1. (**Just for fun**) (2p)

   A space capsule making a reentry into Earth's atmosphere suffers a communication blackout because a plasma is generated by the shock wave in front of the capsule. If the radio operates at a frequency of 300 MHz, what is the minimum plasma density during the blackout?

2. (**Semi-serious algebraic gymnastics**) (6p)

   In the lectures, when deriving the dispersion relation for the X-wave, we arrived at a matrix equation for E_x and E_y (slide 13).

   (a) Determine the matrix coefficients A, B, C and D from the coupled set of differential equations. You should get the answer:

   $$A = \omega^2\left(1 - \frac{\Omega_e^2}{\omega^2}\right) - \omega_p^2 \; ; \; B = i\frac{\omega_p^2\Omega_e}{\omega} \; ; \; C = -i\frac{\omega_p^2\Omega_e}{\omega} \; ; \; D = (\omega^2 - c^2k^2)\left(1 - \frac{\Omega_e^2}{\omega^2}\right) - \omega_p^2$$

   (b) Derive the dispersion relation from the condition $\det(M) = 0$, where M is the matrix. (Hint: Use the definition $\omega_h^2 = \omega_p^2 + \Omega_e^2$ and group the difference $\omega^2 - \omega_h^2$.)

3. (**Physics of mode conversion**) (4p)

   Show that at the resonance the extraordinary wave becomes purely electrostatic, i.e., it loses its electromagnetic component. (Hint: Express E_y as a function of ω and inspect the limit where $\omega \to \omega_h$.)

4. (**More algebraic gymnastics**) (6p)

   Calculate the cut-off frequencies ω_L and ω_R for the X-wave by setting $k \to 0$ in the dispersion relation (see slides 15 and 16).

5. (**Food for thought: Physics of space whistling**)

   What are Whistler waves, and how can you understand them from the physics of electromagnetic waves learned on this course? Return your short write-up in MyCourses before the next lecture.