ELEC-E4130

Lecture 16: Multilayer (Stratified Media)

Beam splitter for a two arm interferometer

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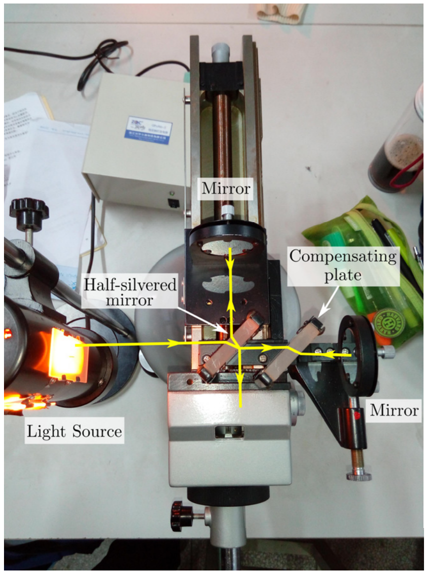
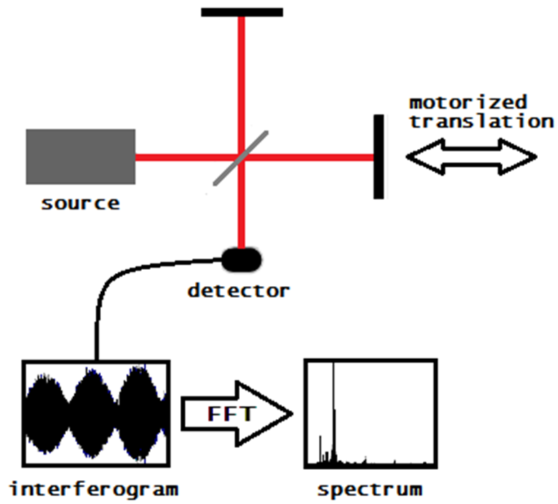
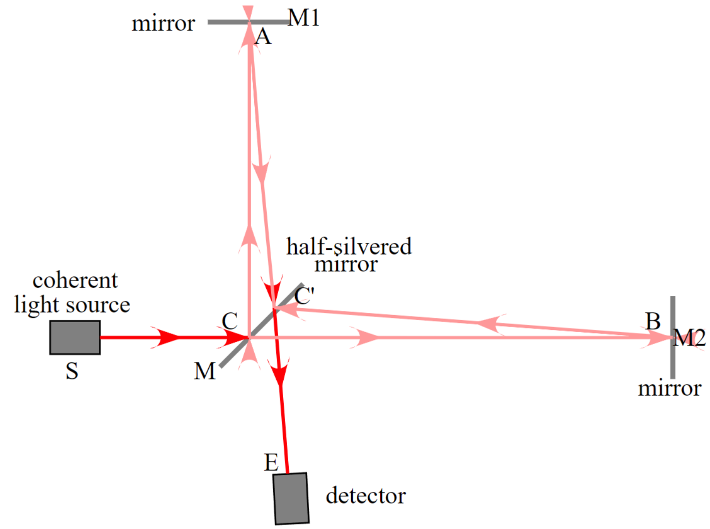
Interferometer (wikipedia)

