Computer Vision

CS-E4850, 5 study credits

Lecturer: Juho Kannala

Lecture 10: Camera calibration & single view metrology

- **Camera calibration** is the process of determining the internal camera parameters, which define the mapping between incoming light rays and image pixels

- **Single view metrology** provides methods for measuring relative lengths from a single image by utilizing certain assumptions

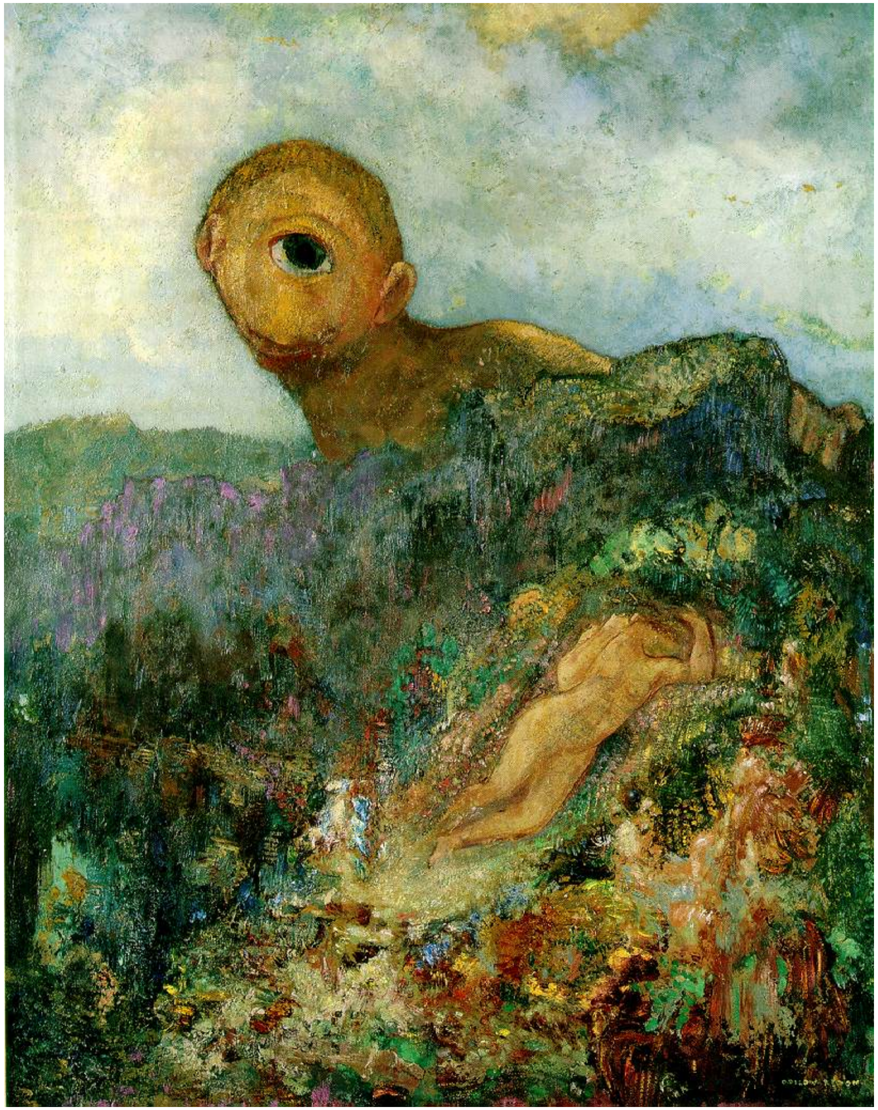
Acknowledgement: many slides from Svetlana Lazebnik, Derek Hoiem, Steve Seitz, and others (detailed credits on individual slides)

Reading

- Szeliski’s book, Sections 6.2 and 6.3 in 1st edition
- Hartley & Zisserman book, Chapters 6, 7, and 8

Source: S. Lazebnik

Calibrating a single camera

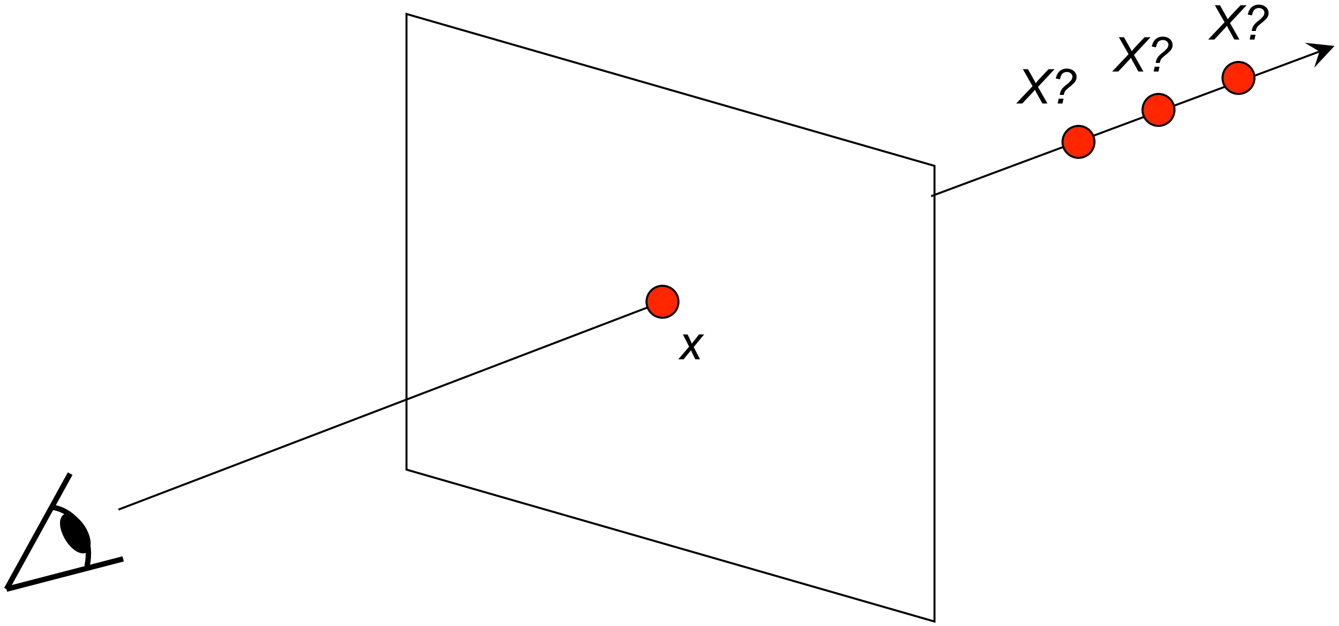


Odilon Redon, *Cyclops*, 1914

Source: S. Lazebnik

Our goal: Recovery of 3D structure

- Recovery of structure from one image is inherently ambiguous

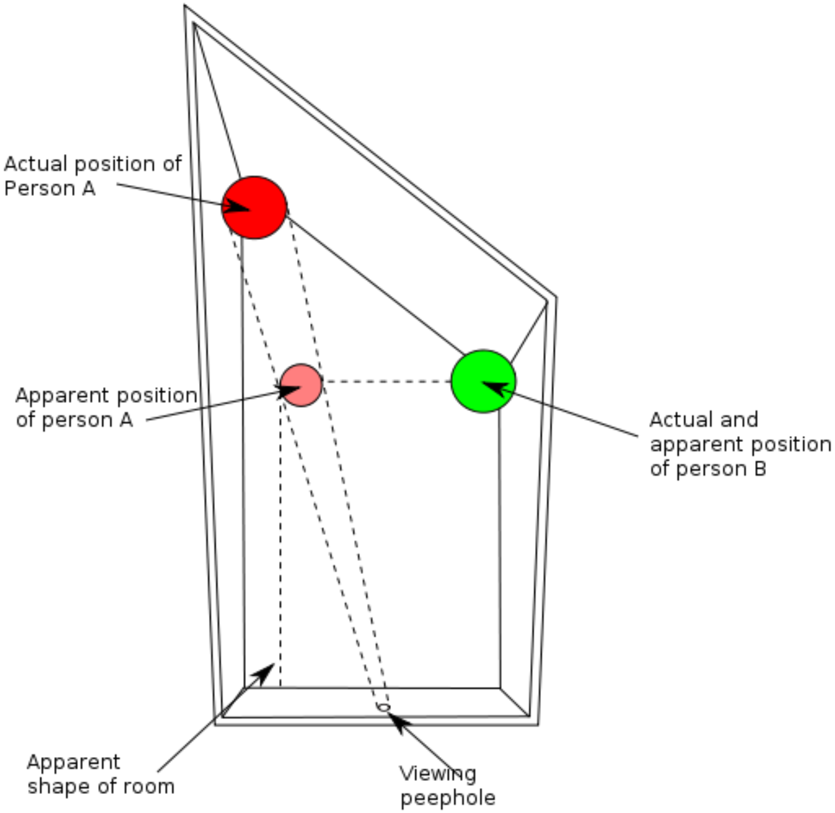


Source: S. Lazebnik

Single-view ambiguity

Source: S. Lazebnik

Single-view ambiguity



Ames room

Source: S. Lazebnik

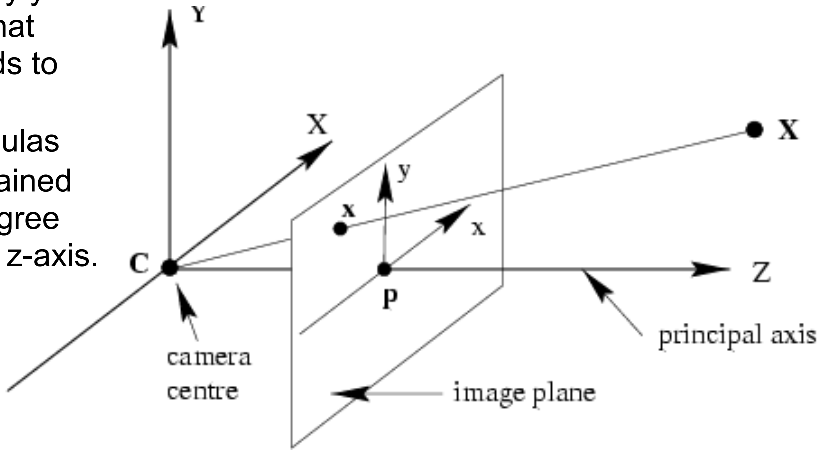
Our goal: Recovery of 3D structure

- We will need *multi-view geometry*

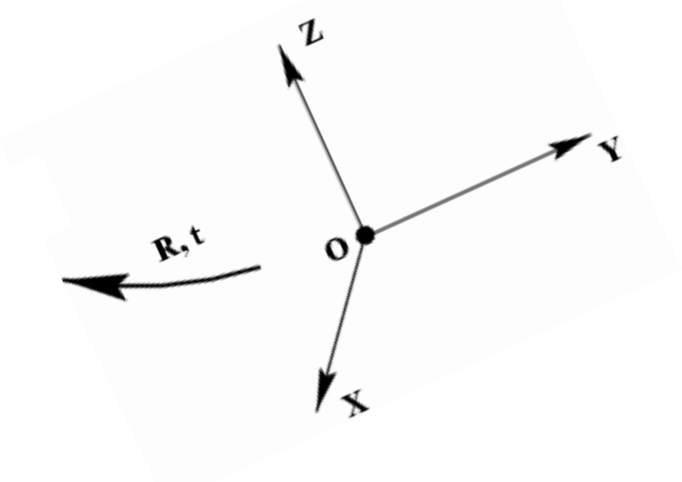
Source: S. Lazebnik

Review: Pinhole camera model

Note that usually y-axis points down. That convention leads to mathematically equivalent formulas and can be obtained here by 180 degree rotation around z-axis. -->



world coordinate system

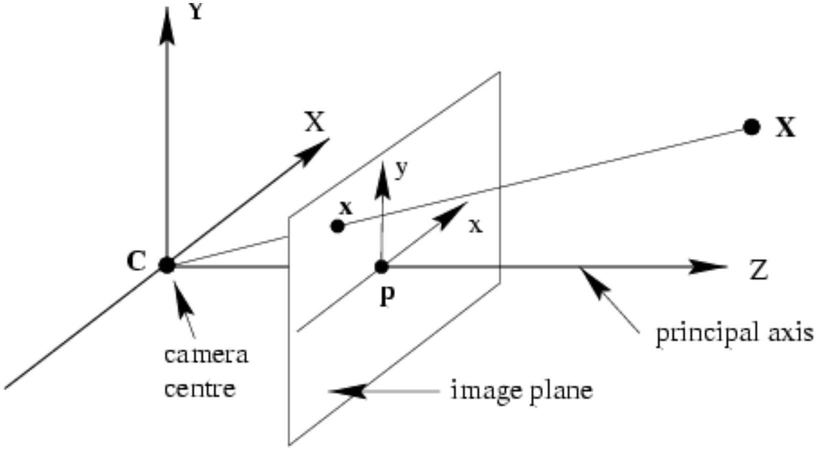


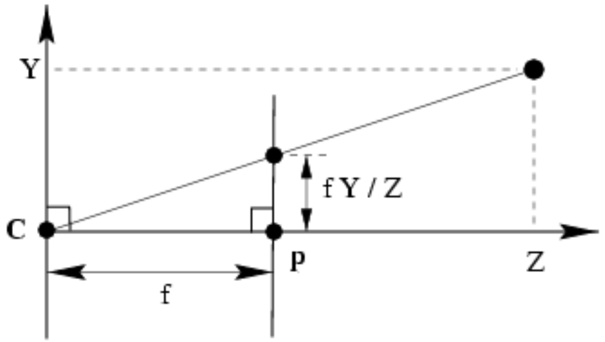
- **Normalized (camera) coordinate system:** camera center is at the origin, the *principal axis* is the z-axis, x and y axes of the image plane are parallel to x and y axes of the world

- Goal of camera calibration: go from *world* coordinate system to *image* coordinate system

