Aalto University
School of Electrical
Engineering

ELEC-E8125 Reinforcement Learning
Model-based RL

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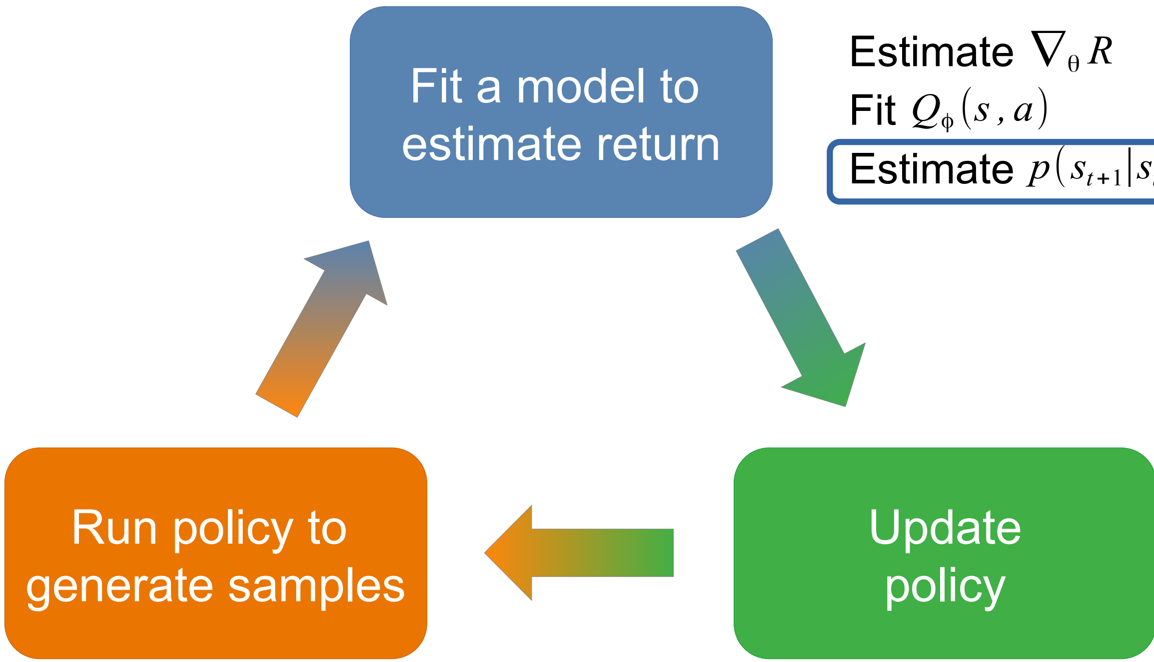
Learning goals

- Understand basic approaches for model-based reinforcement learning.

Anatomy of reinforcement learning
Model-based



Estimate $\nabla_\theta R$	(policy gradient)
Fit $Q_\phi(s,a)$	(Q-learning, actor-critic)
Estimate $p(s_{t+1} \mid s_t, a_t)$	(model-based)

$\theta \leftarrow \theta + \alpha \nabla_\theta R(\theta)$	(policy gradient)
$arg\,max_u Q_\phi(s,a)$	(Q-learning)
Optimize $\pi_\theta(a \mid s)$	(model-based)

Adopted from Sergey Levin.

Motivation (partial recap)

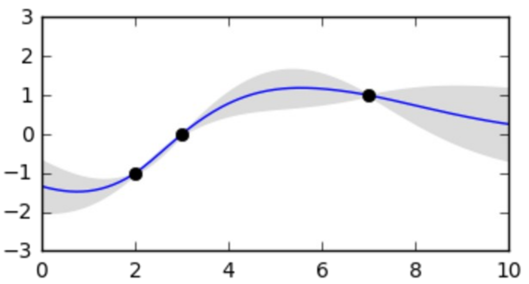
- Reinforcement learning has limited sample efficiency.
- Learned policies are task(reward-function)-specific, learned policies cannot be directly reused.
- Learned dynamics model is reusable and can be used to reason about potential futures.
- Sometimes we know the model, e.g. in games!

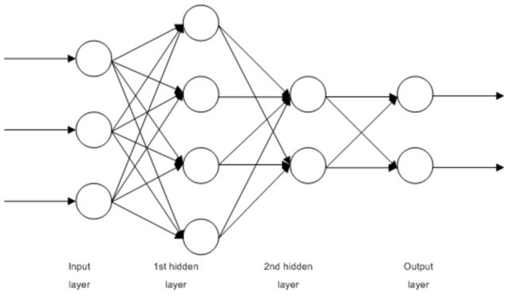
How to specify the model? Are there other situations where we know the model?