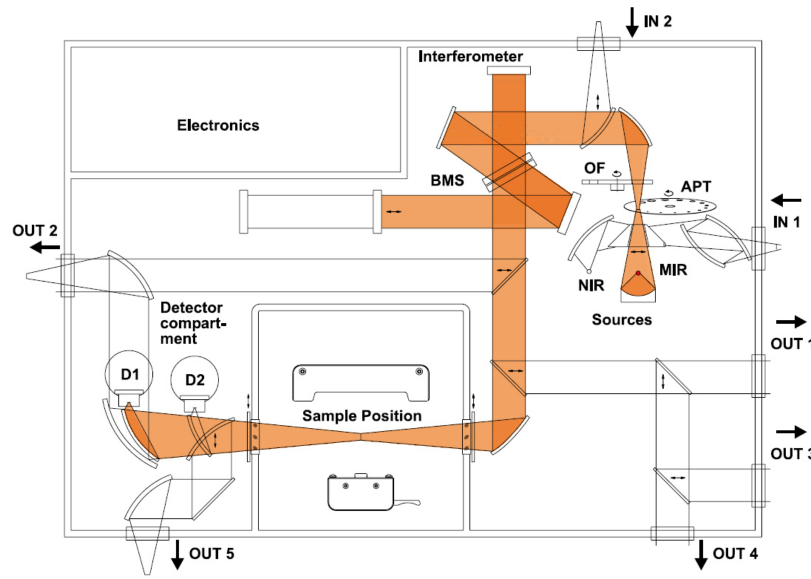






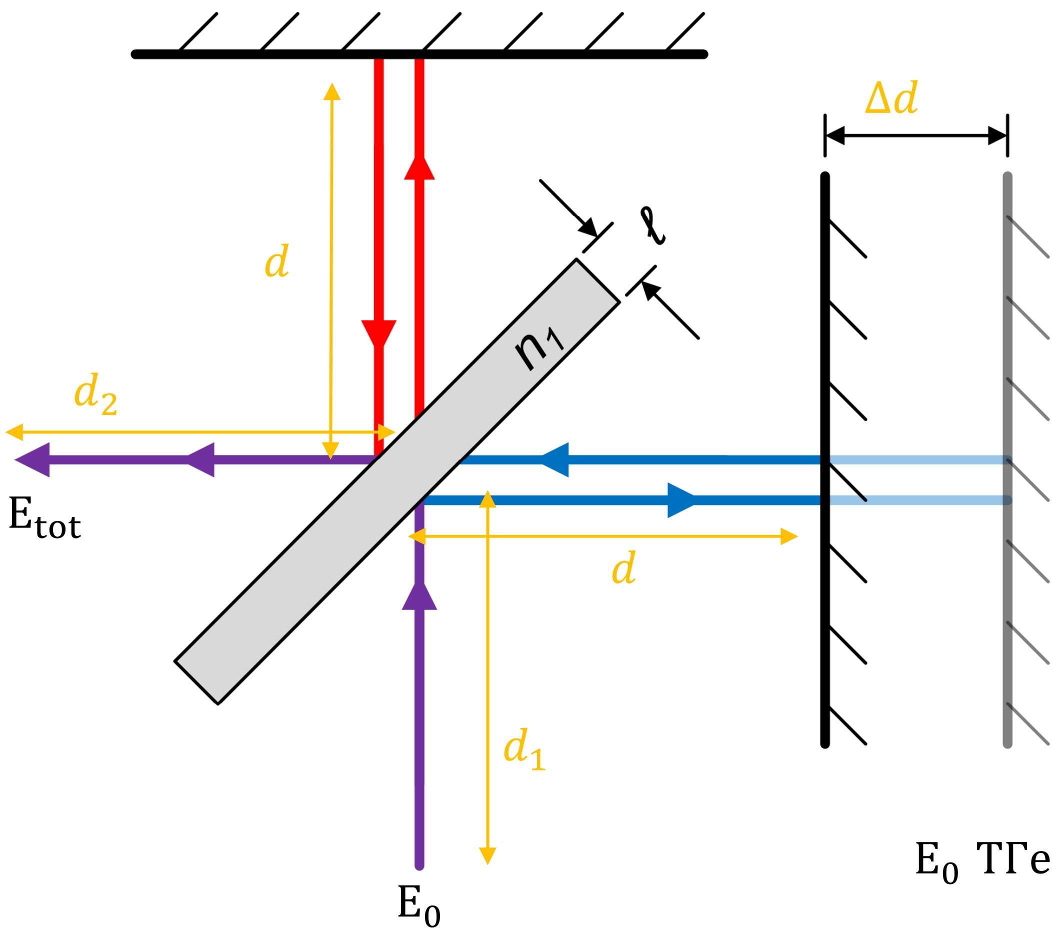
- Interference of light with advanced/delayed copies of itself achieved via splitting light into two arms, modulating the path of one arm while keeping the second arm stationary, recombining light at a detector
- Analysis of interference allows for
  - Determination of light spectrum of is path lengths are well known
  - Determination of sample or mirror spectral response (both source and paths are known)
- **Also see Michelson–Morley experiment**

Interferometer (Bruker)



- Interferometers at very long wave IR (sometimes known as submillimeter wave, 100 GHz – 1 THz) often use self supporting dielectric beam splitters to divide and recombine the beam
- Self supporting dielectric beam splitter means a thin, dielectric film suspended from a ring with no supporting structure
- Lets use multilayer calculations to design a beam splitter for a particular frequency band