Source: S. Lazebnik

Review: Pinhole camera model





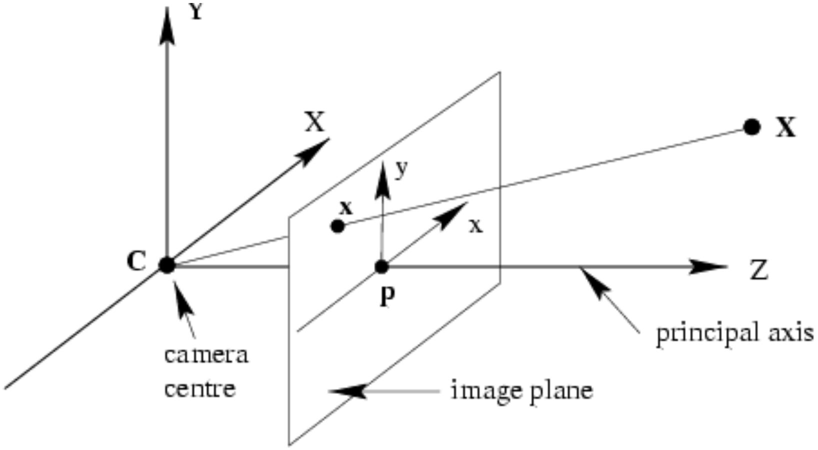
$$(X,Y,Z) \mapsto (fX/Z, fY/Z)$$

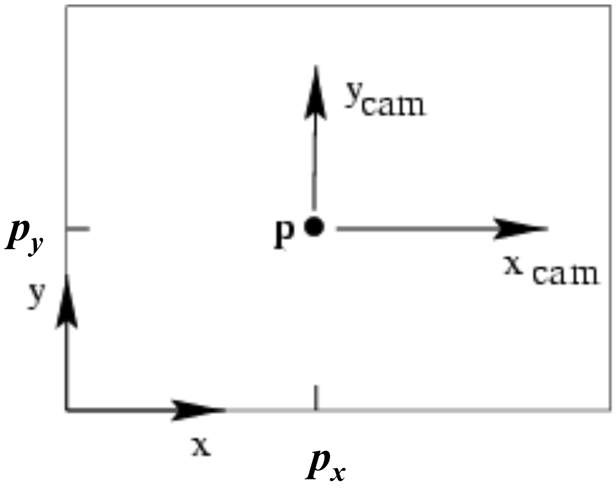
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

Source: S. Lazebnik

Principal point

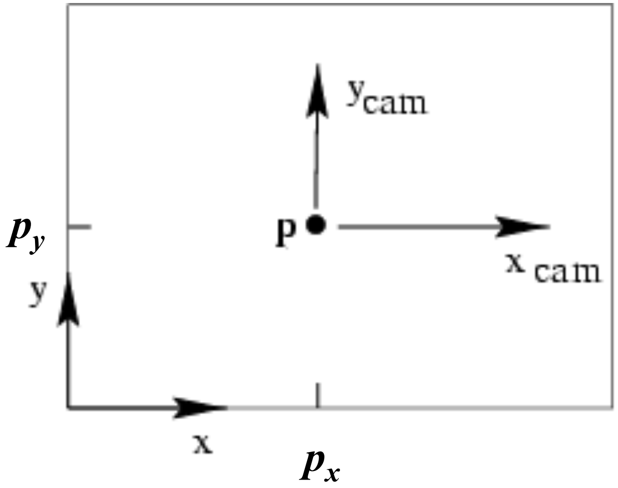




- **Principal point (p):** point where principal axis intersects the image plane
- Normalized coordinate system: origin of the image is at the principal point
- Image coordinate system: origin is in the corner

Source: S. Lazebnik

Principal point offset



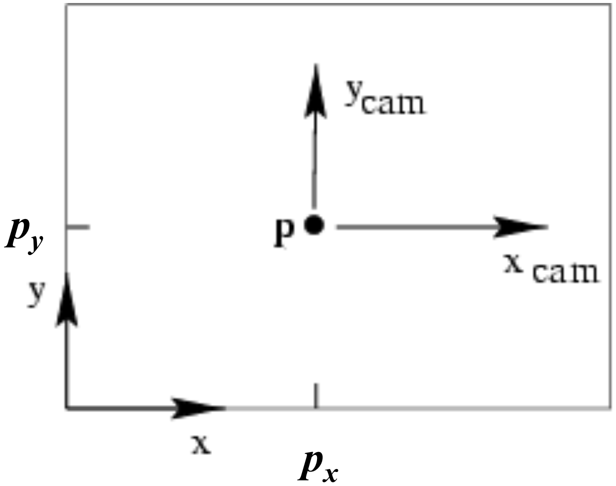
We want the principal point to map to (p_x, p_y) instead of (0,0)

$$(X, Y, Z) \mapsto (f\,X/Z + p_x, f\,Y/Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f\,X + Z\,p_x \\ f\,Y + Z\,p_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Source: S. Lazebnik

Principal point offset



principal point: (p_x, p_y)

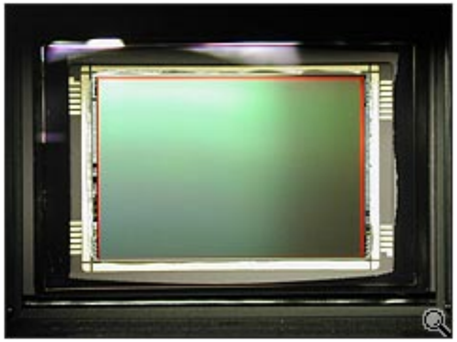
$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$ calibration matrix

$$\mathrm{P} = \mathrm{K}[\mathrm{I} \mid 0]$$

Source: S. Lazebnik

Pixel coordinates



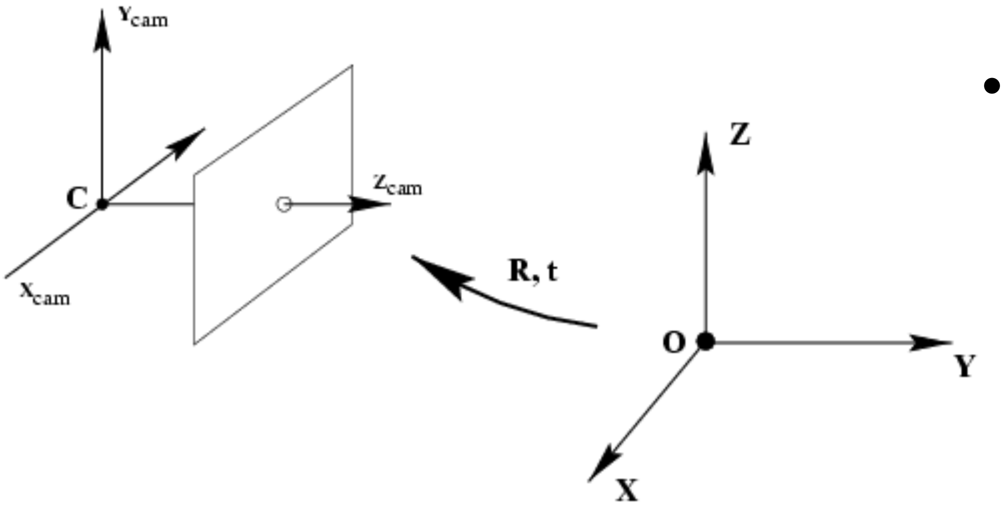
Pixel size: $\frac{1}{m_x} \times \frac{1}{m_y}$

m_x pixels per meter in horizontal direction,
m_y pixels per meter in vertical direction

$$K = \underset{\text{pixels/m}}{\begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix}} \underset{\text{m}}{\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}} = \underset{\text{pixels}}{\begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}}$$

Source: S. Lazebnik

Camera rotation and translation



- In general, the *camera* coordinate frame will be related to the *world* coordinate frame by a rotation and a translation

- Conversion from world to camera coordinate system (in non-homogeneous coordinates):

$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

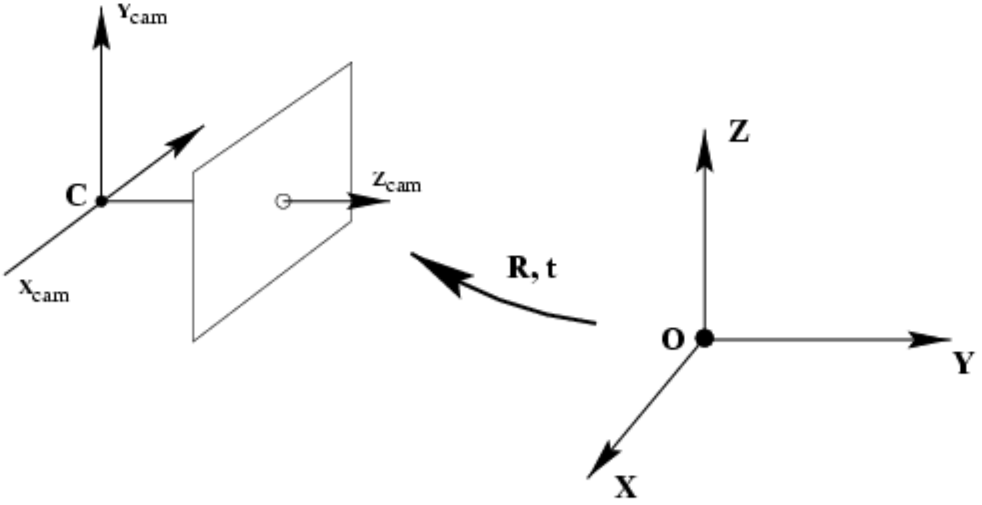
coords. of point in camera frame

coords. of a point in world frame

coords. of camera center in world frame

Source: S. Lazebnik

Camera rotation and translation



$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

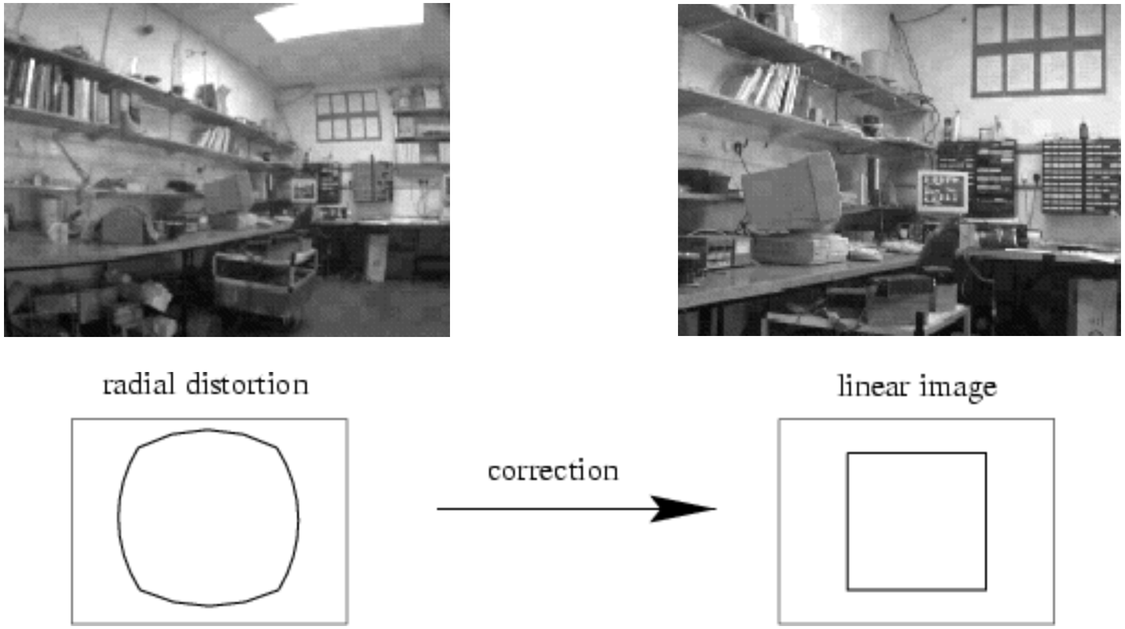
$$X_{cam} = \begin{pmatrix} \tilde{X}_{cam} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$x = K[I \mid 0]X_{cam} = K\left[R \mid -R\tilde{C}\right]X \qquad P = K[R \mid t], \qquad t = -R\tilde{C}$$

Camera parameters $\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*

$$\mathbf{K} = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix}\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & \beta_x \\ & \alpha_y & \beta_y \\ & & 1 \end{bmatrix}$$



Source: S. Lazebnik

Camera parameters

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}$$

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*
- Extrinsic parameters
  - Rotation and translation relative to world coordinate system

$$\mathbf{P} = \mathbf{K}\begin{bmatrix}\mathbf{R} & -\mathbf{R}\widetilde{\mathbf{C}}\end{bmatrix}$$

coords. of
camera center
in world frame

  - What is the projection of the camera center?

$$\mathbf{PC} = \mathbf{K}\begin{bmatrix}\mathbf{R} & -\mathbf{R}\widetilde{\mathbf{C}}\end{bmatrix}\begin{bmatrix}\widetilde{\mathbf{C}} \\ 1\end{bmatrix} = 0$$

The camera center is the *null space* of the projection matrix!

Source: S. Lazebnik

Camera calibration

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

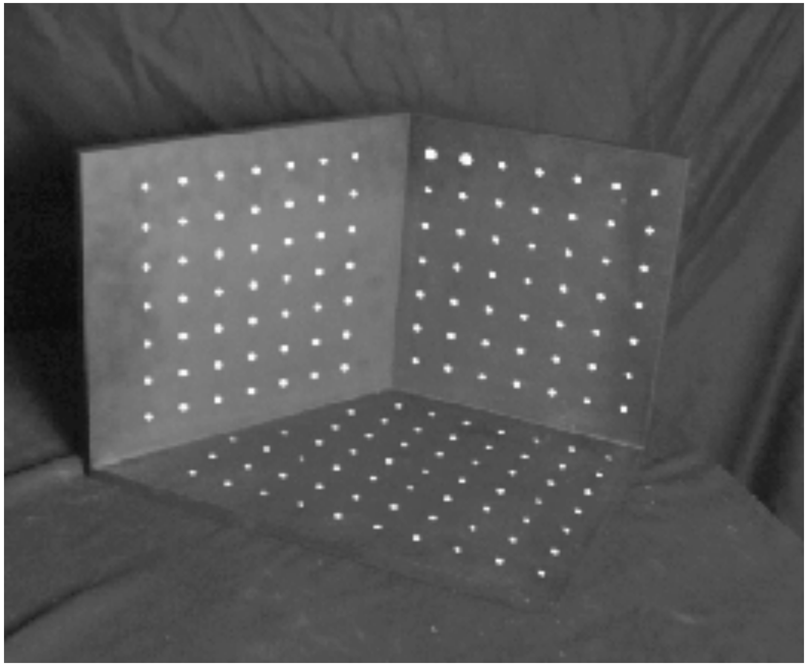
$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