Model definition and types

- Dynamics model $\boldsymbol{s}_{t+1} = f(\boldsymbol{s}_t, \boldsymbol{a}_t)$ or $f(\boldsymbol{s}_{t+1} | \boldsymbol{s}_t, \boldsymbol{a}_t)$
- Reward model $r_{t+1} = r(\boldsymbol{s}_t, \boldsymbol{a}_t)$ or $r(r_{t+1} | \boldsymbol{s}_t, \boldsymbol{a}_t)$
- Models are usually learned.
  - Parametric regression (e.g. neural net) common.
- May be also known (e.g. games, simulators)
  - Even physics based models need to be often calibrated.
- Also other possibilities (active research area)
  - Latent variable models, graph neural networks, non-parametric regression models such as Gaussian processes, ...

Which model to use?





$$Y_i = \beta_0 + \beta_1 \phi_1(X_{i1}) + \cdots + \beta_p \phi_p(X_{ip}) \cdot$$

Gaussian process (GP)

- Data-efficient
- Slow with big datasets
- May be too smooth for non-smooth dynamics

Neural networks

- Expressive
- Unpredictable with sparse data (overfit)

Linear models

- May be used locally
- Do not overfit

Domain specific parametric models (e.g. physics parameters) can also be used.
→ Traditional control engineering approach of model identification + control.

Spectrum of model-based RL

how to act in
current situation
(choose action)

Time of planning

learn to act in
any situation
(learn policy)

On-line
- Act on current state
- Act without learning
- Better in unfamiliar situations

Off-line
- Fast online computation
- Predictable within familiar situations

Spectrum of model-based RL

Time of planning

On-line

Discrete actions
- MCTS
- ...

Continuous actions
- iLQR
- ...

Off-line

Simulate environment
- DYNA
- ...

Assist learning
- Policy backprop
- ...

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We kind of saw this already last week.

Continuous on-line planning: iLQR + learned model

Input: base policy π_0

Run base policy to collect data $D \leftarrow \{(\boldsymbol{s}, \boldsymbol{a}, \boldsymbol{s}')_i\}$

Repeat

Fit dynamics model $f(\boldsymbol{s}, \boldsymbol{a})$ to minimize $\sum_i \|f(\boldsymbol{s}_i, \boldsymbol{a}_i) - \boldsymbol{s}_i'\|^2$

Use model to plan (e.g. iLQR) actions

Execute first planned action, observe resulting state $\boldsymbol{s}'$

Update dataset $D \leftarrow D \cup \{(\boldsymbol{s}, \boldsymbol{a}, \boldsymbol{s}')\}$

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- Sample efficient.
- Computationally expensive for two reasons.
  - Dynamics fitting costly → model may be fitted only periodically (every n steps).
  - Planning costly for long horizons.
- Robust to moderate model errors.
- Choice of regression model is an important design parameter.

This is model-predictive control (MPC) with learned dynamics.
MPC horizon length is limited, can we do something?

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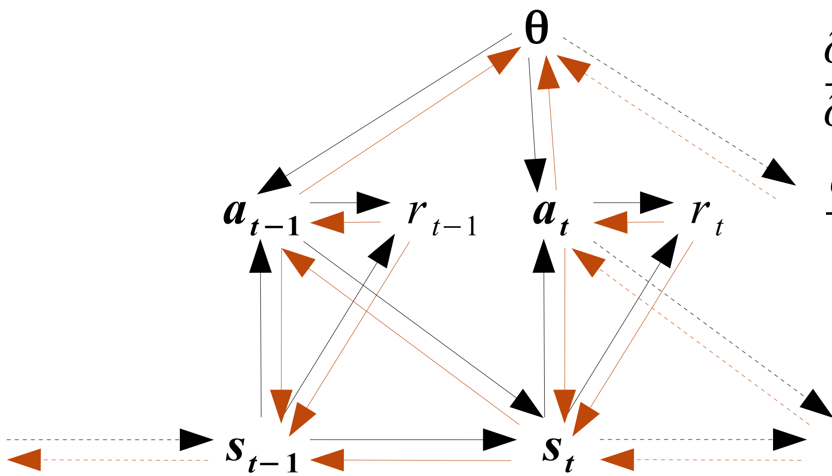
Backprop ~ chain rule of partial derivatives

