1. (**Just for fun**) (3p)

   Show that when the only loss mechanism is recombination (slide 23), the density decays reciprocally

   $$\frac{1}{n(r,t)} = \frac{1}{n_0(r)} + \alpha t$$

   (Hint: Just show that this solution satisfies the corresponding continuity equation.)

2. (**Competing processes**) (3p)

   Consider a 1-dimensional (i.e., slab geometry) weakly ionized plasma with a density distribution $n(x) = n_0 \cos(\pi x/2L)$, where the edges of the plasma are at $x = \pm L$. The plasma density decays by both diffusion and recombination. If $L = 3\,\mathrm{cm}$, $D = 0.4\,\mathrm{m^2\,s^{-1}}$, and $\alpha = 10^{-15}\,\mathrm{m^3\,s^{-1}}$, at what density will the local loss rate due to diffusion be equal to the local loss rate due to recombination? Which one dominates at higher densities?

3. (**Electrons in weakly ionized He gas**) (6p)

   The cross section σ for electrons colliding off neutral helium atoms can be approximated as $6\pi a_0^2$, where a_0 is the radius of the first Bohr orbit in a hydrogen atom. The gas is contained in a column, the pressure is 0.001 atm, the neutral temperature is room temperature and the electron temperature is 2 eV. There is no magnetic field.

   (a) Calculate the electron diffusion coefficient in $\mathrm{m^2\,s^{-1}}$ assuming that for $\langle \sigma v \rangle$ you can simply use $\sigma v_{\mathrm{thermal,e}}$.

   (b) If the ionization ratio is 0.1 % and a current density of $2\,\mathrm{kA\,m^{-2}}$ is driven through the gas, what is the electric field along the column?

4. (**More algebraic gymnastics**) (6p)

   On lecture slide 27, we obtained an expression for the cross-field velocity $\boldsymbol{v}_\perp$, when $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$, but left out the intermediate steps. Carry out all steps for an electron fluid and show that the expression given on the slide is correct.

5. (**Food for thought: About donuts**)

   This week we have seen that the confinement of plasma can be lost via diffusion and/or recombination. Different cross-field drifts were omitted by saying that they can always be aligned with the vessel walls. Consider what happens if we try to take the cylindrical vessel and simply eliminate its end losses by turning it into a torus. Consider the gradient and $\boldsymbol{E} \times \boldsymbol{B}$ drifts and think about what their combined effect would do to the confinement of electrons and ions. Initially, assume only the magnetic field that results from bending the cylinder into a donut, a magnetic field that is now toroidal. Return your short write-up in MyCourses before the next lecture.