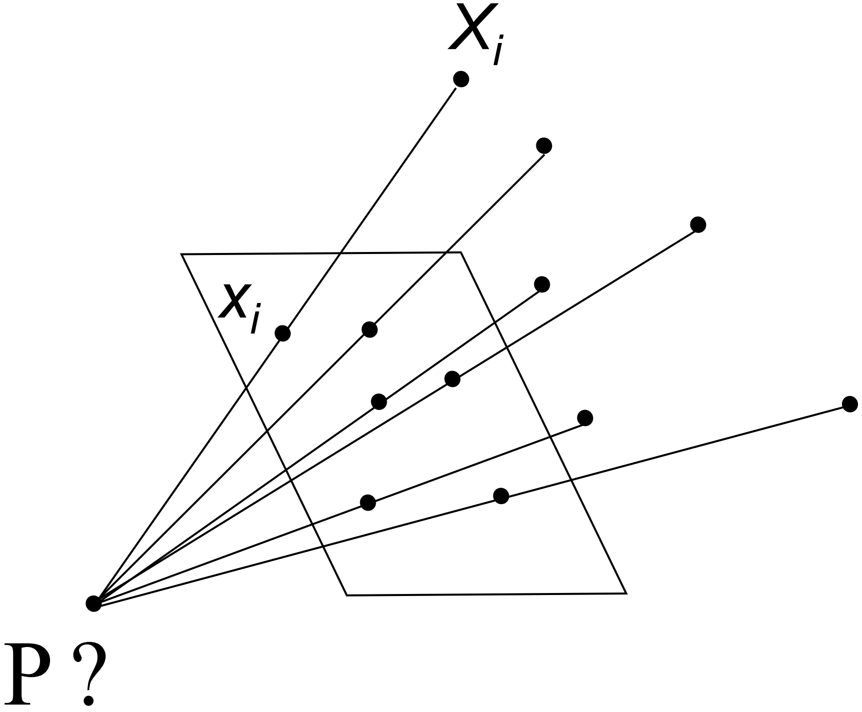
Source: D. Hoiem

Source: S. Lazebnik

Camera calibration

- Given *n* points with known 3D coordinates $\mathbf{X}_i$ and known image projections $\mathbf{x}_i$, estimate the camera parameters





Source: S. Lazebnik

Camera calibration: Linear method

$$\lambda \mathbf{x}_i = \mathbf{P}\mathbf{X}_i \qquad \mathbf{x}_i \times \mathbf{P}\mathbf{X}_i = 0 \qquad \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{P}_1^T \mathbf{X}_i \\ \mathbf{P}_2^T \mathbf{X}_i \\ \mathbf{P}_3^T \mathbf{X}_i \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & -\mathbf{X}_i^T & y_i \mathbf{X}_i^T \\ \mathbf{X}_i^T & 0 & -x_i \mathbf{X}_i^T \\ -y_i \mathbf{X}_i^T & x_i \mathbf{X}_i^T & 0 \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0$$

Two linearly independent equations

Source: S. Lazebnik

Camera calibration: Linear method

$$\begin{bmatrix} 0^T & \mathbf{X}_1^T & -y_1\mathbf{X}_1^T \\ \mathbf{X}_1^T & 0^T & -x_1\mathbf{X}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{X}_n^T & -y_n\mathbf{X}_n^T \\ \mathbf{X}_n^T & 0^T & -x_n\mathbf{X}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0 \qquad \mathbf{Ap = 0}$$

- P has 11 degrees of freedom
- One 2D/3D correspondence gives us two linearly independent equations
  - 6 correspondences needed for a minimal solution
- Homogeneous least squares: find p $(\|p\|=1)$ minimizing $\|Ap\|^2$
  - Solution given by eigenvector of $A^T A$ with smallest eigenvalue

Source: S. Lazebnik

Camera calibration: Linear method

$$\begin{bmatrix} 0^T & \mathbf{X}_1^T & -y_1\mathbf{X}_1^T \\ \mathbf{X}_1^T & 0^T & -x_1\mathbf{X}_1^T \\ \cdots & \cdots & \cdots \\ 0^T & \mathbf{X}_n^T & -y_n\mathbf{X}_n^T \\ \mathbf{X}_n^T & 0^T & -x_n\mathbf{X}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0 \qquad \mathbf{Ap} = \mathbf{0}$$

- Note: for coplanar points that satisfy $\mathbf{\Pi}^T\mathbf{X}=0$, we will get degenerate solutions $(\mathbf{\Pi},\mathbf{0},\mathbf{0})$, $(\mathbf{0},\mathbf{\Pi},\mathbf{0})$, or $(\mathbf{0},\mathbf{0},\mathbf{\Pi})$

Source: S. Lazebnik

Camera calibration: Linear method

- The linear method only estimates the entries of the projection matrix:

$$\begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

- What we ultimately want is a decomposition of this matrix into the intrinsic and extrinsic parameters:

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

- This can be achieved via the RQ matrix decomposition (see Sec. 6.2.4 of H&Z book)

Camera calibration: Linear method

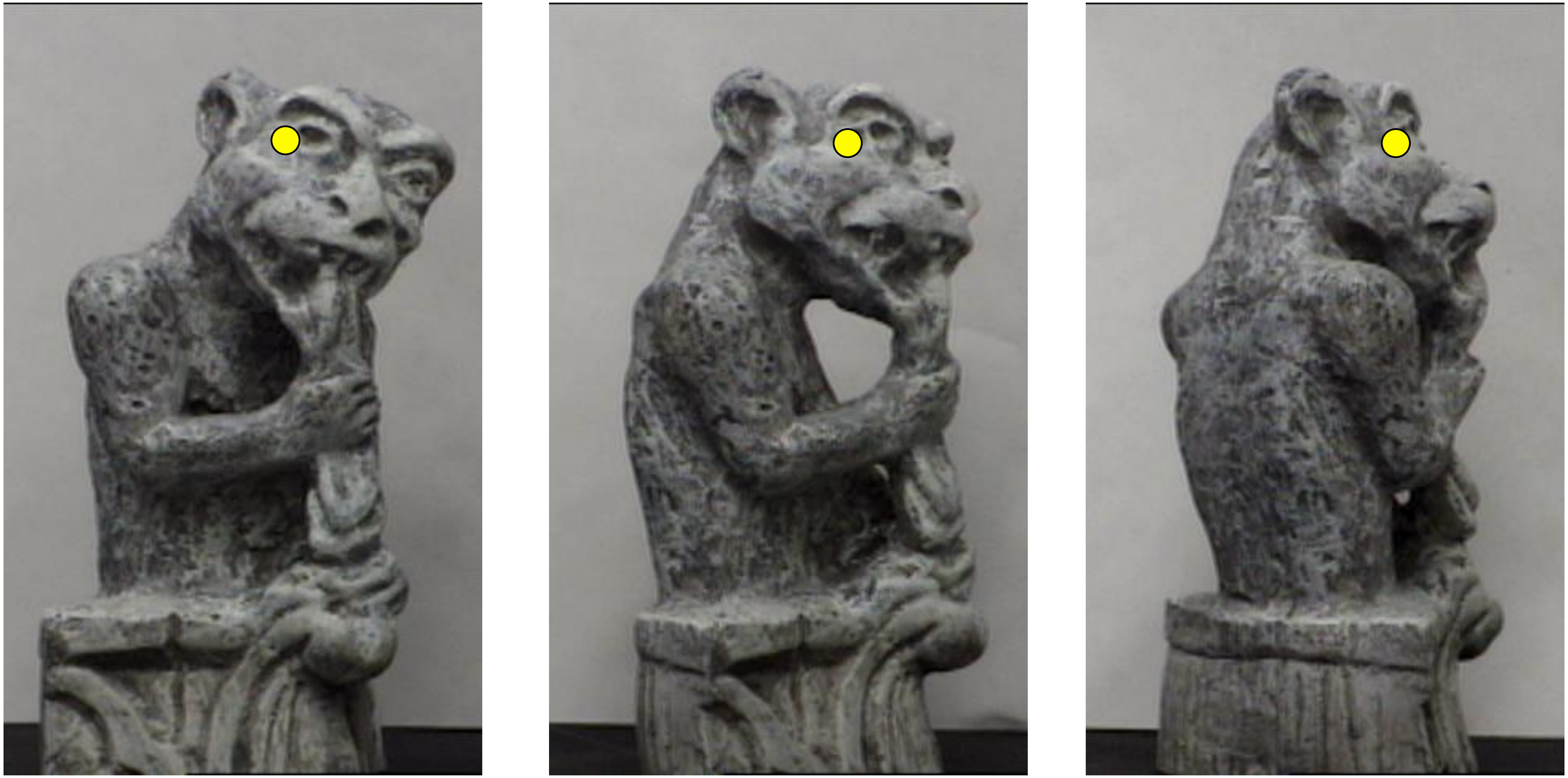
- Advantages: easy to formulate and solve
- Disadvantages
  - Doesn’t directly tell you camera parameters
  - Doesn’t model radial distortion
  - Can’t impose constraints, such as known focal length and orthogonality

- Non-linear methods are preferred
  - Define error as sum of squared distances between measured 2D points and estimated projections of 3D points
  - Minimize error using Newton’s method or other non-linear optimization
  - The iterative optimization by non-linear methods can be initialized with the solution provided by the linear method

Source: D. Hoiem

A taste of multi-view geometry: Triangulation

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point

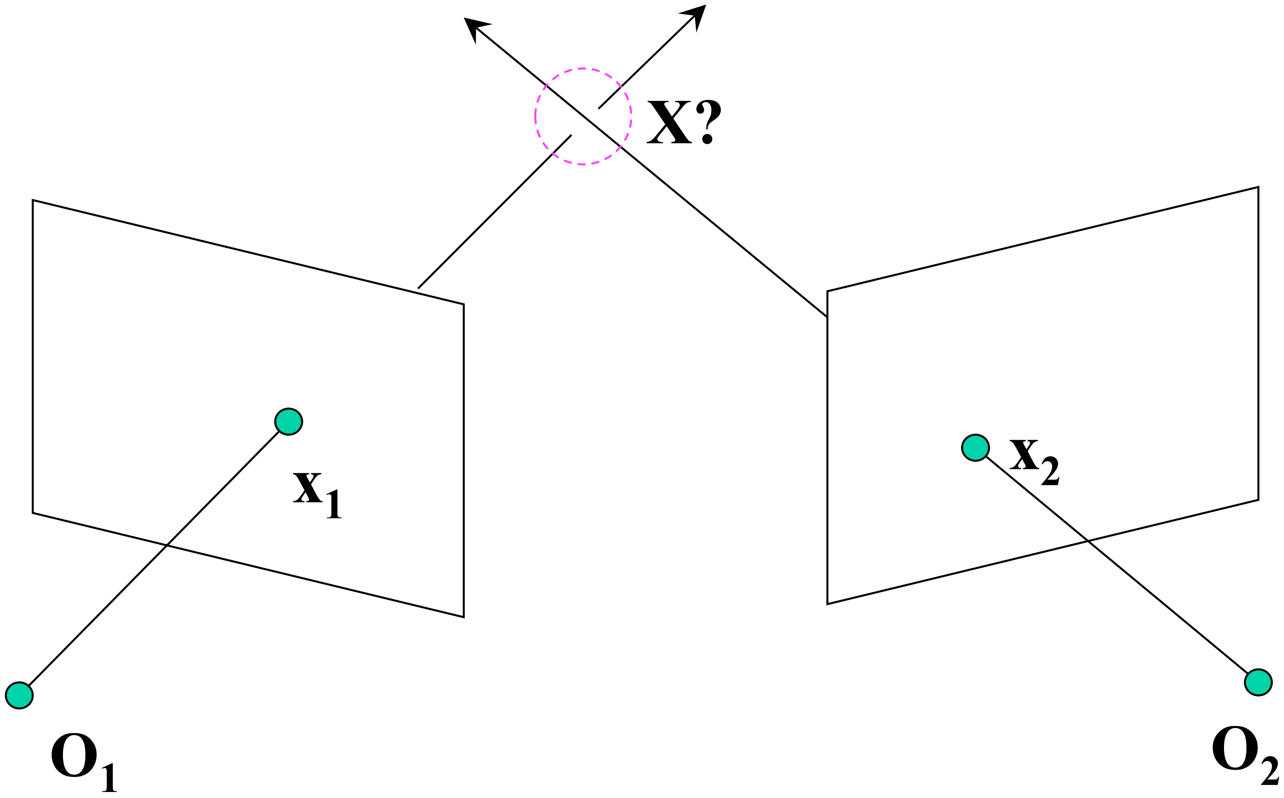


Source: S. Lazebnik

Source: S. Lazebnik

Triangulation

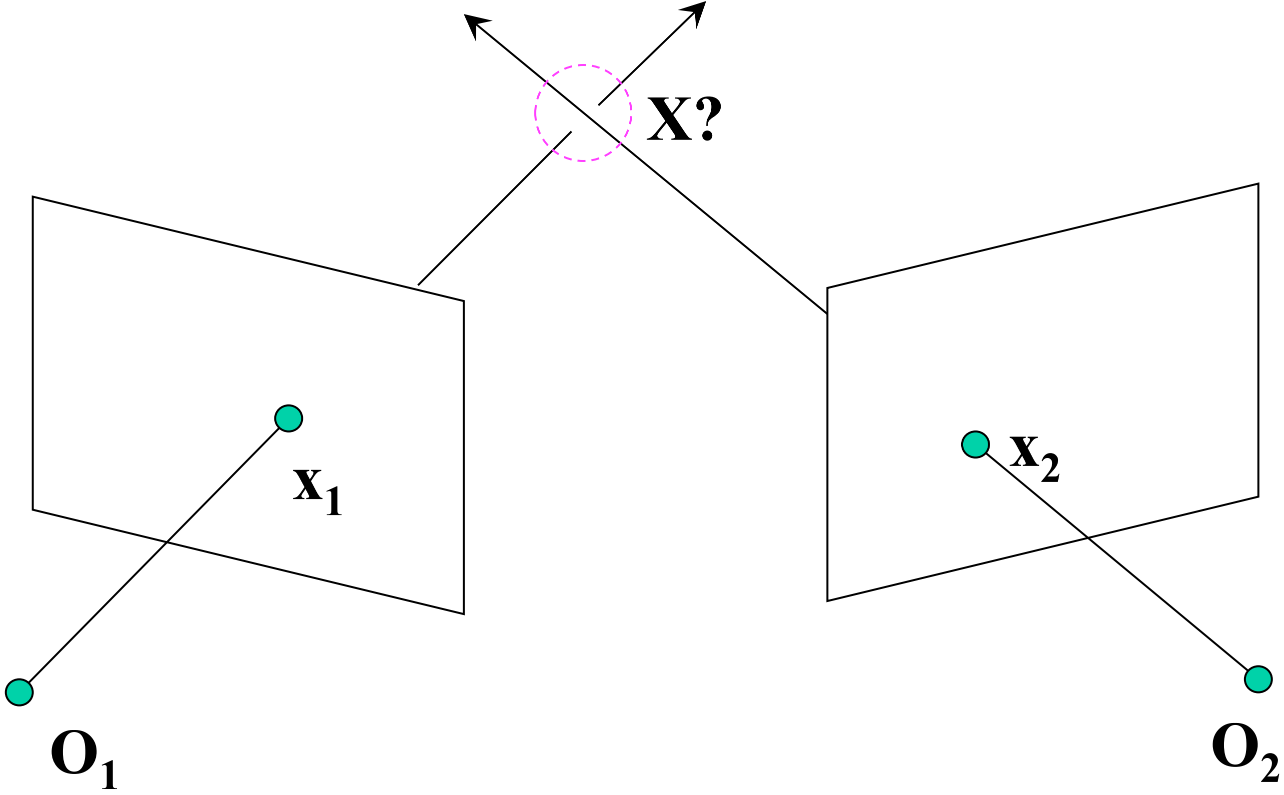
- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



