

1 Helpful physics formulas

$$\text{Debye length} \quad \lambda_D^2 = \frac{\epsilon_0 T_e}{e^2 n_e} \quad (1)$$

$$\text{Plasma parameter} \quad \Lambda = \frac{4}{3} n \pi \lambda_D^3 \quad (2)$$

$$\text{Plasma frequency} \quad \omega_p^2 = \frac{e^2 n}{m \epsilon_0} \quad (3)$$

$$\text{Larmor frequency} \quad \Omega = \frac{qB}{m} \quad (4)$$

$$\text{Larmor radius} \quad r_L = \frac{mv_{\perp}}{qB} \quad (5)$$

$$\text{Magnetic moment} \quad \mu = \frac{mv_{\perp}^2}{2B} \quad (6)$$

$$\mathbf{E} \times \mathbf{B} \text{ drift} \quad \mathbf{v}_{\mathbf{E} \times \mathbf{B}} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \quad (7)$$

$$\text{Gradient drift} \quad \mathbf{v}_{\nabla B} = \pm \frac{1}{2} v_{\perp} r_L \frac{\mathbf{B} \times \nabla B}{B^2} \quad (8)$$

$$\text{Diamagnetic drift} \quad \mathbf{v}_D = -\frac{\nabla p \times \mathbf{B}}{qnB^2} \quad (9)$$

$$\text{Maxwell-Boltzmann} \quad f(\mathbf{v}) = \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{mv^2}{2T}\right) \quad (10)$$

$$h(E) = \frac{2}{T^{3/2}} \sqrt{\frac{E}{\pi}} e^{-E/T} \quad (11)$$

$$\text{Convective derivative} \quad \frac{dn(\mathbf{r}, t)}{dt} = \frac{\partial n(\mathbf{r}, t)}{\partial t} + \mathbf{v} \cdot \nabla n(\mathbf{r}, t) \quad (12)$$

$$\text{Collision frequency} \quad \nu = \frac{e^4 \ln \Lambda}{4\pi \epsilon_0^2 \sqrt{m}} \frac{n}{T^{3/2}} \quad (13)$$

$$\text{Gauss's law} \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (14)$$

$$\text{Gauss's law for magnetism} \quad \nabla \cdot \mathbf{B} = 0 \quad (15)$$

$$\text{Maxwell-Faraday equation} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (16)$$

$$\text{Ampère's circuital law} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (17)$$

$$\text{Sound speed in ideal gases} \quad v_s = \sqrt{\frac{E_{\text{therm}}}{m}} \quad (18)$$

$$\text{Sound speed in plasma} \quad v_s = \sqrt{\frac{E_{\text{therm}, e}}{M} + \frac{\gamma_i E_{\text{therm}, i}}{M}} \quad (19)$$

2 Constants

- vacuum permittivity $\epsilon_0 = 8.9 \cdot 10^{-12}$ C/Vm
- magnetic permeability $\mu_0 = 4\pi \times 10^{-7}$ Tm/A
- speed of light $c = 3 \times 10^8$ m/s

- elementary charge $e = 1.6 \cdot 10^{-19}$ C
- electron mass $m_e = 9.1 \cdot 10^{-31}$ kg
- proton mass $m_p = 1.7 \times 10^{-27}$ kg

3 Vector identities

1. $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \times \mathbf{A}$
2. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{C} \times \mathbf{B}) \times \mathbf{A} = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$
3. $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$
4. $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = \mathbf{C}(\mathbf{A} \times \mathbf{B} \cdot \mathbf{D}) - \mathbf{D}(\mathbf{A} \times \mathbf{B} \cdot \mathbf{C})$
5. $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$
6. $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$
7. $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$
8. $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$
9. $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$
10. $\nabla \cdot (\mathbf{A}\mathbf{B}) = (\nabla \cdot \mathbf{A})\mathbf{B} + (\mathbf{A} \cdot \nabla)\mathbf{B}$
11. $\nabla \cdot (f\mathbf{T}) = \nabla f \cdot \mathbf{T} + f\nabla \cdot \mathbf{T}$
12. $\nabla \cdot (\nabla \times \mathbf{A}) = 0$
13. $\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$
14. $\nabla(fg) = f\nabla g + g\nabla f$
15. $\nabla \cdot (\nabla f \times \nabla g) = 0$
16. $\nabla \cdot \nabla f = \nabla^2 f$
17. $\nabla \times \nabla f = 0$
18. $\int_V \nabla f \, dV = \int_S f \, d\mathbf{S}$
19. $\int_V \nabla \times \mathbf{A} \, dV = \oint_S d\mathbf{S} \times \mathbf{A}$
20. $\int_S d\mathbf{S} \cdot \nabla \times \mathbf{A} = \oint_C d\mathbf{l} \cdot \mathbf{A}$
21. $\oint_C d\mathbf{l} \times \mathbf{A} = \int_S (d\mathbf{S} \times \nabla) \times \mathbf{A}$

4 Curvilinear coordinates

Cylindrical Coordinates (r, φ, z)

1. $\nabla f = \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \varphi} \hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$
2. $\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (rA_r) + \frac{1}{r} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$
3. $\nabla \times \mathbf{A} = \left(\frac{1}{r} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right) \hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \hat{\boldsymbol{\varphi}} + \frac{1}{r} \left(\frac{\partial}{\partial r} (rA_\varphi) - \frac{\partial A_r}{\partial \varphi} \right) \hat{\mathbf{z}}$
4. $\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical Coordinates (r, θ, φ)

5. $\nabla f = \frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \hat{\boldsymbol{\varphi}}$
6. $\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\varphi}{\partial \varphi}$
7. $\nabla \times \mathbf{A} = \frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (\sin \theta A_\varphi) - \frac{\partial A_\theta}{\partial \varphi} \right) \hat{\mathbf{r}} + \frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r} (rA_\varphi) \right) \hat{\boldsymbol{\theta}} + \frac{1}{r} \left(\frac{\partial}{\partial r} (rA_\theta) - \frac{\partial A_r}{\partial \theta} \right) \hat{\boldsymbol{\varphi}}$
8. $\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \varphi^2}$

5 Gaussian Integrals

Definite integral relations of Gaussian integrals

1. $\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a} \right)^{1/2}$
2. $\int_{-\infty}^\infty e^{-ax^2} dx = \left(\frac{\pi}{a} \right)^{1/2}$
3. $\int_{-\infty}^\infty e^{-ax^2} e^{-2bx} dx = \left(\frac{\pi}{a} \right)^{1/2} e^{-\frac{b^2}{a}}$ for $a > 0$
4. $\int_{-\infty}^\infty x e^{-a(x-b)^2} dx = b \left(\frac{\pi}{a} \right)^{1/2}$
5. $\int_{-\infty}^\infty x^2 e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a^3} \right)^{1/2}$
6. $\int_0^\infty x^n e^{-ax^2} dx = \begin{cases} \frac{1}{2} \Gamma \left(\frac{n+1}{2} \right) / a^{(n+1)/2} & a > 0 \\ \frac{(2k-1)!!}{2^{k+1} a^k} \sqrt{\frac{\pi}{a}} & n = 2k, a > 0 \\ \frac{k!}{2a^{k+1}} & n = 2k+1, a > 0 \end{cases}$

6 Useful Fourier transforms

1. $\mathcal{F} \left[\frac{\partial f}{\partial t} \right] = -i\omega \tilde{f}$
2. $\mathcal{F} [\nabla f] = i\mathbf{k} \tilde{f}$