Interferometer



$$E_0$$

$$E_0\ e^{-jkd_1}$$

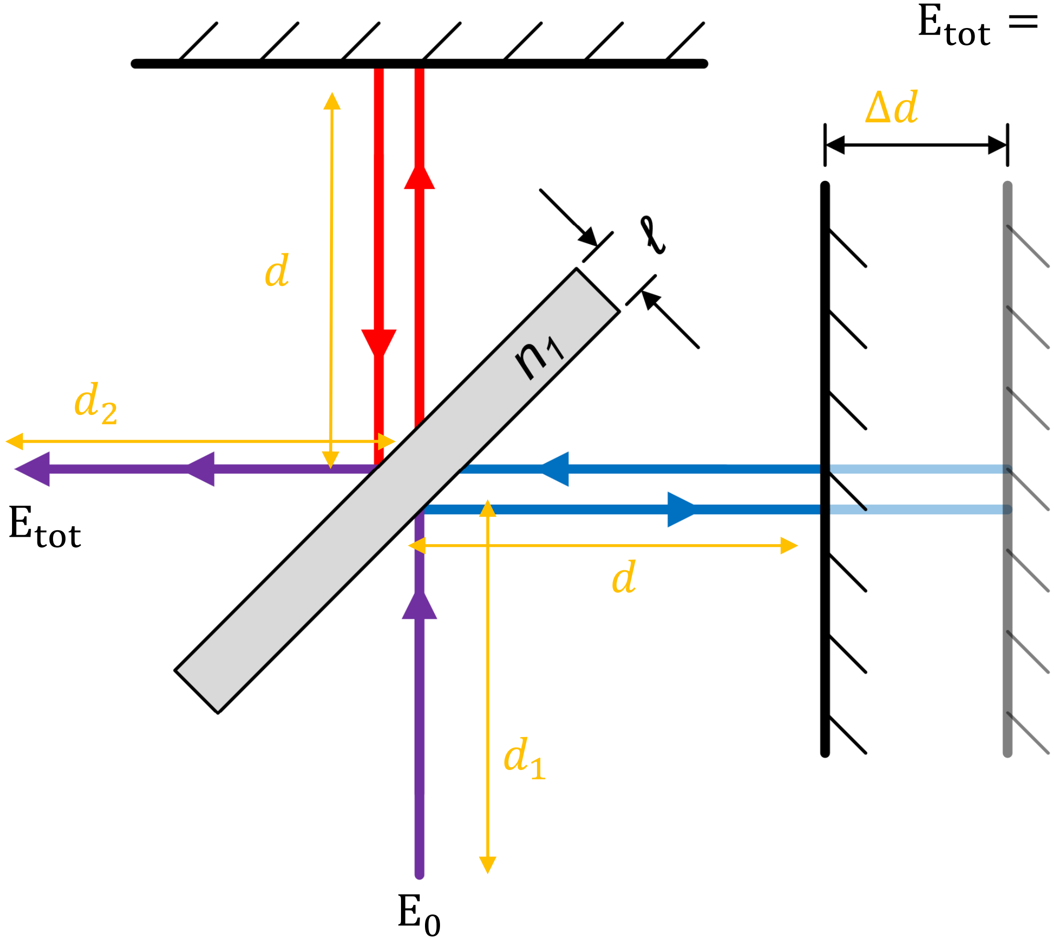
$$E_0\ \Gamma e^{-jkd_1} \qquad\qquad E_0\ T e^{-jkd_1}$$

$$E_0\ \Gamma e^{-jkd_1} e^{-j2kd} \qquad\qquad E_0\ T e^{-jkd_1} e^{-j2k(d+\Delta d)}$$

$$E_0\ T\Gamma e^{-jkd_1} e^{-j2kd} \qquad\qquad E_0\ \Gamma T e^{-jkd_1} e^{-j2k(d+\Delta d)}$$

$$E_0\ T\Gamma e^{-jkd_1} e^{-j2kd} e^{-jkd_2} \qquad\qquad E_0\ \Gamma T e^{-jkd_1} e^{-j2k(d+\Delta d)} e^{-jkd_2}$$

Interferometer



$$E_{tot} = E_0\ \Gamma T e^{-jkd_1} e^{-j2kd} e^{-jkd_2}\left(1 + e^{-j2k\Delta d}\right)$$