Source: S. Lazebnik

Triangulation

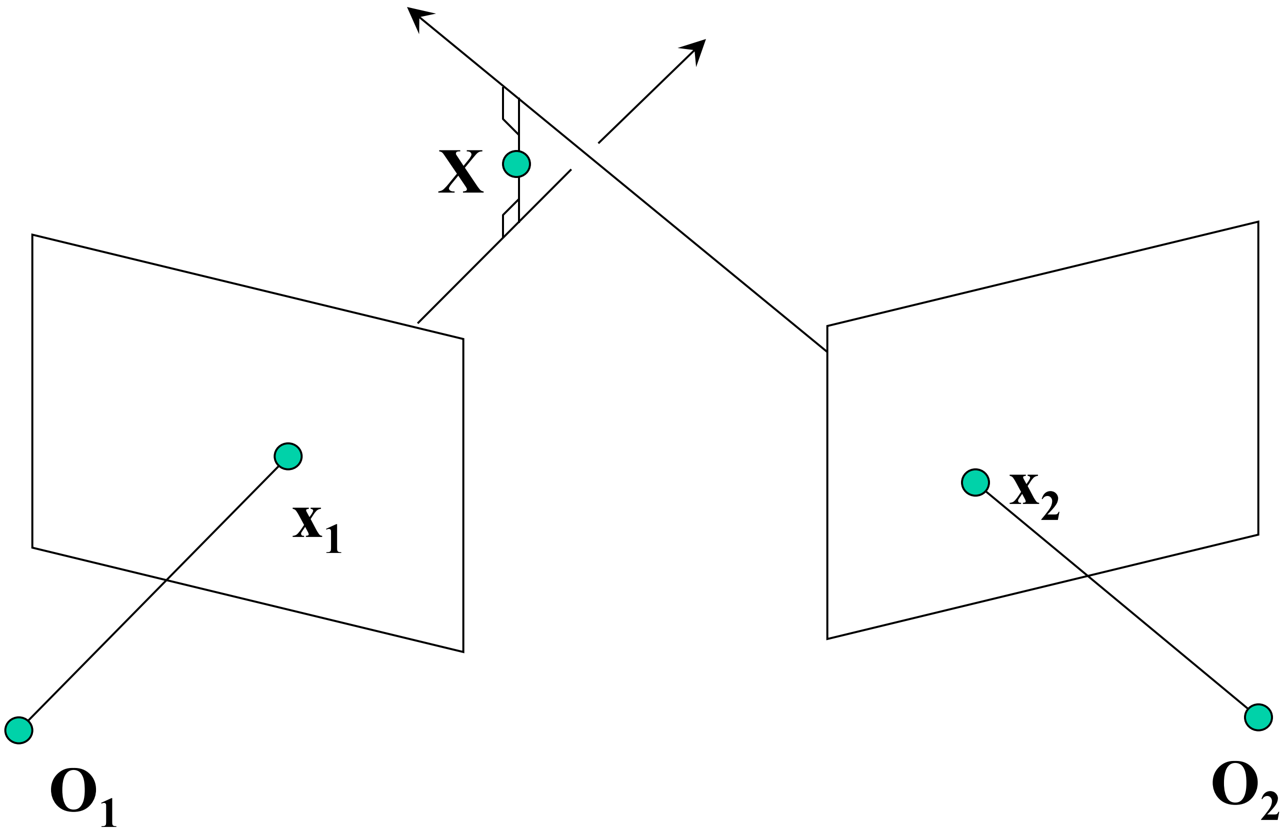
- We want to intersect the two visual rays corresponding to $\mathbf{x}_1$ and $\mathbf{x}_2$, but because of noise and numerical errors, they don't meet exactly



Source: S. Lazebnik

Triangulation: Geometric approach

- Find shortest segment connecting the two viewing rays and let **X** be the midpoint of that segment

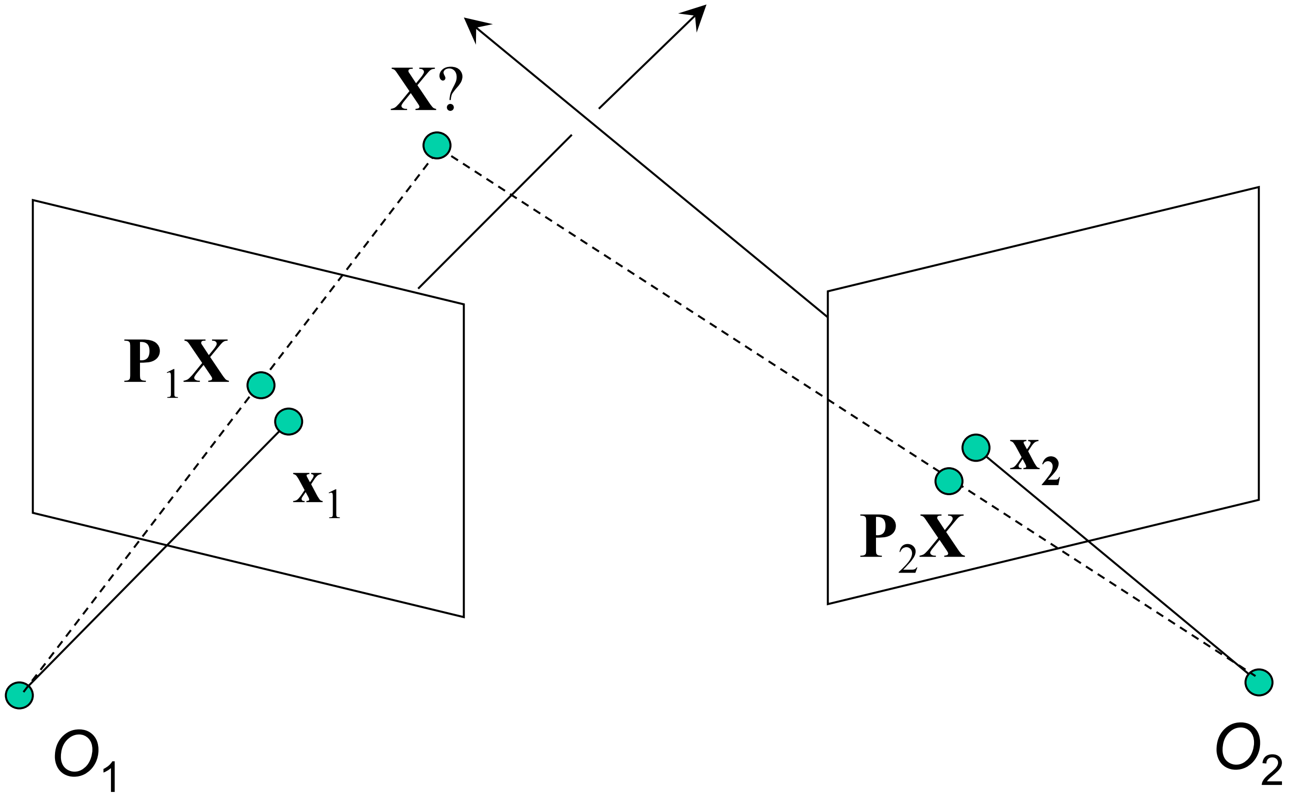


Source: S. Lazebnik

Triangulation: Nonlinear approach

Find X that minimizes

$$d^2(\mathbf{x}_1, \mathbf{P}_1\mathbf{X}) + d^2(\mathbf{x}_2, \mathbf{P}_2\mathbf{X})$$



Source: S. Lazebnik

Triangulation: Linear approach

$$\lambda_1 \mathbf{x_1} = \mathbf{P_1 X} \qquad \mathbf{x_1} \times \mathbf{P_1 X} = \mathbf{0} \qquad [\mathbf{x_{1\times}}]\mathbf{P_1 X} = \mathbf{0}$$

$$\lambda_2 \mathbf{x_2} = \mathbf{P_2 X} \qquad \mathbf{x_2} \times \mathbf{P_2 X} = \mathbf{0} \qquad [\mathbf{x_{2\times}}]\mathbf{P_2 X} = \mathbf{0}$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Source: S. Lazebnik

Triangulation: Linear approach

$$\lambda_1 \mathbf{x_1} = \mathbf{P_1 X} \qquad \mathbf{x_1} \times \mathbf{P_1 X} = \mathbf{0} \qquad [\mathbf{x_{1\times}}]\mathbf{P_1 X} = \mathbf{0}$$

$$\lambda_2 \mathbf{x_2} = \mathbf{P_2 X} \qquad \mathbf{x_2} \times \mathbf{P_2 X} = \mathbf{0} \qquad [\mathbf{x_{2\times}}]\mathbf{P_2 X} = \mathbf{0}$$

Two independent equations each in terms of the 4 elements of **X** (but only 3 degrees of freedom since scale is ambiguous and can be fixed)

This is again a linear least-squares problem which can be solved as shown previously

Source: S. Lazebnik

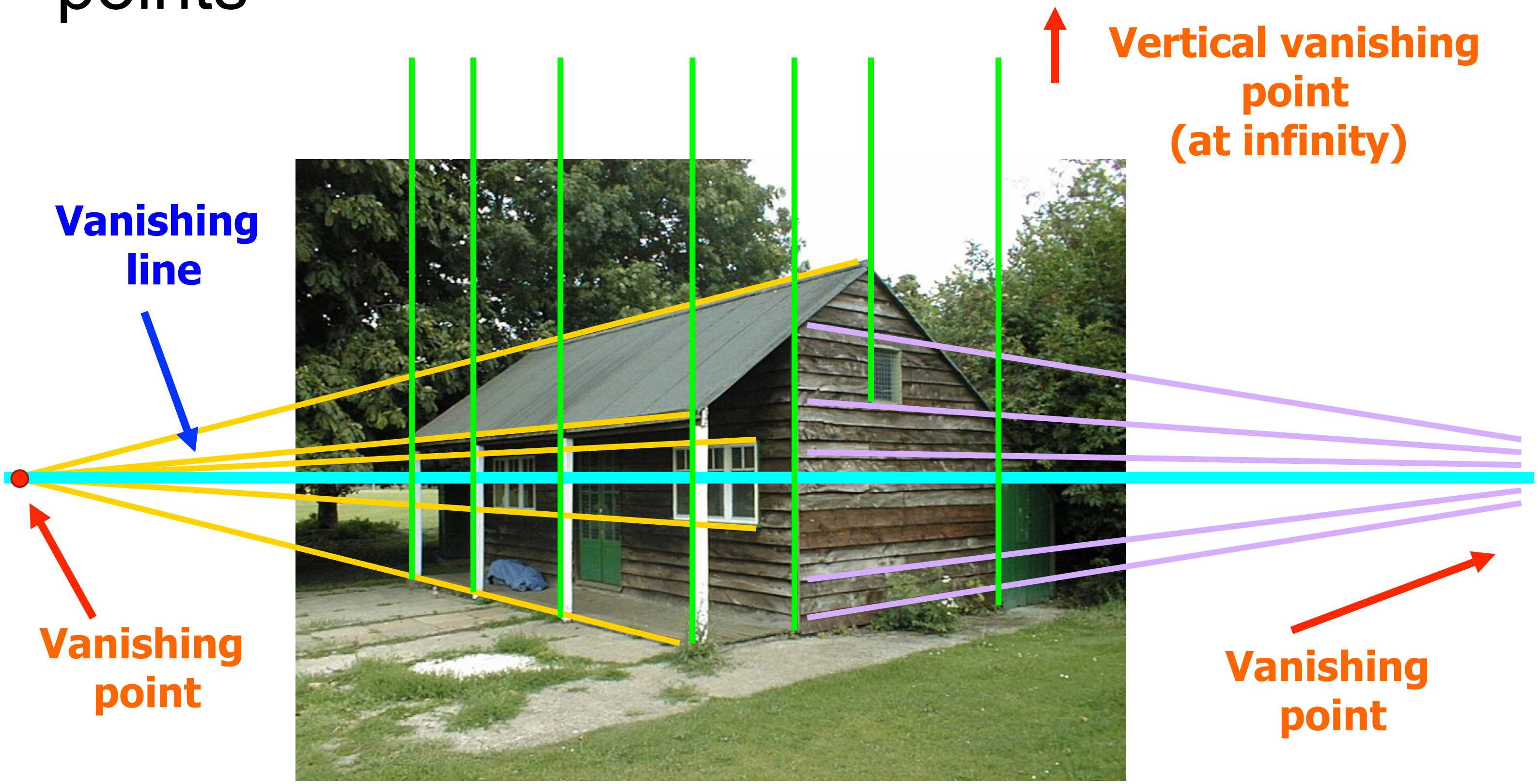
Single-view metrology

Magritte, *Personal Values*, 1952

Many slides from S. Seitz, D. Hoiem

Camera calibration revisited

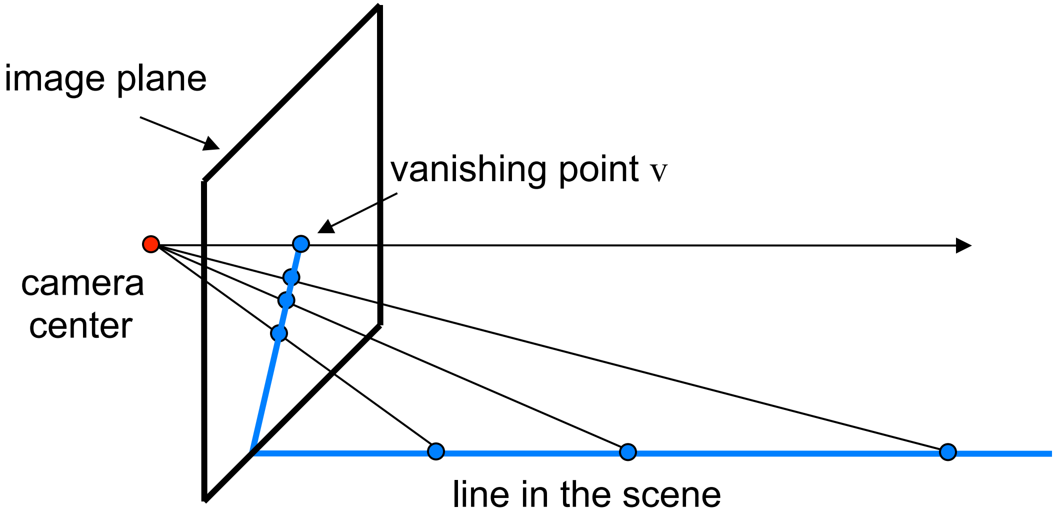
- What if world coordinates of reference 3D points are not known?
- We can use scene features such as vanishing points