Combining parametric policy with learned dynamics by backpropagation



$$\frac{\partial r_t}{\partial \theta} = \frac{\partial r_t}{\partial a_t}\frac{\partial a_t}{\partial \theta} + \frac{\partial r_t}{\partial s_t}\frac{\partial s_t}{\partial \theta}$$

$$\frac{\partial s_t}{\partial \theta} = \frac{\partial s_t}{\partial s_{t-1}}\frac{\partial s_{t-1}}{\partial \theta} + \frac{\partial s_t}{\partial a_{t-1}}\frac{\partial a_{t-1}}{\partial \theta}$$

policy	reward	dynamics
$\nabla_\theta \pi(s_t, a_t)$	$\nabla_a r(s_t, a_t)$	$\nabla_s f(s_{t-1}, a_{t-1})$
	$\nabla_s r(s_t, a_t)$	$\nabla_a f(s_{t-1}, a_{t-1})$

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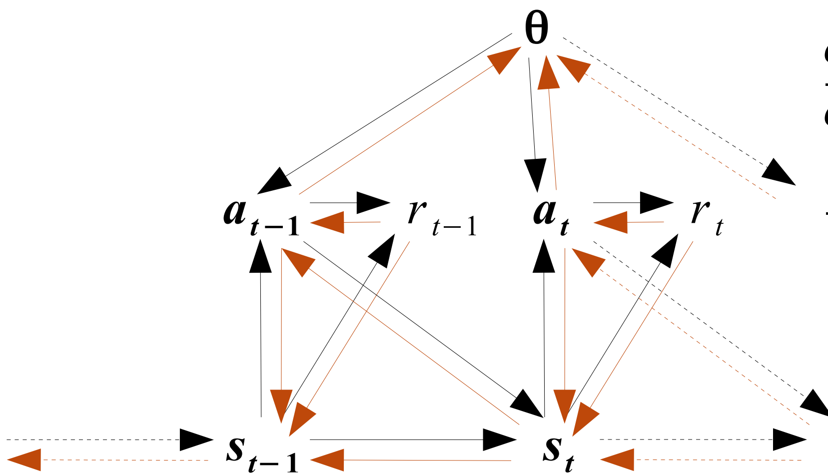

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Run base policy to collect data $D \leftarrow \{(s, a, s')_i\}$

Repeat

- Fit dynamics model $f_\phi(s, a)$ to minimize $\sum_i \|f_\phi(s_i, a_i) - s_i'\|^2$
- Calculate policy gradient update by backpropagating through dynamics
- Execute updated policy (1 or more steps), collect data
- Update dataset $D \leftarrow D \cup \{(s, a, s')\}$

Tools handle this automatically by automatic differentiation.

Continuous on-line planning: iLQR + learned model

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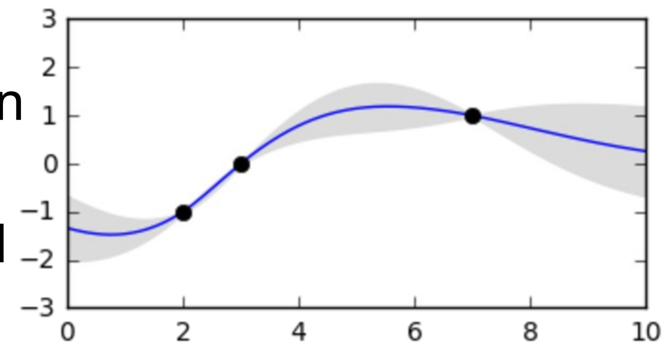
Example
PILCO (Deisenroth&Rasmussen, 2011)



- Dynamics learning: Use Gaussian process models to include model uncertainty. Known quadratic reward.
- Simulation: Simulate trajectory with learned model, including uncertainty.
- Policy: Radial basis function.
- Policy update: Calculate analytically policy gradient using learned dynamics and optimize with quasi-Newton optimizer (BFGS).
- GP → Very sample efficient. Cannot handle large dataset.

Reward function can also be learned using GP, e.g. BlackDROPS (2017).