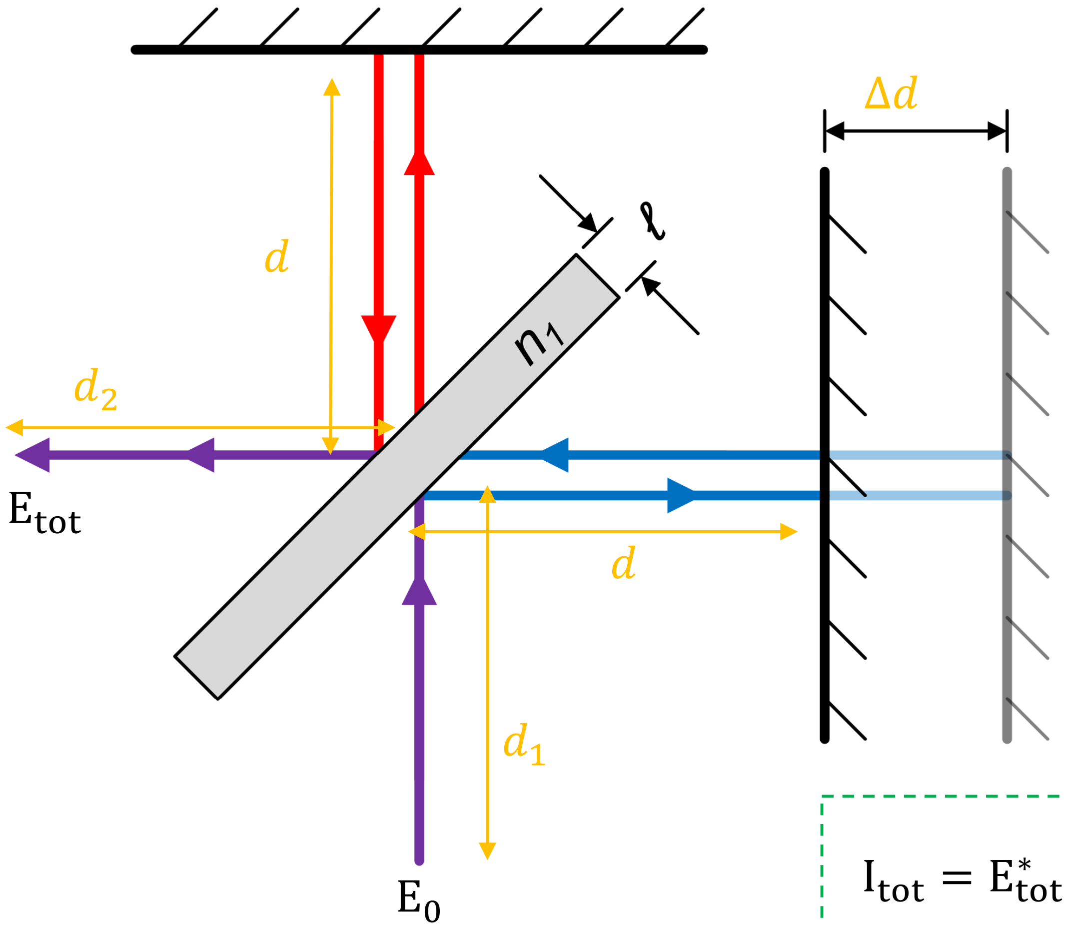
$$E_{tot} = E_0\ \Gamma T e^{-jkd_1} e^{-j2kd} e^{-jkd_2} e^{-jk\Delta d}\left(e^{jk\Delta d} + e^{-jk\Delta d}\right)$$

$$D = d_1 + d_{12} + 2d + \Delta d$$

$$E_{tot} = E_0\ \Gamma T e^{-jkD}\left(e^{jk\Delta d} + e^{-jk\Delta d}\right)$$