Slide from Efros, Photo from Criminisi

Source: S. Lazebnik

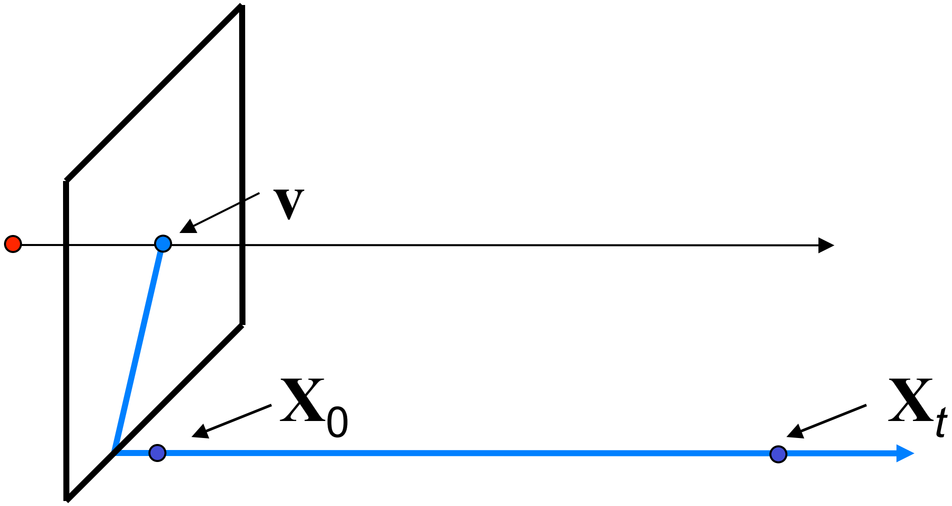
Recall: Vanishing points



- All lines having the same direction share the same vanishing point

Source: S. Lazebnik

Computing vanishing points



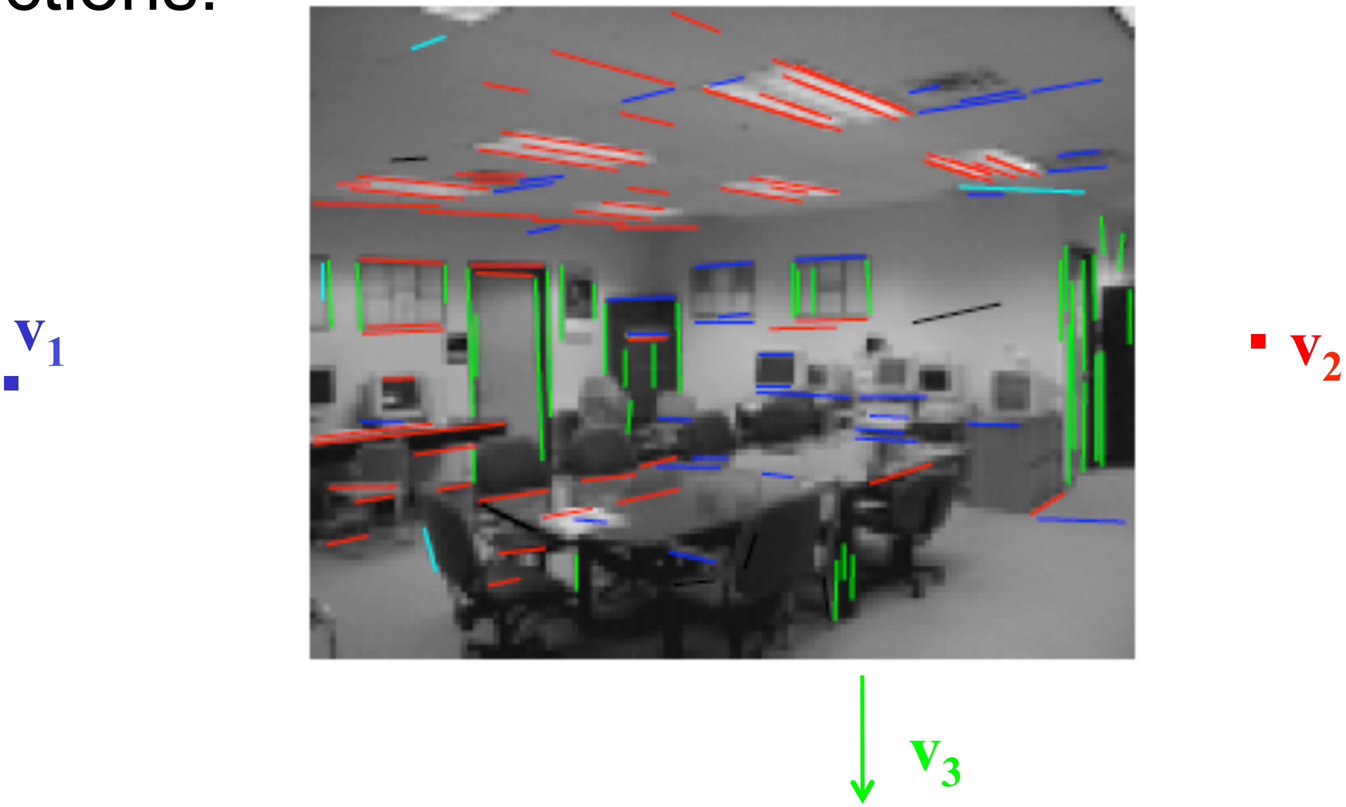
$$\mathbf{X}_t = \begin{bmatrix} x_0 + td_1 \\ y_0 + td_2 \\ z_0 + td_3 \\ 1 \end{bmatrix} = \begin{bmatrix} x_0 / t + d_1 \\ y_0 / t + d_2 \\ z_0 / t + d_3 \\ 1/t \end{bmatrix} \qquad \mathbf{X}_\infty = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \end{bmatrix}$$

- $\mathbf{X}_\infty$ is a *point at infinity,* $\mathbf{v}$ is its projection: $\mathbf{v} = \mathbf{PX}_\infty$
- The vanishing point depends only on *line direction*
- All lines having direction $\mathbf{d}$ intersect at $\mathbf{X}_\infty$

Source: S. Lazebnik

Calibration from vanishing points

- Consider a scene with three orthogonal vanishing directions:

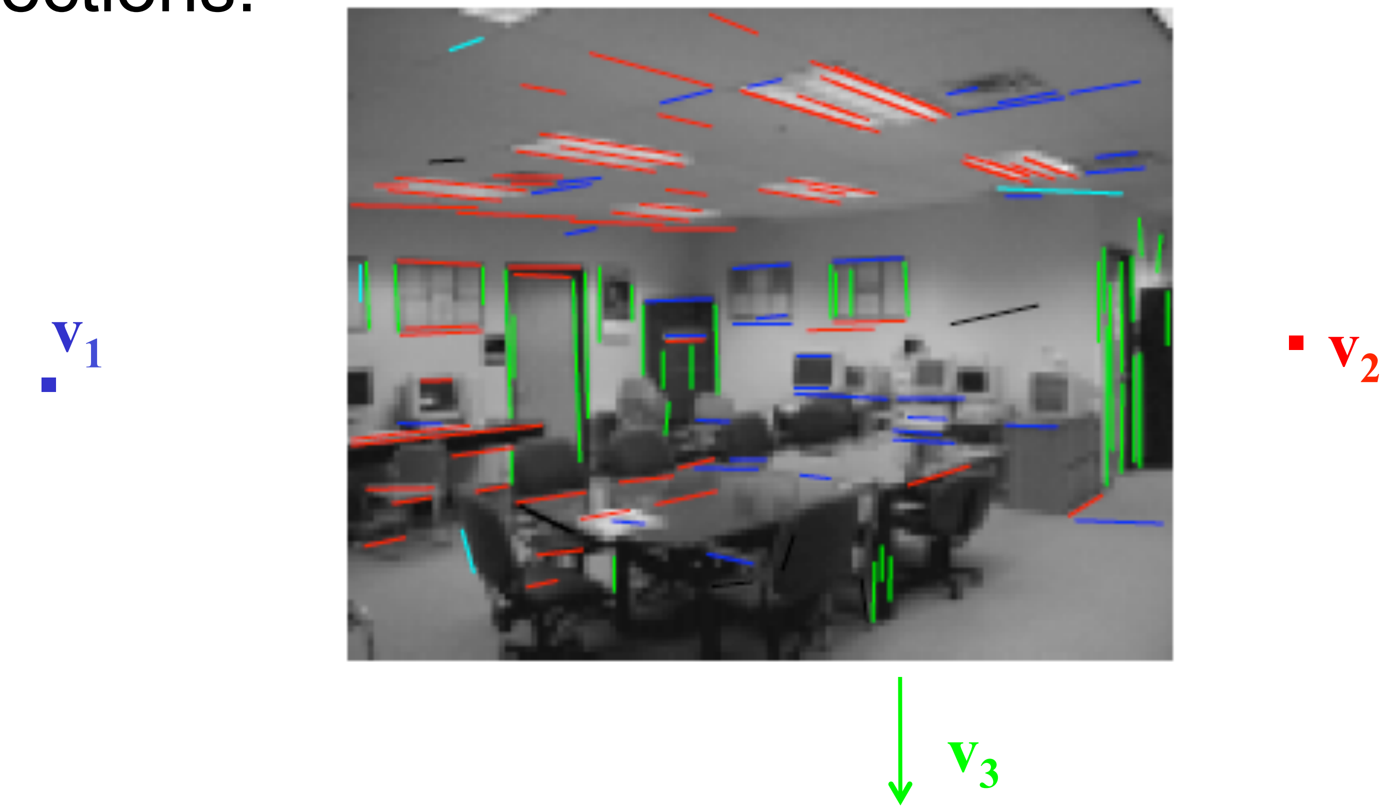


- Note: $\mathbf{v}_1$, $\mathbf{v}_2$ are *finite* vanishing points and $\mathbf{v}_3$ is an *infinite* vanishing point

Source: S. Lazebnik

Calibration from vanishing points

- Consider a scene with three orthogonal vanishing directions:



- We can align the world coordinate system with these directions

Source: S. Lazebnik

Calibration from vanishing points

$$\mathbf{P} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 & \mathbf{p}_4 \end{bmatrix}$$

- $\mathbf{p}_1 = \mathbf{P}(1,0,0,0)^T$ – the vanishing point in the x direction
- Similarly, $\mathbf{p}_2$ and $\mathbf{p}_3$ are the vanishing points in the y and z directions
- $\mathbf{p}_4 = \mathbf{P}(0,0,0,1)^T$ – projection of the origin of the world coordinate system
- Problem: we can only know the four columns up to independent scale factors, additional constraints needed to solve for them

Source: S. Lazebnik

Calibration from vanishing points

- Let us align the world coordinate system with three orthogonal vanishing directions in the scene:

$$\mathbf{e_1} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \quad \mathbf{e_2} = \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \mathbf{e_3} = \begin{bmatrix}0\\0\\1\end{bmatrix} \qquad \lambda_i \mathbf{v}_i = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]\begin{bmatrix}\mathbf{e}_i\\0\end{bmatrix} = \mathbf{KRe}_i$$

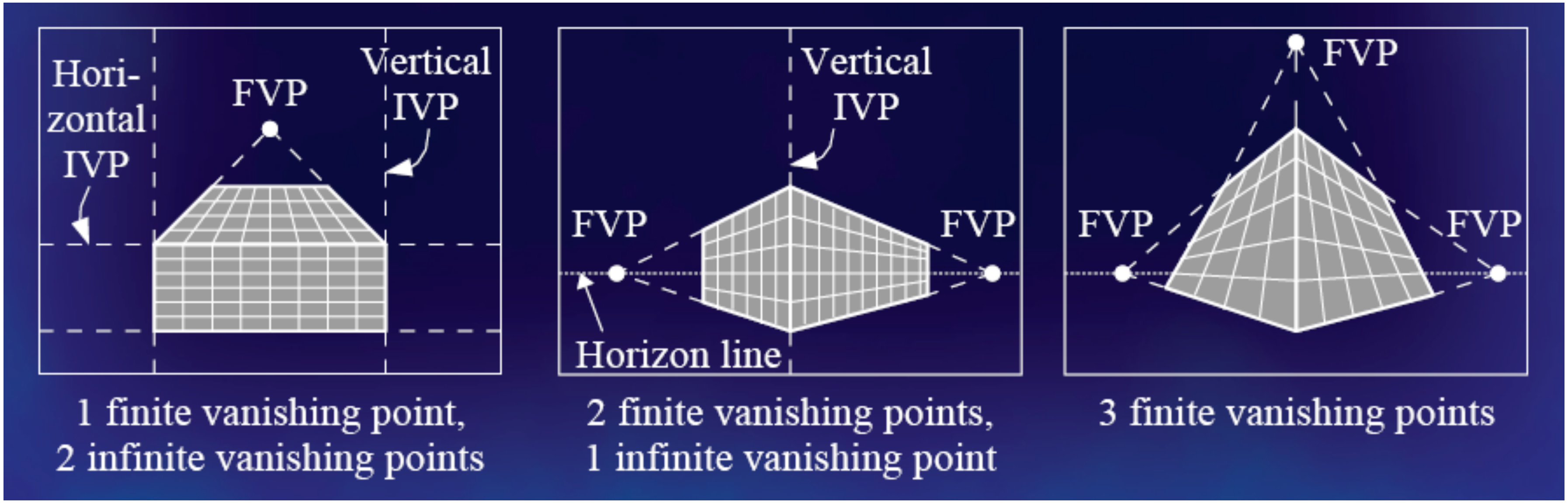
$$\mathbf{e}_i = \lambda_i \mathbf{R}^T \mathbf{K}^{-1} \mathbf{v}_i, \quad \mathbf{e}_i^T \mathbf{e}_j = 0$$

$$\mathbf{v}_i^T \mathbf{K}^{-T} \mathbf{R} \mathbf{R}^T \mathbf{K}^{-1} \mathbf{v}_j = \mathbf{v}_i^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{v}_j = 0$$

- Each pair of vanishing points gives us a constraint on the focal length and principal point

