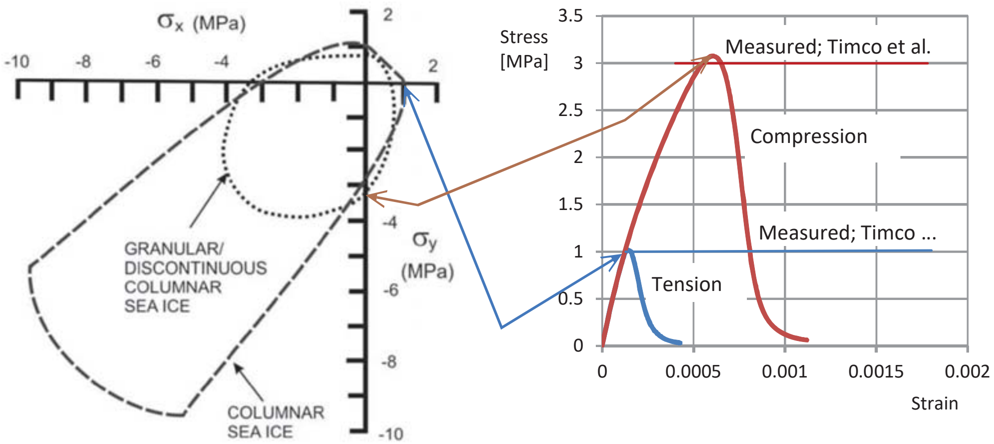
In Eq. (6) the quantities k^2, k^1, Q_{re} and k are the material parameters.
Eq. (6) describes how grain boundary sliding generates microcrack nuclei.

Numerical results

Creep-damage material model was applied to columnar-grained S2 sea ice.
Material model was coded as an Abaqus VUMAT subroutine.

Timco & Weeks (2010) $\dot{\varepsilon} = 2 \cdot 10^{-4}$ 1/s

Compressive curve is mirrored.
Computed curves at $\dot{\varepsilon} = 2 \cdot 10^{-4}$ 1/s.

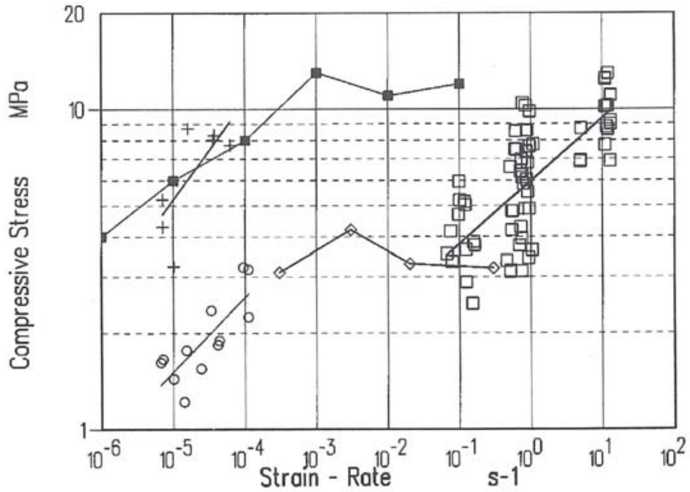


Peak values are:
$\sigma^{utens} = 1.02$ MPa and
$\sigma^{ucomp} = 3.07$ MPa.

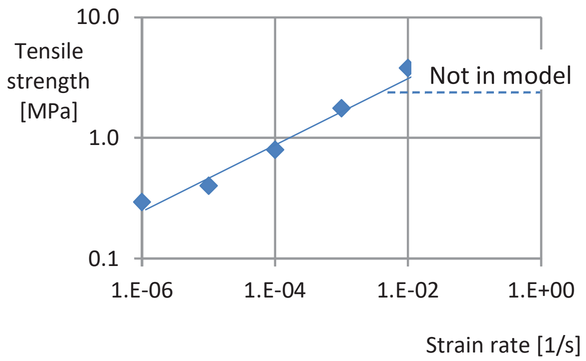
Values fit well.

Compressive strength vs Strain rate

Jones (1997, Fig. 3):



Numerical simulations with Santaoja model:



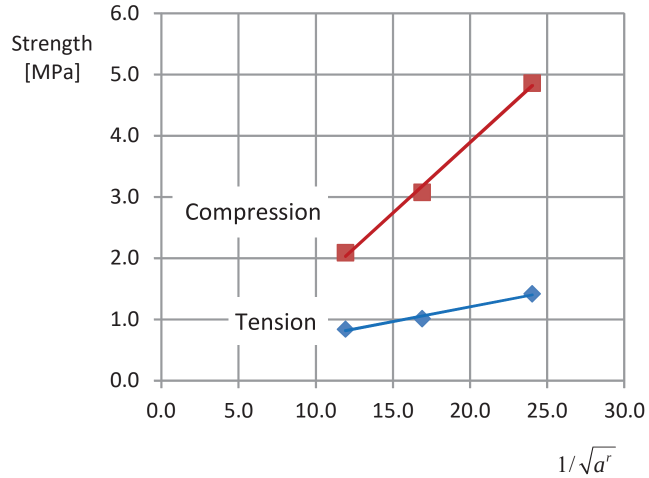
Both show linear relation in log-log axes.

Lee and Schulson have shown that strength of ice σ^{u} follows Hall-Petch relation. Hall-Petch relation reads

$$\sigma^{u} = a + b/\sqrt{a^{r}}\ , \tag{7}$$

where a^{r} is the grain size.
a and b are material parameters.

Numerical simulations with Santaoja model:



Thus, Santaoja model follows the Hall-Petch relation.

Thank you for your attention!