$$E_{tot} = E_0\ \Gamma T e^{-jkD}(2\cos(k\Delta d))$$

Interferometer



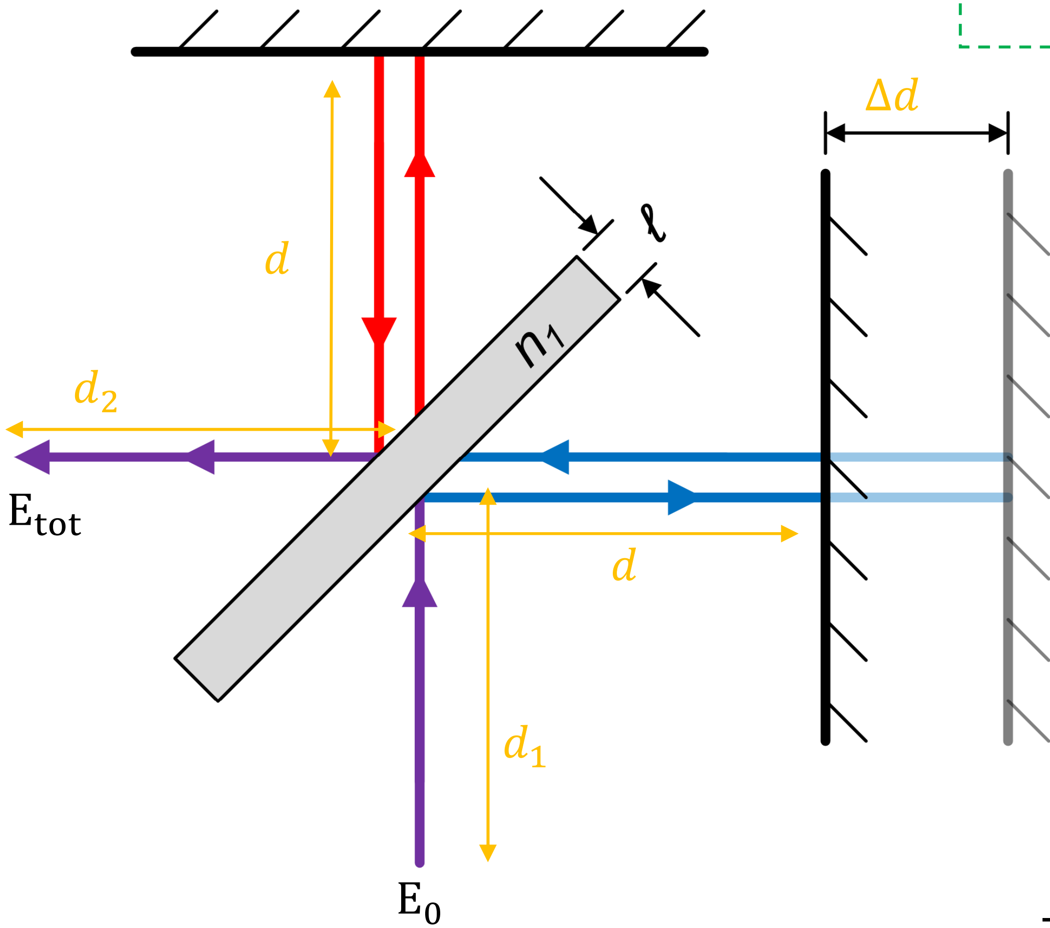
$$E_{tot} = E_0\ \Gamma T e^{-jkD}(2\cos(k\Delta d))$$

- The signal at the detector is a function of the signal intensity
- **The magnitude squared of the incoming field**

$$I_{tot} = E_{tot}^{*}E_{tot} = 2|E_0|^2|\Gamma|^2|T|^2(1+\cos(2k\Delta d))$$

Interferometer

$$I_{tot} = E^*_{tot}E_{tot} = 2|E_0|^2|\Gamma|^2|T|^2(1 + \cos(2k\Delta d))$$



- The total received signal will be modified by the power reflection and transmission of the dielectric thin film beam splitter
- **Assuming no loss in the beam splitter, How do we maximize signal?**