Source: S. Lazebnik

Calibration from vanishing points

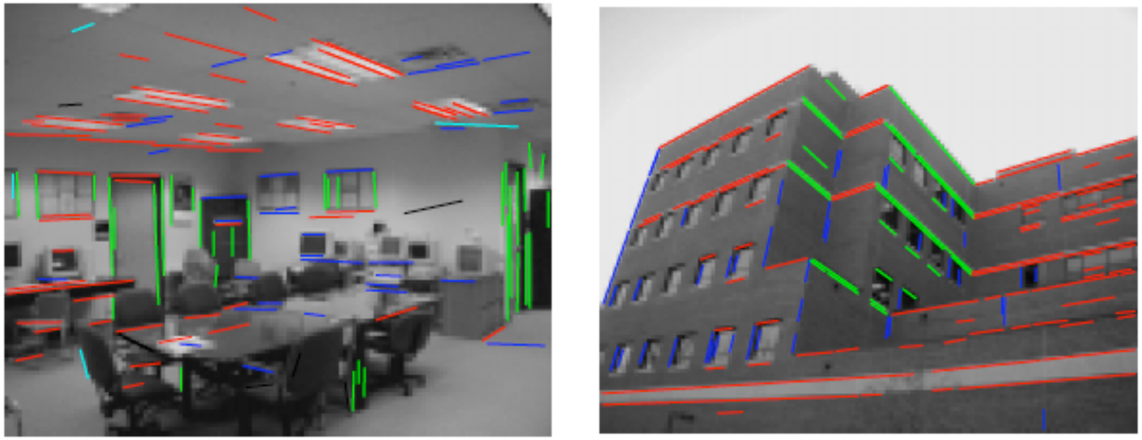


1 finite vanishing point,
2 infinite vanishing points

2 finite vanishing points,
1 infinite vanishing point

3 finite vanishing points

Cannot recover focal length, principal point is the third vanishing point



Can solve for focal length, principal point

Source: S. Lazebnik

Rotation from vanishing points

$$\lambda_i \mathbf{v}_i = \mathbf{K}\left[\mathbf{R} \mid \mathbf{t}\right]\begin{bmatrix}\mathbf{e}_i \\ 0\end{bmatrix} = \mathbf{K}\mathbf{R}\mathbf{e}_i$$

$$\lambda_1 \mathbf{K}^{-1}\mathbf{v}_1 = \mathbf{R}\mathbf{e}_1 = [\ \mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{r}_3\ ]\begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix} = \mathbf{r}_1$$

Thus, $\lambda_i \mathbf{K}^{-1}\mathbf{v}_i = \mathbf{r}_i$.

Get λ_i by using the constraint $\|\mathbf{r}_i\|^2=1$.

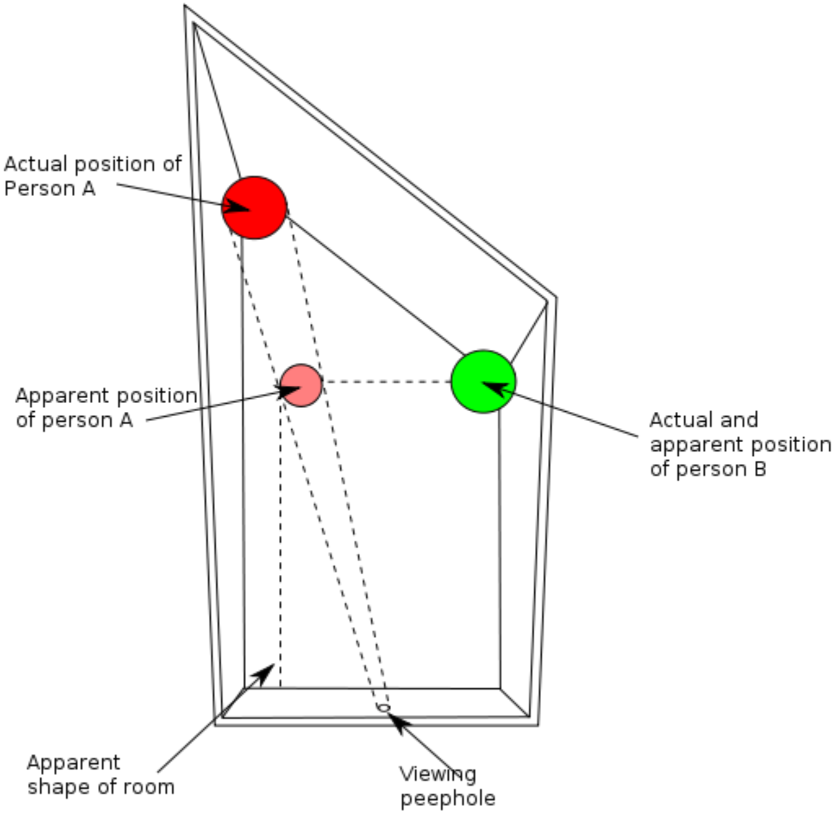
Calibration from vanishing points: Summary

- Solve for K (focal length, principal point) using three orthogonal vanishing points
- Get rotation directly from vanishing points once calibration matrix is known

- Advantages
  - No need for calibration chart, 2D-3D correspondences
  - Could be completely automatic
- Disadvantages
  - Only applies to certain kinds of scenes
  - Inaccuracies in computation of vanishing points
  - Problems due to infinite vanishing points

Source: S. Lazebnik

Making measurements from a single image

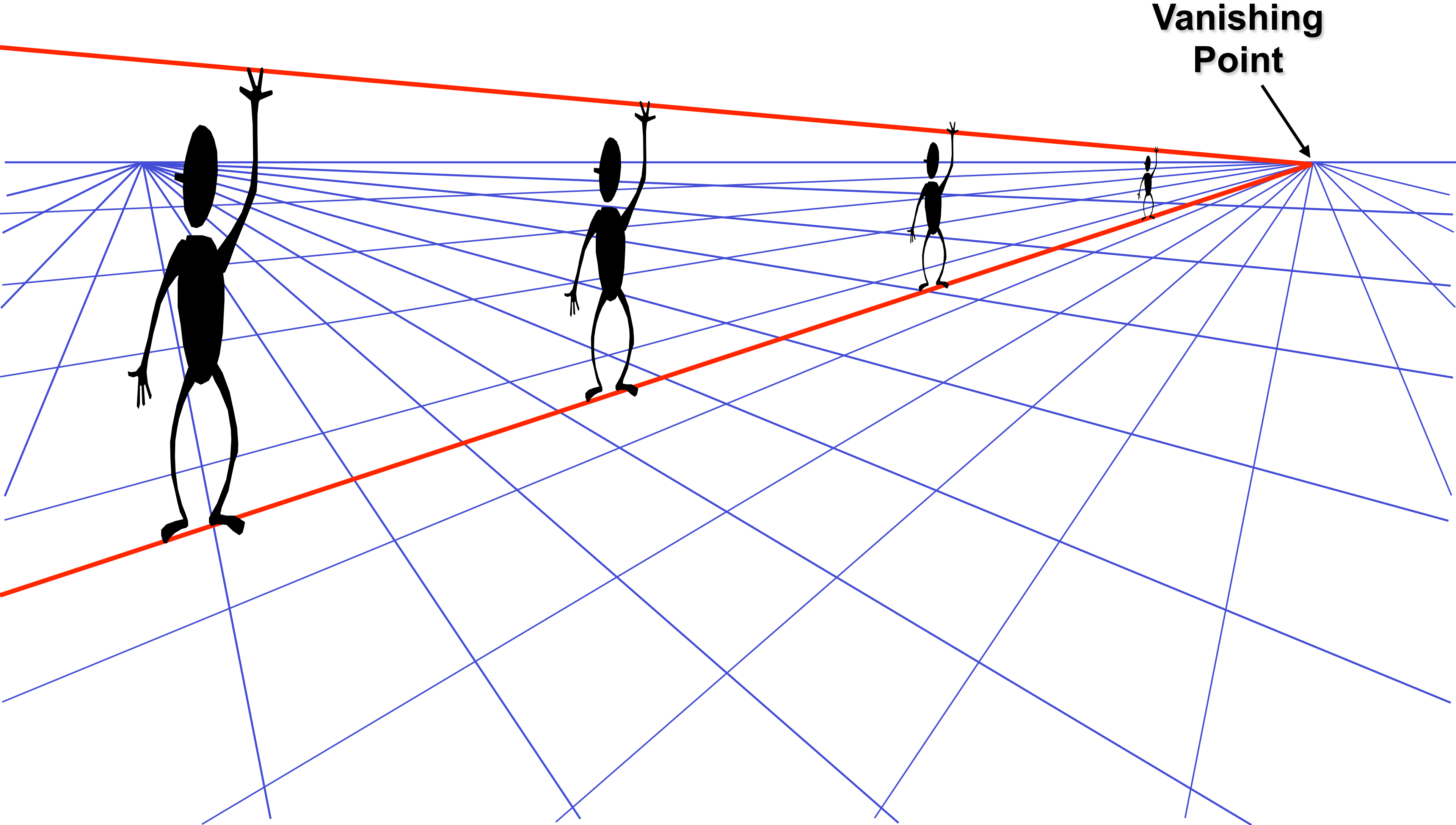


http://en.wikipedia.org/wiki/Ames_room

Source: S. Lazebnik

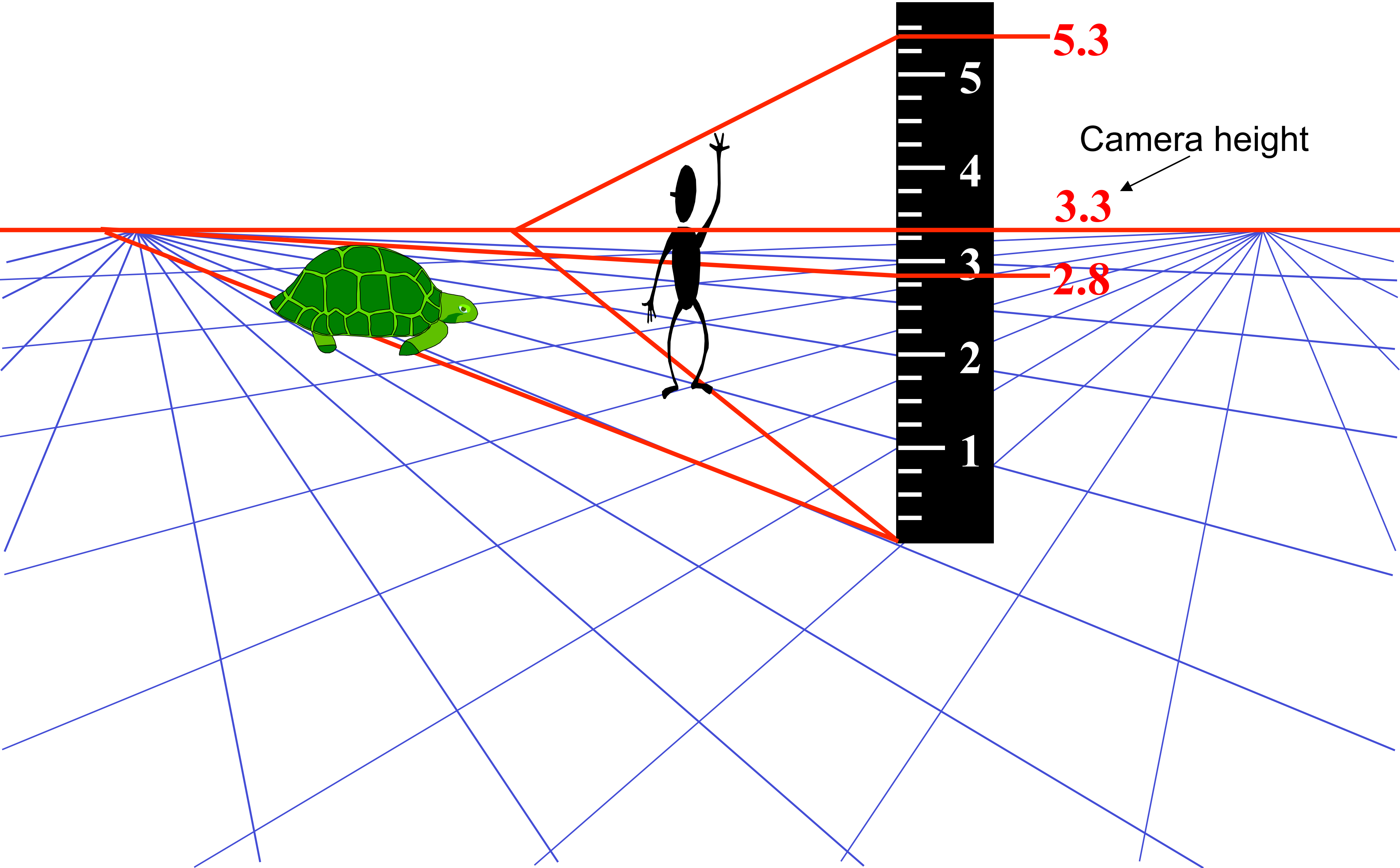
Slide by Steve Seitz

Comparing heights



Slide by Steve Seitz

Measuring height

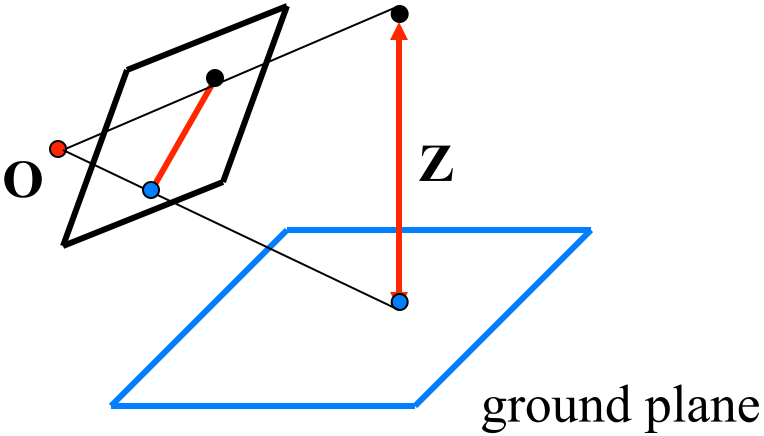


Camera height

Which is higher – the camera or the man in the parachute?