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Simulate environment to generate additional data: DYNA

Tabular Dyna-Q

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Loop forever:
(a) $S \leftarrow$ current (nonterminal) state
(b) $A \leftarrow \varepsilon$-greedy(S, Q)
(c) Take action A; observe resultant reward, R, and state, S'
(d) $\boxed{Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]}$
(e) $\boxed{Model(S,A) \leftarrow R, S' \text{ (assuming deterministic environment)}}$
(f) Loop repeat n times:
  $S \leftarrow$ random previously observed state
  $A \leftarrow$ random action previously taken in S
  $\boxed{R, S' \leftarrow Model(S,A)}$
  $\boxed{Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]}$

Update using experience

Learn dynamics model

Generate data by simulating dynamics

Update using simulated experience

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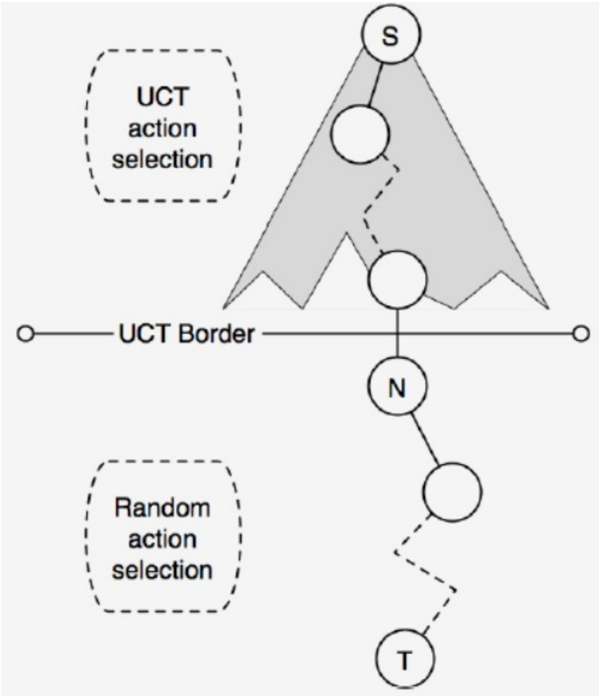
Monte Carlo tree search

- Search method for optimal decision making.
- State-of-the-art for playing games (e.g. Alpha Go).

- Iteratively builds a search tree.
- Phases:
  - Selection: Choose a promising node to expand.
  - Expansion: Add a new node.
  - Simulation: Simulate value for new node.
  - Backup: Back-up value to root (update values for parents).