$$R_0 = |\Gamma|^2$$

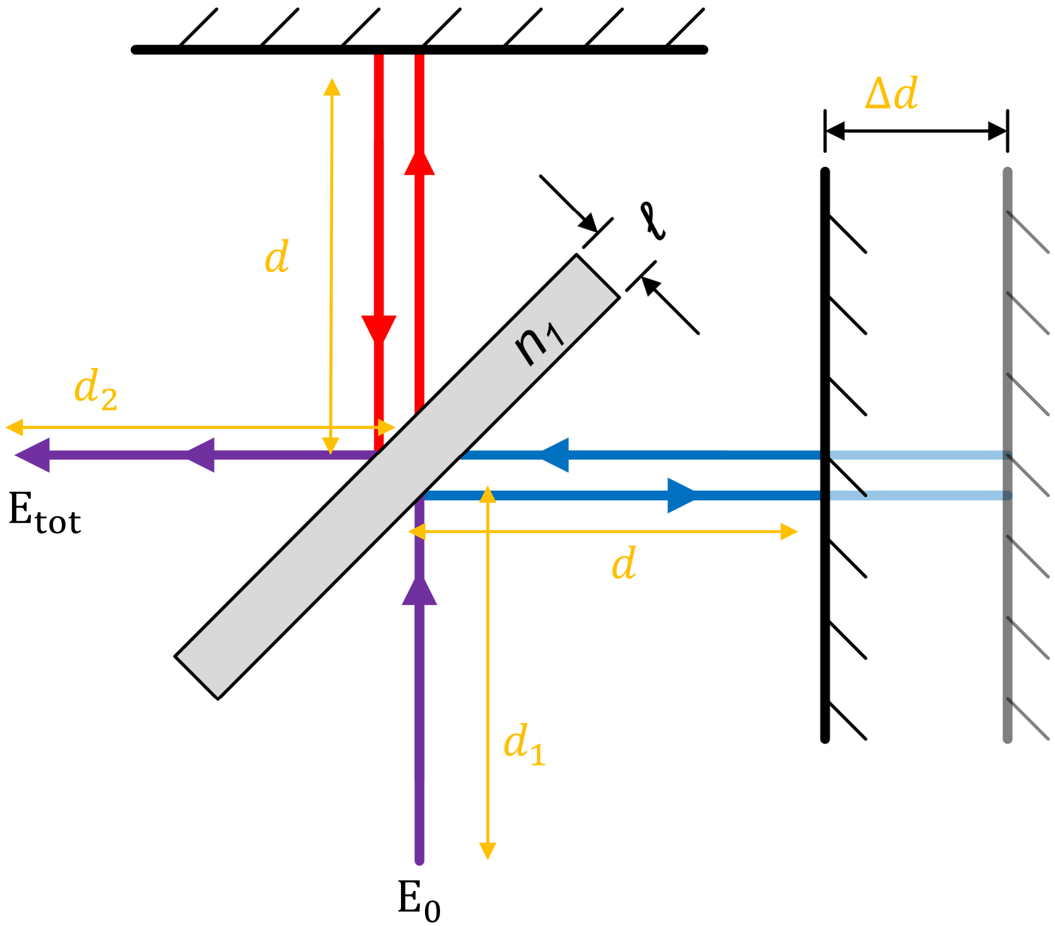
$$T_0 = |T|^2$$

$$R_0 + T_0 = 1$$

$$R_0 T_0 = R_0 (1 - R_0)$$

$$\frac{d}{dR_0} R_0 (1 - R_0) = 0 \longrightarrow R_0 = 0.5$$

Interferometer

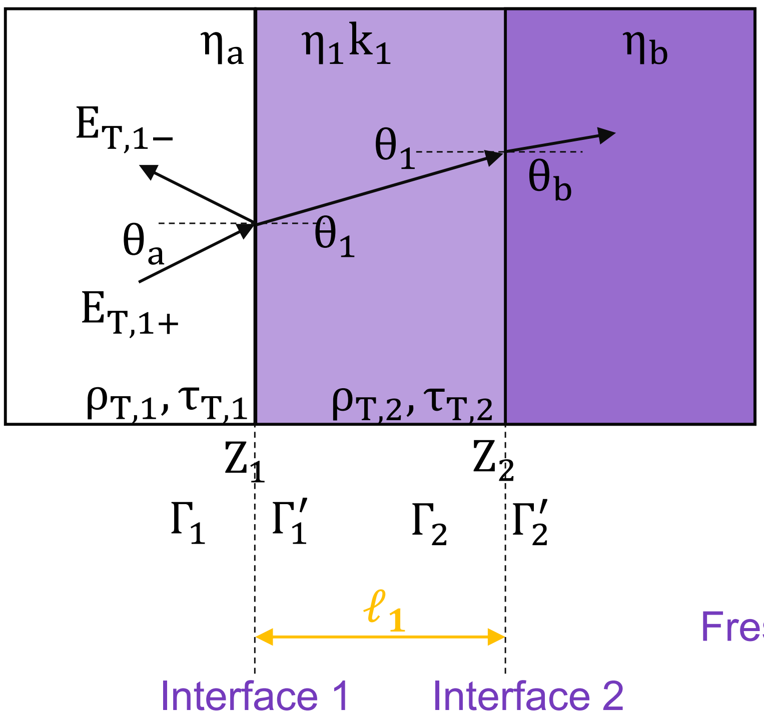


$R_0 = 0.5$ → $R_0 + T_0 = 1$

$T_0 = 0.5$ → $R_0 = T_0$

- Maximum signal occurs when power reflectivity and power transmissivity are equal