Source: S. Lazebnik

Measuring height without a ruler



Compute Z from image measurements

- Need more than vanishing points to do this

Source: S. Lazebnik

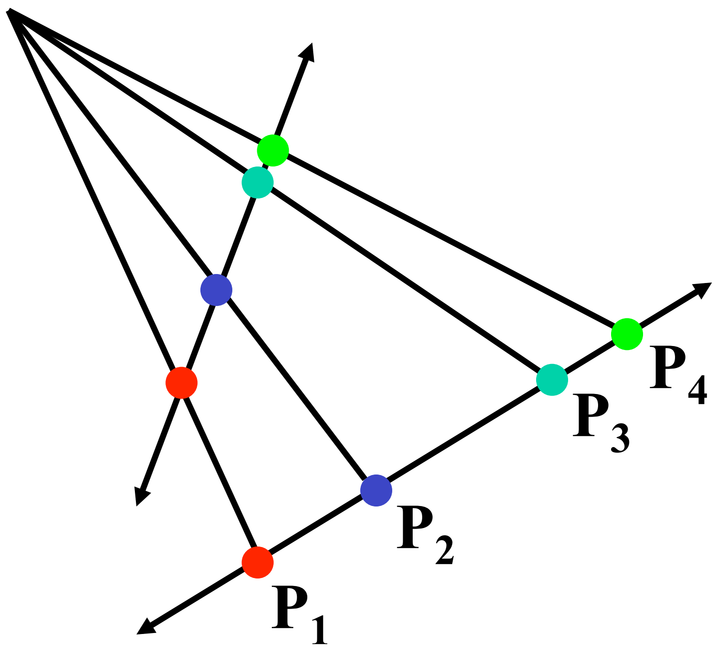
Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)

Source: S. Lazebnik

Projective invariant

- We need to use a *projective invariant*: a quantity that does not change under projective transformations (including perspective projection)
- The *cross-ratio* of four points:



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

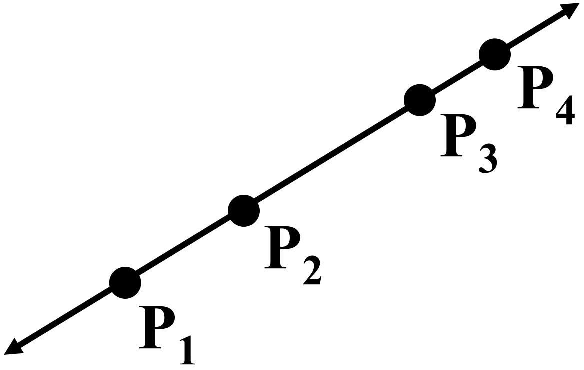
Slide by Steve Seitz

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Can permute the point ordering

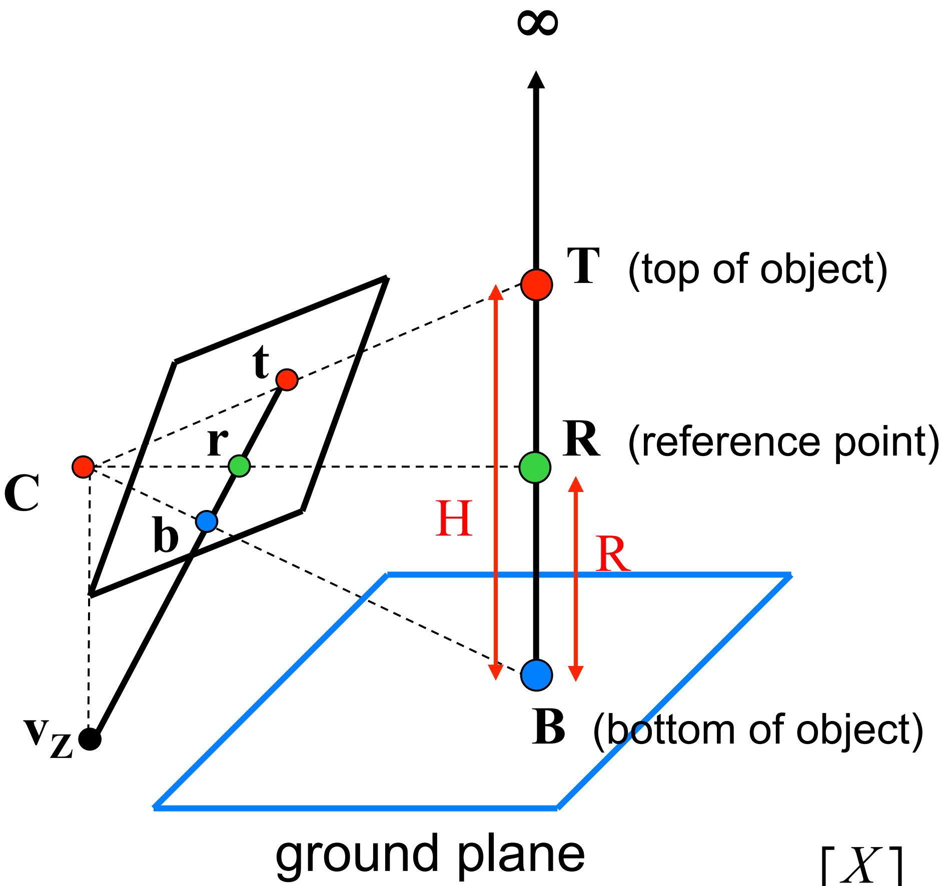
$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \, \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \, \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- 4! = 24 different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Slide by Steve Seitz

Measuring height



$$\frac{\|\mathbf{B}-\mathbf{T}\|\,\|\infty-\mathbf{R}\|}{\|\mathbf{B}-\mathbf{R}\|\,\|\infty-\mathbf{T}\|}=\frac{H}{R}$$

scene cross ratio

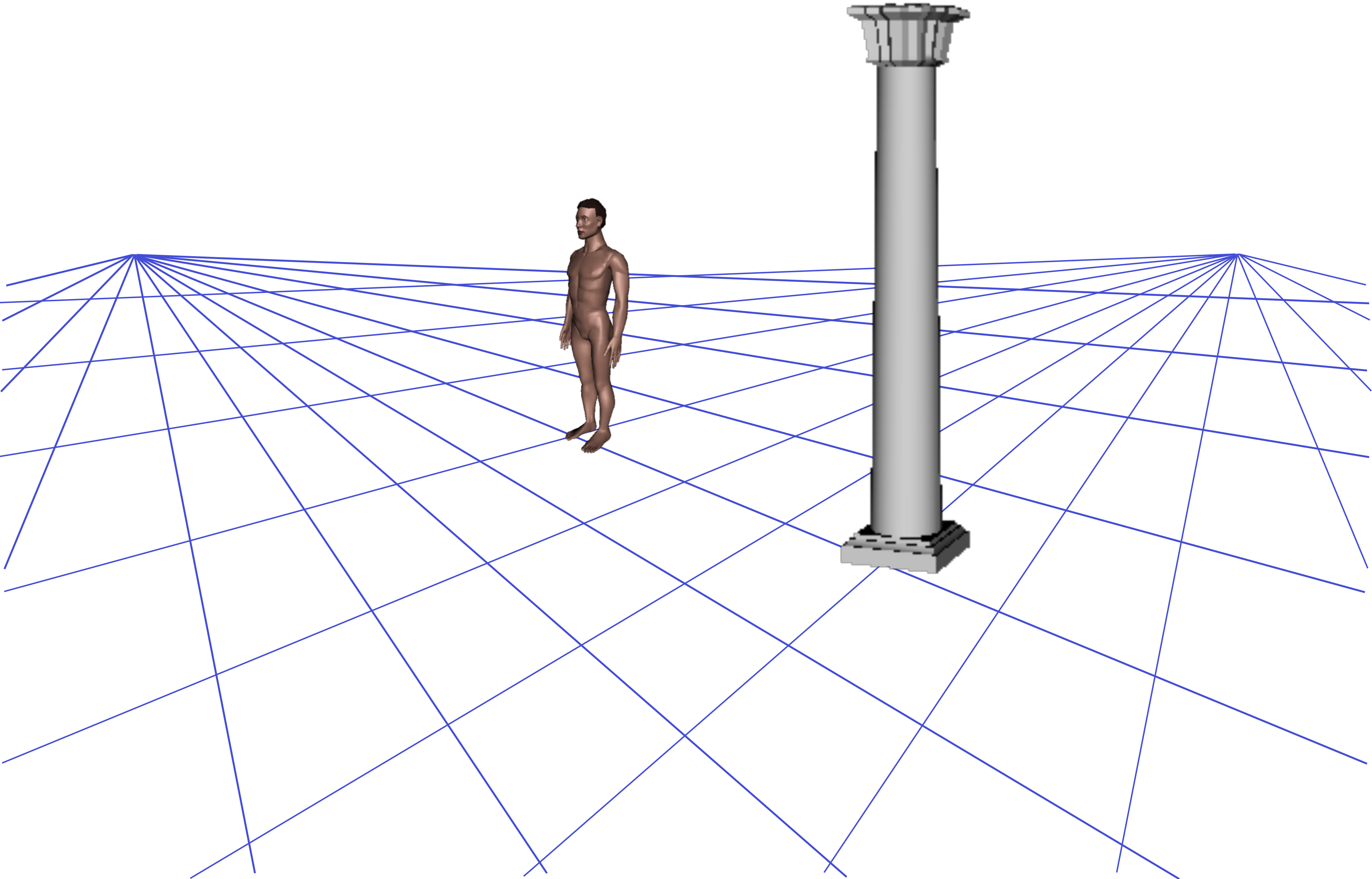
$$\frac{\|\mathbf{b}-\mathbf{t}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{b}-\mathbf{r}\|\,\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

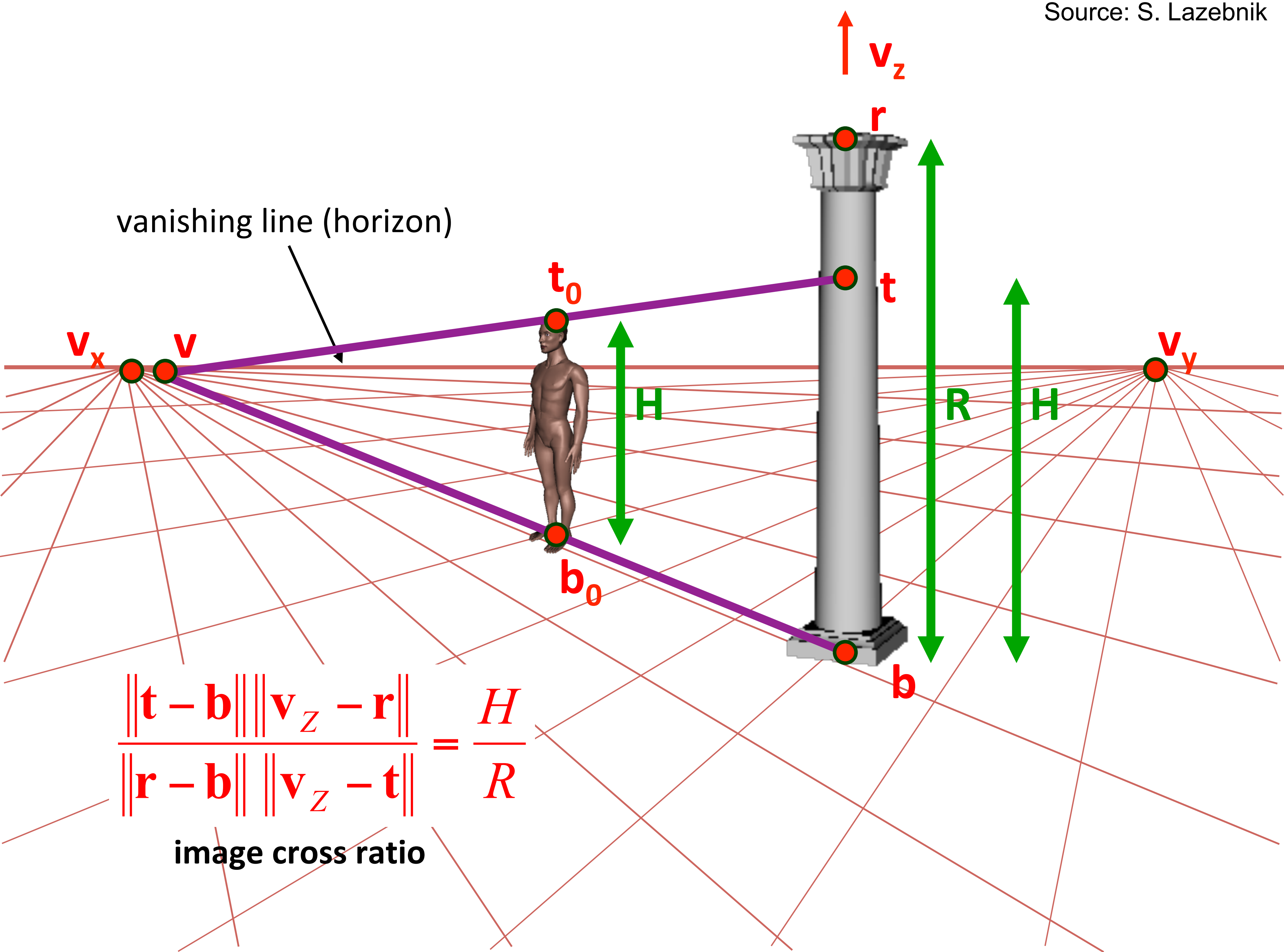
scene points represented as $\mathbf{P}=\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$ image points as $\mathbf{p}=\begin{bmatrix}x\\y\\1\end{bmatrix}$