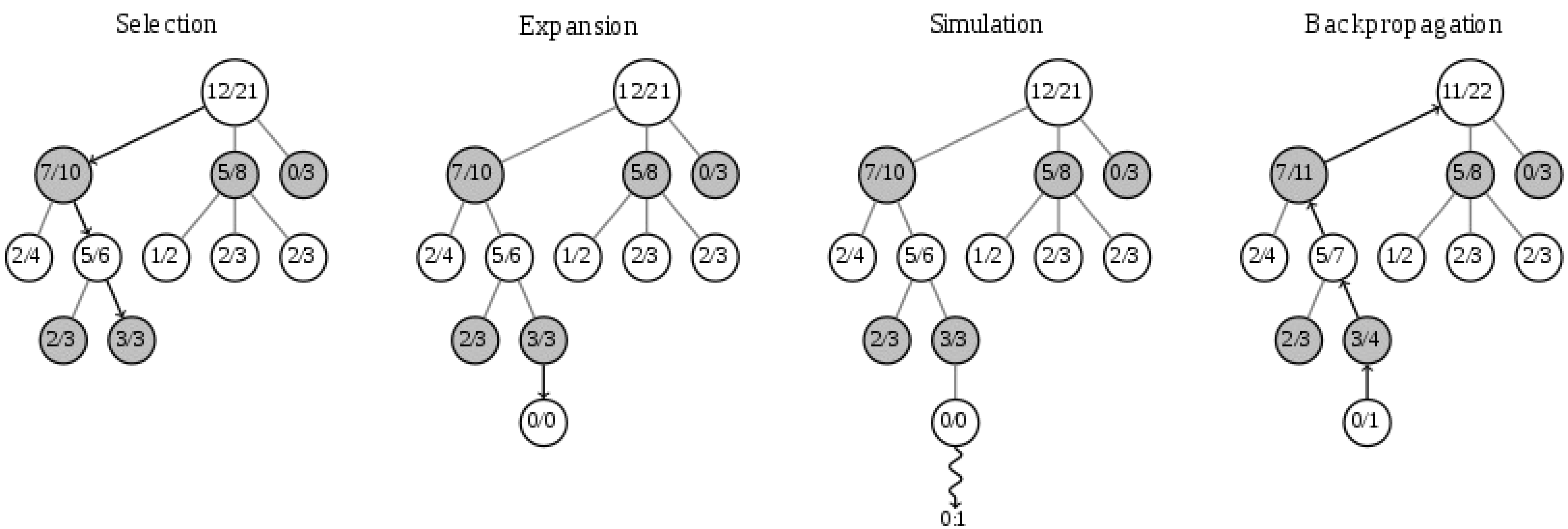
MCTS operation

- From start node *S* choose actions to walk down tree until reaching a leaf node.
- Choose an action and create a child node *N* for that action.
- Perform a **random** roll-out (take random actions) until end of episode (or for a fixed horizon).
- Record returns as value for *N* and back up value to root.



MCTS: Example in game playing

- Value: number of won/simulated games.



Node selection in MCTS

- Node selection in search has to balance between exploration and exploitation (note difference to RL, here exploration & exploitation only using simulation).
- Idea: Explore when uncertain of outcome.
- Upper confidence bound 1 (UCB1) on trees (UCT).
  - A bound for value of a node (Kocsis&Szepesvari, 2006).

$$Q^{+}(s,a)=Q(s,a)+c\sqrt{\frac{\log N(s)}{N(s,a)}}$$

Positive exploration constant

Visitation count

MCTS simulation phase

- Perform one or several roll-outs from leaf node using random action selection.
- Stop at terminal state or until a discount horizon is reached.
- Estimate value of state as mean return of the *N* simulations: $V(s) = \frac{1}{N} \sum_i G_i$

MCTS backpropagation

- After simulation phase backpropagate values to the root node
- Estimate value of state as mean return of the *N* simulations:

$$V(s) = \sum_a \frac{N_a}{N} Q(s,a)$$

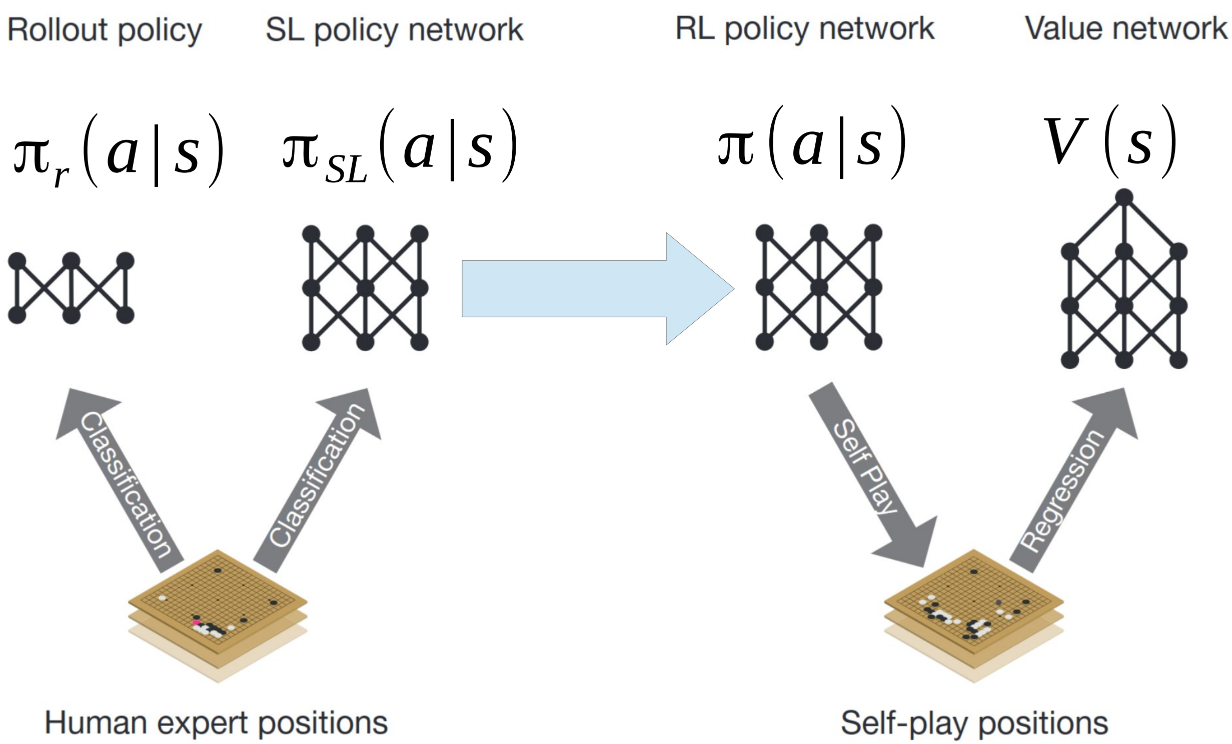
$$Q(s,a) = E_{s' \sim p(.|s,a)}[V(s')]$$