Oblique Incidence: Single slab



Total reflection and transmission

$$\Gamma_{T,1} = \frac{\rho_{T,1} + \rho_{T,2} e^{-j2\delta_m}}{1 + \rho_{T,1}\rho_{T,2} e^{-j2\delta_m}}$$

$$T_{T,2} = \frac{\tau_{T,1}\tau_{T,2} e^{-j\delta_m}}{1 + \rho_{T,1}\rho_{T,2} e^{-j2\delta_m}}$$

Optical path

$$\delta_m = k_m \ell_m \cos(\theta_m)$$

Refractive index

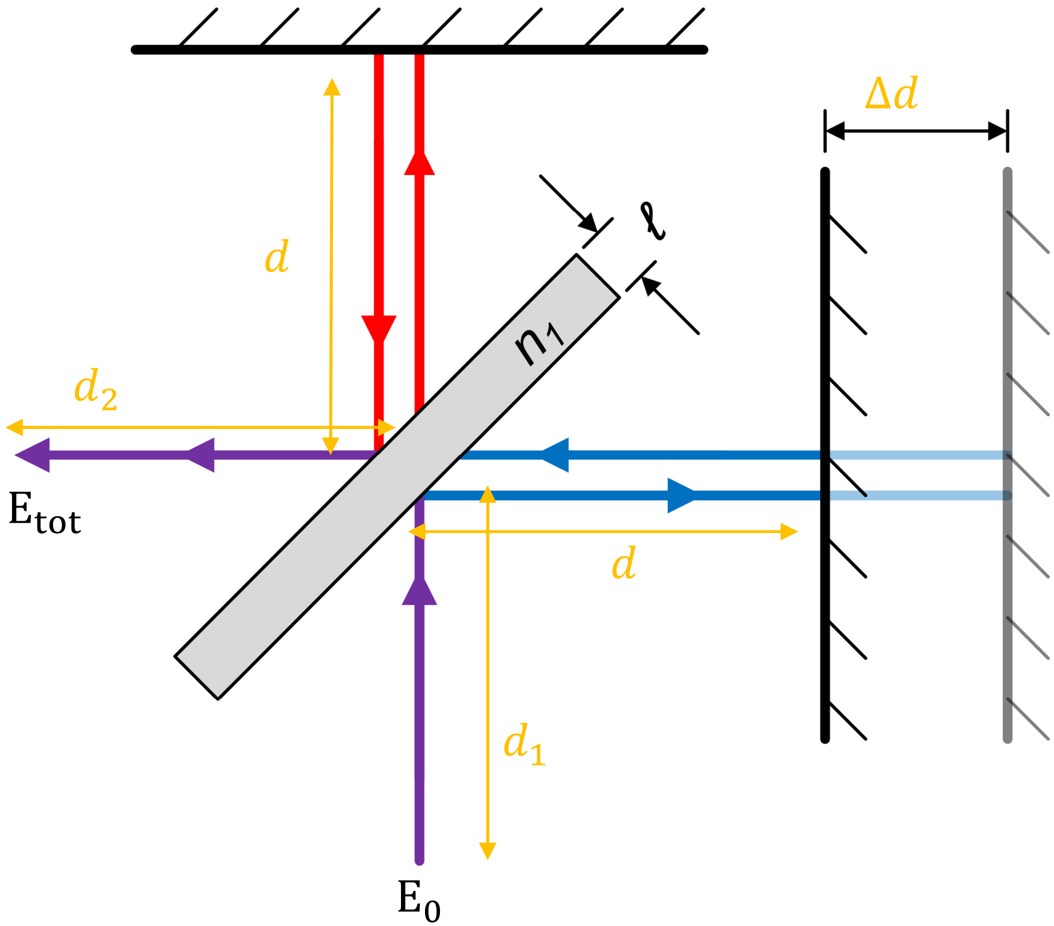
$$n_{T,i} = \begin{cases} \dfrac{n_i}{\cos(\theta_{T,i})} & \parallel, \mathrm{TM} \\ n_i \cos(\theta_{T,i}) & \perp, \mathrm{TE} \end{cases}$$

