Theorems of the fourier transform	**Function**	**Transform**
Linearity	$ax(t) + by(t)$	$aX(f) + bY(f)$
Time delay or time shift	$x(t-a)$	$X(f)e^{-j2\pi fa}$
Scale change	$x(at)$	$\frac{1}{\lvert a\rvert}X(\frac{f}{a})$
Conjugation	$x^*(t)$	$X^*(-f)$
Duality	$X(t)$	$x(-f)$
Frequency shift	$x(t)e^{j2\pi at}$	$X(f-a)$
Linear modulation	$x(t)\cos(2\pi at + b)$	$\frac{e^{jb}X(f-a)+e^{-jb}X(f+a)}{2}$
Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
Integration	$\int_{-\infty}^{t} x(u)du$	$\frac{X(f)}{j2\pi f}$
Convolution	$x(t) \otimes y(t)$	$X(f)Y(f)$
Multiplication	$x(t)y(t)$	$X(f) \otimes Y(f)$
Multiplication by t^n	$t^n x(t)$	$-\frac{1}{j2\pi}\frac{d^n X(f)}{df^n}$

Fourier transforms	**Function**	**Transform**
Rectangular pulse	$\text{rect}(t/a)$	$a \cdot \text{sinc}(af)$
Triangular pulse	$\text{tria}(t/a)$	$a \cdot \text{sinc}^2(af)$
Gaussian pulse	$e^{-\pi(\frac{t}{a})^2}$	$a \cdot e^{-\pi(af)^2}$
One sided exponential pulse	$e^{-t/a}u(t)$	$\frac{a}{1+j2\pi fa}$
Two sided exponential pulse	$e^{-\lvert t\rvert/a}$	$\frac{2a}{1+(2\pi fa)^2}$
Sinc pulse	$\text{sinc}(at)$	$\frac{1}{a}\text{rect}(f/a)$
Constant	a	$a \cdot \delta(f)$
Phasor	$e^{j(2\pi at+b)}$	$e^{jb}\delta(f-a)$
Cosine wave	$\cos(2\pi at + b)$	$\frac{e^{jb}\delta(f-a)+e^{-jb}\delta(f+a)}{2}$
Delayed impulse	$\delta(t-a)$	$e^{-j2\pi fa}$
Step	$u(t)$	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$

$$T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$

$$e^{j\phi} = \cos(\phi) + j\sin(\phi)$$

$$\sin^2\phi + \cos^2\phi = 1$$

$$\cos(\phi) = \sin(\phi + \pi/2)$$

$$\sin(\phi) = \cos(\phi - \pi/2)$$

$$\cos^2(\phi) = \frac{1}{2}\Big[1 + \cos(2\phi)\Big]$$

$$\sin^2(\phi) = \frac{1}{2}\Big[1 - \sin(2\phi)\Big]$$

$$\cos^3(\phi) = \frac{1}{4}\Big[3\cos(\phi) + \cos(3\phi)\Big]$$

$$\sin^3(\phi) = \frac{1}{4}\Big[3\sin(\phi) - \sin(3\phi)\Big]$$

$$\sin(\alpha)\cos(\beta) = \frac{\sin(\alpha-\beta) + \sin(\alpha+\beta)}{2}$$

$$\sin(\alpha)\sin(\beta) = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2}$$

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta) + \cos(\alpha+\beta)}{2}$$

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{j2\pi k f_0 t} = \frac{\alpha_0}{2} + \sum_{k=1}^{\infty}\Big[\alpha_k\cos(2\pi k f_0 t) + \beta_k \sin(2\pi k f_0 t)\Big]$$

$$x_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-j2\pi k f_0 t}dt$$

$$\alpha_k = 2\cdot \mathrm{Re}\{x_k\}, \quad \text{when } x(t)\in\mathbb{R}$$

$$\beta_k = -2\cdot \mathrm{Im}\{x_k\}, \quad \text{when } x(t)\in\mathbb{R}$$

$$X(f) = \mathcal{F}\Big\{x(t)\Big\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f t}dt$$

$$x(t) = \mathcal{F}^{-1}\Big\{X(f)\Big\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi f t}df$$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

$$x_k = \frac{1}{N}\sum_{n=0}^{N-1} X_n e^{j2\pi kn/N}$$

$$f_0 = \frac{1}{N\cdot T_s} = \frac{f_s}{N}$$

$$s = \sigma + j\omega = \sigma + j2\pi f$$

$$X(s) = \mathcal{L}\Big\{x(t)\Big\} = \int_0^{\infty} x(t)\cdot e^{-st}dt$$

$$d_n = \frac{u_n}{u_1}$$

$$d_{\text{tot}} = \sqrt{\sum_{n=2}^{\infty} d_n^2}$$