MCTS extensions

- AlphaGo (2016)
  - Learn initial policy from expert demonstrations
  - Update policy using self-play and MCTS
- AlphaZero (2017, 2018)
  - No expert demonstrations needed
- MuZero (2020)
  - Similar to AlphaZero but interleaves model learning and MCTS
  - Does not require a known model

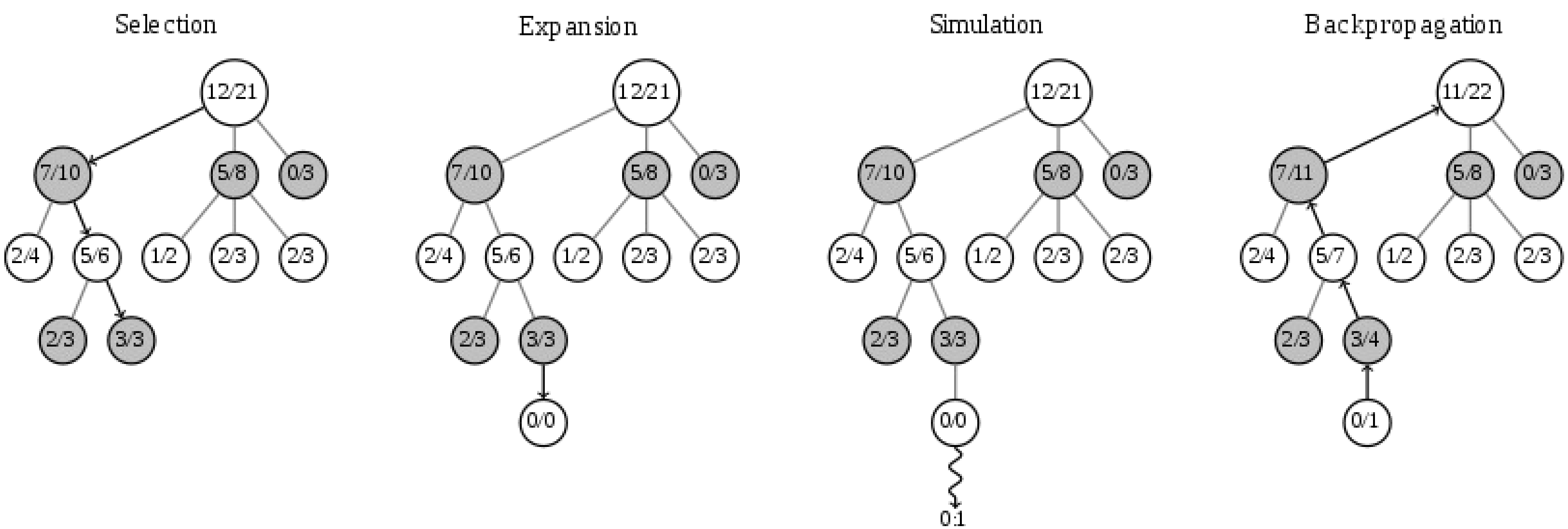
Example: Alpha Go (2016)

- Policy learned initially to imitate human players.
- Updated through policy gradient and self-play.



Example: Alpha Go (2016)

- Action chosen by bandit using Q(s,a) and policy
- Leaf-node value: estimated value V(s) plus roll-out value



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The ideas can also be combined!

Summary

- Model-based RL requires typically less data than value-based or policy gradient approaches.
- Learned dynamics can be transferred across tasks.
- Potentially suboptimal: models do not optimize for task performance and policy optimization may be prone to local minima.
- Sometimes models are harder to learn than policy.
- Often require explicit choices (e.g. time horizon).

Next: Guest Lectures

- Next week: Two! guest lectures!
  - Safety and constraints (Gökhan Alcal)
  - Entropy Regularization in Reinforcement Learning (Riad Akrour)