Fresnel's Coefficients

$$\rho_{T,1} = \frac{n_{T,a} - n_{T,1}}{n_{T,a} + n_{T,1}} \qquad \tau_{T,1} = 1 + \rho_{T,1}$$

$$\rho_{T,2} = \frac{n_{T,1} - n_{T,b}}{n_{T,1} + n_{T,b}} \qquad \tau_{T,2} = 1 + \rho_{T,2}$$

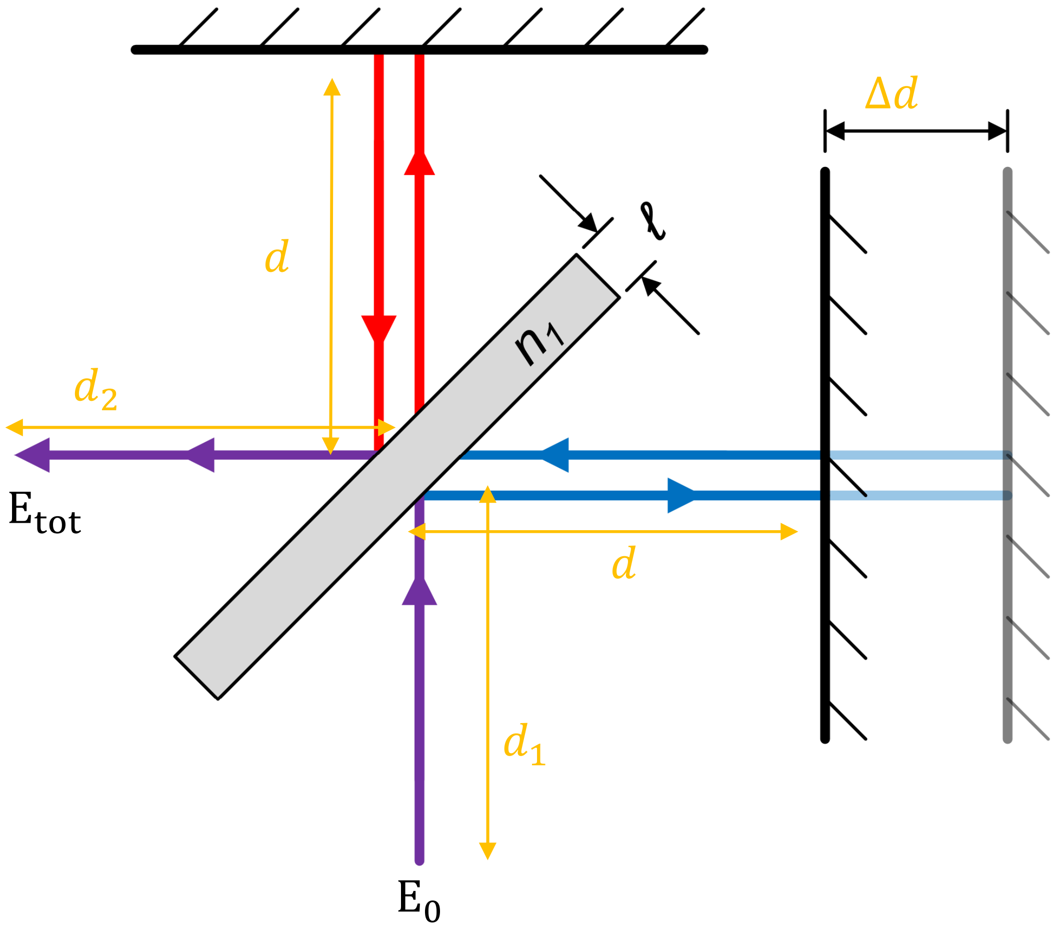
Interferometer



$$\left|\frac{\rho_{T,1}+\rho_{T,2}e^{-j2\delta_m}}{1+\rho_{T,1}\rho_{T,2}e^{-j2\delta_m}}\right|^2 = \left|\frac{\tau_{T,1}\tau_{T,2}e^{-j\delta_m}}{1+\rho_{T,1}\rho_{T,2}e^{-j2\delta_m}}\right|^2$$

- Find the correct **thickness** and **refractive** index for a given wavelength

Interferometer



Nulls

$$\delta_{\mathrm{m}} = \mathrm{k_m} \ell_{\mathrm{m}} \cos(\theta_m) = n\pi$$

Peaks

$$\delta_{\mathrm{m}} = \mathrm{k_m} \ell_{\mathrm{m}} \cos(\theta_m) = \frac{n}{2}\pi$$