Source: S. Lazebnik

Measuring height without a ruler



Source: S. Lazebnik



$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

2D lines in homogeneous coordinates

- Line equation: $ax + by + c = 0$

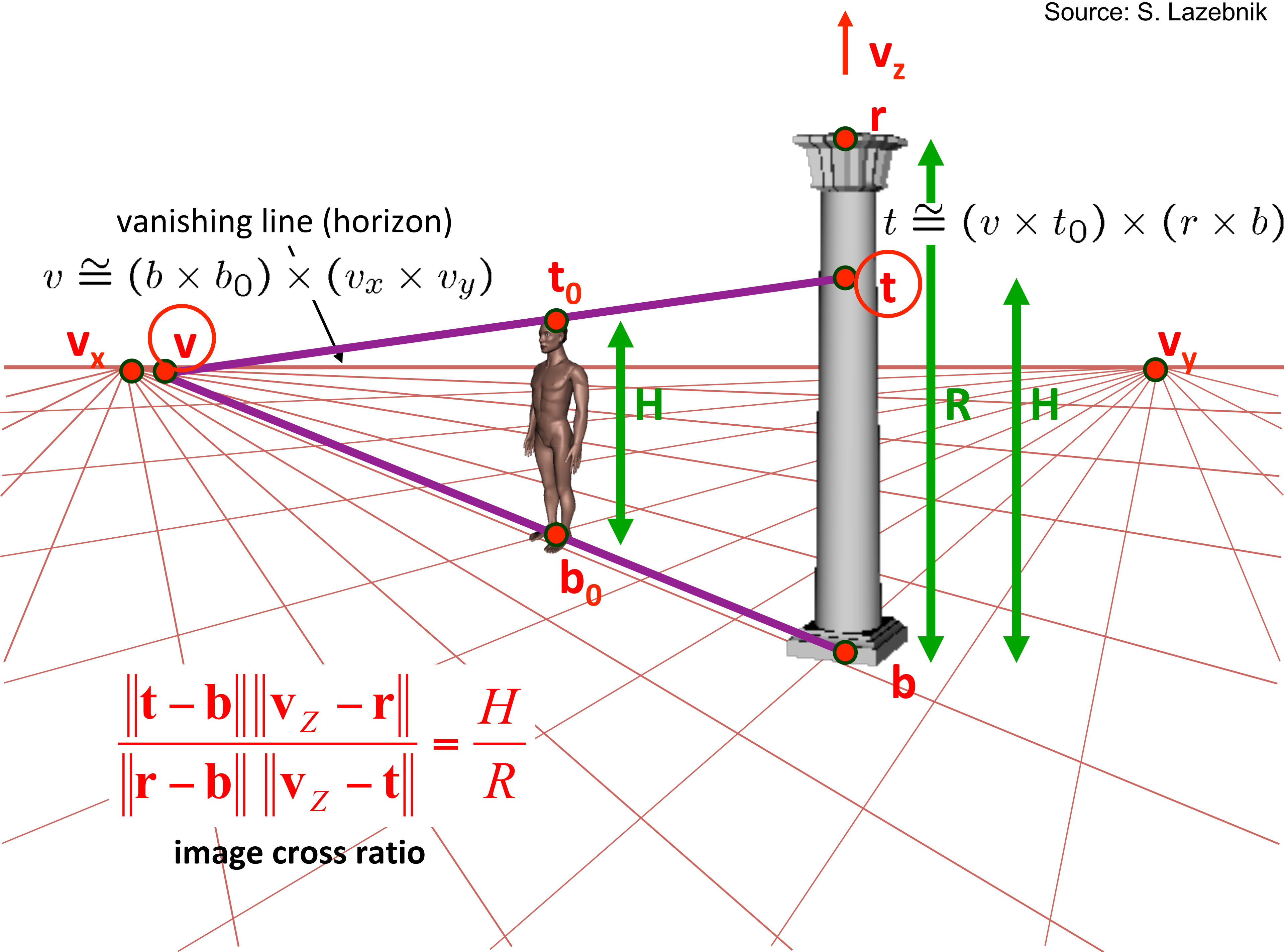
$$\mathbf{l}^T \mathbf{x} = 0 \quad \text{where} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Line passing through two points: $\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$

- Intersection of two lines: $\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$
  - What is the intersection of two parallel lines?

Source: S. Lazebnik

Source: S. Lazebnik



vanishing line (horizon)

$$v \cong (b \times b_0) \times (v_x \times v_y)$$

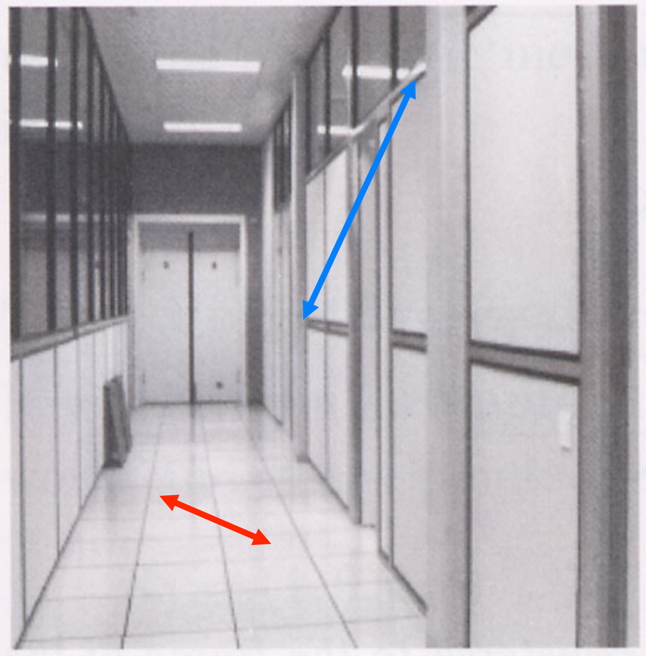
$$t \cong (v \times t_0) \times (r \times b)$$

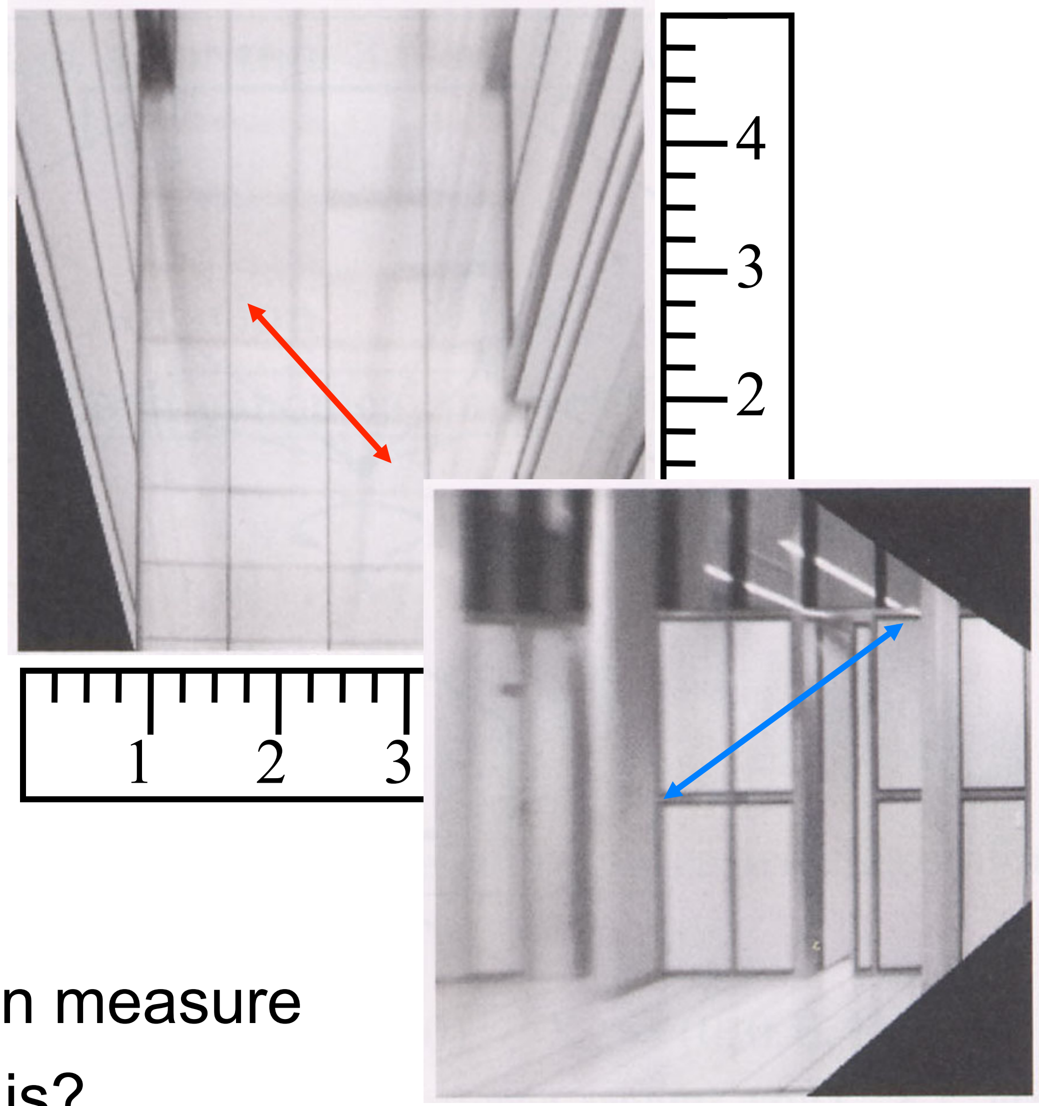
$$\frac{\|\mathbf{t}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{r}\|}{\|\mathbf{r}-\mathbf{b}\|\|\mathbf{v}_Z-\mathbf{t}\|}=\frac{H}{R}$$

image cross ratio

Source: S. Lazebnik

Measurements on planes



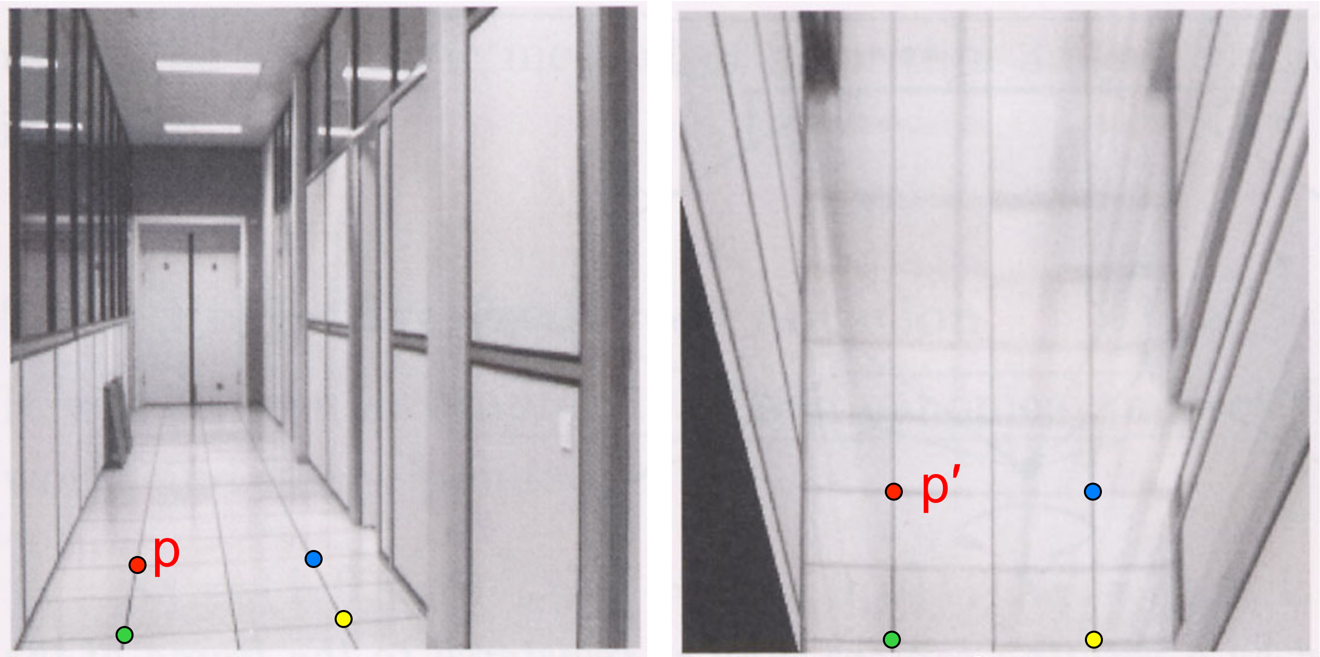


Approach: unwarp then measure

What kind of warp is this?

Source: S. Lazebnik

Image rectification



To unwarp (rectify) an image

- solve for homography H given p and p′

– how many points are necessary to solve for H?

Source: S. Lazebnik

Image rectification: example

Piero della Francesca*, Flagellation,* ca. 1455

Application: 3D modeling from a single image

J. Vermeer, Music Lesson, 1662

A. Criminisi, M. Kemp, and A. Zisserman,
Bringing Pictorial Space to Life: computer techniques for the analysis of paintings, *Proc. Computers and the History of Art*, 2002

http://research.microsoft.com/en-us/um/people/antcrim/ACriminisi_3D_Museum.wmv

Application: 3D modeling from a single image

D. Hoiem, A.A. Efros, and M. Hebert, "Automatic Photo Pop-up", SIGGRAPH 2005.

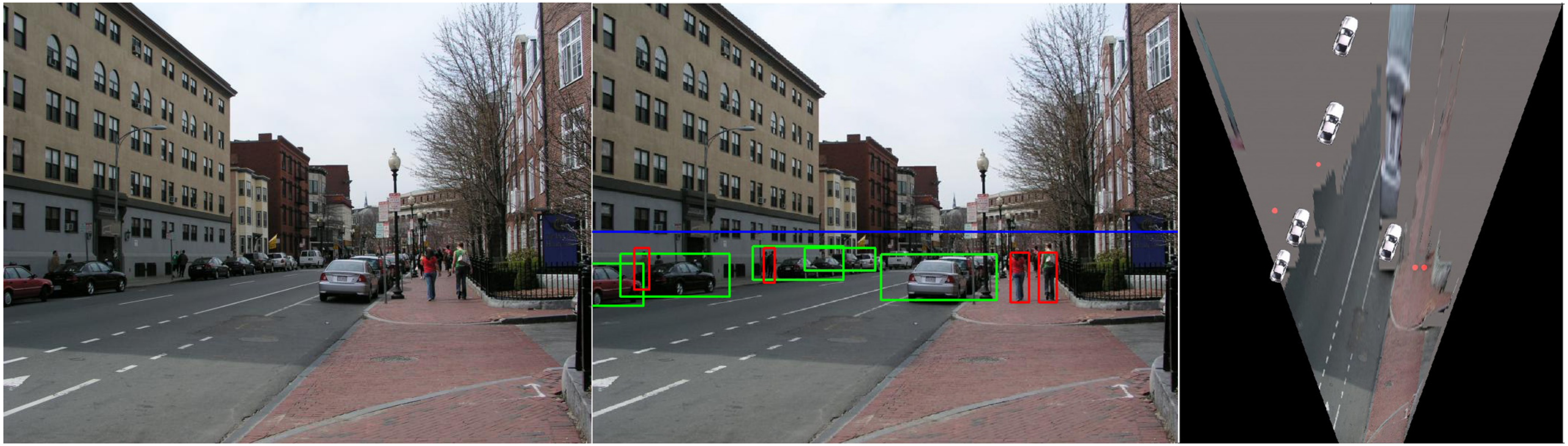
http://dhoiem.cs.illinois.edu/projects/popup/popup_movie_450_250.mp4

Application: Image editing

Inserting synthetic objects into images:
http://vimeo.com/28962540

K. Karsch and V. Hedau and D. Forsyth and D. Hoiem, “Rendering Synthetic Objects into Legacy Photographs,” *SIGGRAPH Asia* 2011

Application: Object recognition



D. Hoiem, A.A. Efros, and M. Hebert, "Putting Objects in Perspective", CVPR 2006