Combinatorial Network Analysis - Introduction

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- Example 1. The world wide web is a network: the elements are the web pages and the connections are the hyperlinks that point from one web page to another.
- Example 2. The network made by the behaviour of the reindeer in a herd in a given day. Each reindeer is an actor, and we place a weighted interaction between any two reindeer: the weight is n if they come closer than a given threshold (say, 1 meter) n times in a day.

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- Example of question addressed by network science: which elements of the network are the most important?
- Example of question addressed by network science: are there any naturally identified subnetworks that emerge from the main network?

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- The underlying application – the mathematical model should be as realistic as possible;
- Mathematical issues – it should be complex enough to capture as much as possible of the reality, but also simple enough that mathematicians can do something with it;
- Computational issues – real-life networks are often huge; they may evolve with time; practitioners may want very quick answers on them.

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- Edges = abstract connections. Information spreading among a group of people; airports linked by airline routes.

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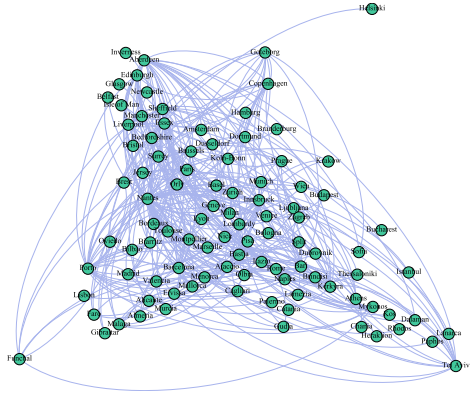
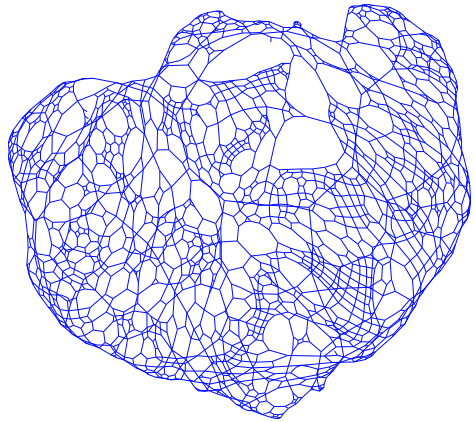
- Edges = other kinds of relations. Exchange of goods/services in an economic network; correlation between stocks in the market exchange.

Some real-world examples



(a)



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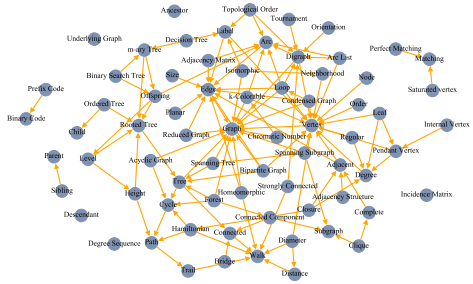

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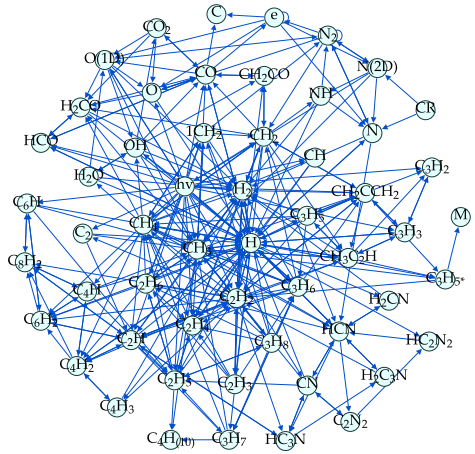

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